Homotopy transfer for conserved currents and rigid symmetries in gauge theories

(work in progress)

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PDEs

- A PDE is $\mathcal{E} \hookrightarrow J^\infty F$, where $F \to M$ is a (ghost) graded super-bundle of fields over spacetime M, with $n = \dim M$.
- The Cartan distribution on $J^\infty F$ gives rise to d_V, d_H and evolutionary vector fields (evfs), $\mathfrak{X}_{\mathrm{ev}}(J^\infty F)$, which commute with d_H and d_V.
- The Cartan distribution should be involutive on $\mathcal{E}$ and local symmetries are evfs tangent to $\mathcal{E}$, $\mathfrak{X}_{\mathrm{ev}}(\mathcal{E})$.
- We mostly consider horizontal forms $\Omega^{\bullet,\bullet} = \Omega^{\text{ghost}\#,\text{horiz.deg}}(J^\infty F \text{ or } \mathcal{E})$.

- **Sign Conventions:** Any formula can be written for purely even (odd) objects; general signs recovered by introducing formal parity changing parameters $(\epsilon_1, \epsilon_2, \ldots)$.
  Adopt $|\mathscr{L}_{(-)}| = |[-,-]_{\mathscr{L}}| = $ even, $|\mathrm{d}_V| = |\mathrm{d}_H| = |\iota_{(-)}| = $ odd, $\#\mathscr{L}_{(-)} = \#[-,-]_{\mathscr{L}} = \#\mathrm{d}_V = \#\mathrm{d}_H = \#\iota_{(-)} = 0$.

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BV-BRST

- A (sufficiently regular) variational PDE $\mathcal{E}$ has a BV-BRST description.
  - In the BV-BRST extension, fiber coordinates over M come in field Φ^I (F_{BRST}), antifield Φ^*_I pairs ($F_{BV} \to F_{BRST}$):

$$\begin{array}{ccccccc} F_{BV} = T^*_V[-1]F_{BRST} & \longrightarrow & F_{BRST} & \longrightarrow & F & \longrightarrow & M \\ \uparrow & & \uparrow & & \uparrow & & \\ J^\infty F_{BV} & \longrightarrow & J^\infty F_{BRST} & \longrightarrow & J^\infty F & & \\ & & \hookuparrow & & \hookuparrow & & \\ & & \mathcal{E}_{BRST} & \longrightarrow & \mathcal{E} & & \end{array}$$

  - Antifields give rise to a local shifted symplectic density $\Omega = \mathrm{d}_V\Phi^*_I \mathrm{d}_V\Phi^I$, together with a homological evfs $Q \in \mathfrak{X}_{\mathrm{ev}}(J^\infty F_{BV})$, $Q_{BRST} \in \mathfrak{X}_{\mathrm{ev}}(\mathcal{E}_{BRST})$, with ghost $\#Q = -\#\Omega = 1$, such that $H^{\#<0}(C^\infty(J^\infty F_{BV}), \mathbf{s} = \mathscr{L}_Q) = 0$ and $H^0(C^\infty(J^\infty F_{BV}), \mathbf{s}) = H^0(C^\infty(\mathcal{E}_{BRST}), \mathscr{L}_{Q_{BRST}}) = C^\infty(\mathcal{E})^{\mathscr{G}}$ with $\mathscr{G}$ generated by infinitesimal gauge symmetries.
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L_∞-algebras

- **Def:** On a $(\mathbb{Z}, \mathbb{Z}_2)$-graded vector space V, the $(1, \text{odd})$-degree brackets $[\mathcal{S}(V)] \to V$ are an L_∞-algebra when $[e^B[e^B]] = 0$ for any even $B \in V$, while $[1] = 0$ and $\epsilon[(\cdots)] = (-)^{|\epsilon|}[\epsilon(\cdots)]$, for odd ϵ.
- Writing $\mathbf{s}B := [B]$ and decoding the **higher Jacobi** identities, $\mathbf{s}^2 B = 0$, $2[B\mathbf{s}B] + \mathbf{s}[B^2] = 0, \quad 3[B^2\mathbf{s}B] + 3[B[B^2]] + \mathbf{s}[B^3] = 0, \quad \ldots,$

$$\sum_{k=1}^{n} \frac{n!}{(n-k)!k!}[B^{n-k}[B^k]] = 0.$$

- **Ex:** For a dg-Lie algebra $(\mathfrak{g}, \mathbf{s}, [-,-])$, the L_∞-algebra is

$$(\mathfrak{g}[-1, \text{odd}], 0 \oplus \mathbf{s} \oplus [-,-] \oplus 0 \oplus \cdots).$$

- **Dually:** L_∞-algebra $(V, [-]) \iff (\mathcal{S}(V^*), D = [-]^*)$ dgca.
- **(non-)Ex:** $(C^\infty(J^\infty F_{BV}), \mathbf{s}_{BV})$ is an L_∞-algebr(-oid) in the dual picture. But it is **not** the L_∞-structure that I will talk about!

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Higher Structure Constants in Extended BV

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  Extended antifield formalism Nucl Phys B**510** 640–656
  - Start with a BV description of a gauge theory and consider a basis S_A of representatives for $H^{\bullet<0,n}(\mathbf{s}|\mathrm{d}_H)$ in local functionals, $\int b(\Phi, \Phi^*)\mathrm{d}^n x$.
  - Then take some antibrackets $(-,-)$, then some more antibrackets, ...

    At each stage, you get new structure constants $f_{AB}^D, f_{ABC}^D, \ldots$, and new local functionals $S_{AB}, S_{ABC}, \ldots$.
  - Putting the dg-Lie properties of $\mathbf{s}$ and $(-,-)$ to full use, after some magic, they end up with ...

$$\iff \quad \sum_{k=1}^{n} \frac{n!}{(n-k)!k!}[B^{n-k}[B^k]] = 0 \quad L_\infty\text{-algebra identities!}$$

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Homotopy Transfer

- Dually, an L_∞-morphism $\lambda^*\colon (\mathcal{S}(V_2^*), D_2) \to (\mathcal{S}(V_1^*), D_1)$ is a dgca-morphism, with $\lambda(1) = 0$. Equivalently, for even $B \in V_1$:

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Noether's Theorem

"symmetries" $\simeq$ "conserved currents"

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  - Recall the hierarchy $\overset{\text{antifields}}{F_{BV}} \to \overset{\text{ghosts}}{F_{BRST}} \to \overset{\text{fields}}{F} \to M$ (topologically trivial M, F).
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- The antibracket $\left(\int b(\Phi,\Phi^*), \int c(\Phi,\Phi^*)\right)$ is traditionally defined on local functionals or $H^{\bullet,n}(\mathrm{d}_H)$ classes $[b]$, $[c]$.
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$$(b,c)_{\mathrm{loc}} = \mathscr{L}_\beta c \quad \text{or} \quad -\mathscr{L}_\gamma b \quad \text{or} \quad \iota_\beta \iota_\gamma \Omega$$

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  Proof. For Jacobi, use $\Omega = \mathrm{d}_V(\Phi^*_I \mathrm{d}_V \Phi^I)$ and $(b,c)_{\mathrm{loc}} \rightsquigarrow [\beta,\gamma]$ (Barnich-Henneaux'96), while $\tilde{\mathbf{s}}(-) = (L + J + \cdots, -)_{\mathrm{loc}}$, with $\mathrm{d}_H J = (L,L)_{\mathrm{loc}}$.

L_∞-zigzags (WIP)

$$\begin{array}{ccccc}
(\mathfrak{X}^\bullet_{\mathrm{ev},\Omega}(J^\infty F_{BV}), [Q,-] \oplus [-,-]_{\mathrm{ev}}) & \twoheadleftarrow & (\Omega^{\bullet+\bullet<n}, \mathrm{d}_H + \tilde{\mathbf{s}} \oplus (-,-)_{\mathrm{loc}}) & \hookleftarrow & (\Omega^{\bullet+\bullet<n}(\mathcal{E}_{BRST}), \mathrm{d}_H \oplus [-,-]_D) \\
\downarrow & & \downarrow & & \uparrow \\
(\mathfrak{X}^\bullet_{\mathrm{ev}}(\mathcal{E}_{BRST}), [Q_{CE},-] \oplus [-,-]_{\mathrm{ev}}) & & (H^{\bullet<0,n}(\mathrm{d}_H), \mathbf{s} \oplus (-,-)) & & (\Omega^{\bullet<n}(\mathcal{E}), \mathrm{d}_H \oplus [-,-]_D) \\
\uparrow & & \uparrow & & \uparrow \\
(H^\bullet_{\mathcal{E}_{BRST}}([Q_{CE},-]_{\mathrm{ev}}), 0 \oplus [-]_{\mathrm{ev}}) & & (H^{\bullet<0,n}(\mathbf{s}|\mathrm{d}_H), 0 \oplus [-]_{BHW}) & & (H^{\bullet<n}_{\mathcal{E}}(\mathrm{d}_H), 0 \oplus [-]_D)
\end{array}$$

- Arrows indicate dg-Lie morphisms, or inclusion of dg-vector cocycles. All arrows should be dg-vector quasi-isomorphisms. (Work In Progress)
- (conj.) **Extended Noether's theorem:**

$$(H^\bullet_{\mathcal{E}_{BRST}}([Q_{CE},-]_{\mathrm{ev}}), 0 \oplus [-]_{\mathrm{ev}}) \xleftrightarrow{L_\infty \text{ equiv.}} (H^{\bullet<n}_{\mathcal{E}}(\mathrm{d}_H), 0 \oplus [-]_D)$$

- Topologically non-trivial $\mathcal{E} \to M$: $H^\bullet_{\mathcal{E}}(\mathrm{d}_H) \rightsquigarrow H^\bullet_{\mathcal{E}}(\mathrm{d}_H)/H^\bullet_{\mathcal{E}}(\mathrm{d})$.
  $\rightsquigarrow$ Central L_∞-extension?
- **Bonus:** $(\Omega^\bullet(\mathcal{E}), \mathrm{d}_H, \wedge)$ is a dgca.
  $\rightsquigarrow$ Homotopy Transfer of C_∞-algebra structure?

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Discussion

- L_∞ Homotopy Transfer interpretation of constant ghost/antifield extended BV (*Brandt-Henneaux-Wilch'98*).
  Cf. talk in Prague Mathematical Physics Seminar by *Hiroaki Matsunaga (05.2021)*.
- Successful dg-Lie local lift $(-,-)_{\mathrm{loc}}$ of antibracket.
- Geometric interpretation of $H^{\bullet<0,n}(\mathbf{s}|\mathrm{d}_H)$ via higher symmetries and conserved currents.
- L_∞-extension of Noether's theorem.

Thank you for your attention!

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