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Excessive Extrapolations in Cosmology

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Abstract—The current standard cosmological model is based on the normalized Friedmann equation $1 = \Omega_M + \Omega_\Lambda + \Omega_K$, where Ω_M is the mass density of dark and baryonic matter, Ω_Λ the vacuum energy density, and Ω_K is the curvature parameter. We show that the Friedmann equation was derived under excessive extrapolations from Einstein's equations, which are not scale invariant and are "verified" on much smaller scales. We explain why these extrapolations are incorrect, why the unrestricted use of the term "verified" is questionable, and why dark matter may exist only by definition.

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1. INTRODUCTION

In 1922 Alexander Friedmann [19] derived from Einstein's equations for a perfectly symmetric space, which is homogeneous and isotropic at each fixed time instant, a nonlinear differential equation of the first order for an unknown *expansion function* $a = a(t) > 0$ describing the expansion of the universe,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}, \qquad (1)$$

where the dot stands for the time derivative, $\rho = \rho(t) > 0$ denotes the mean mass density of the universe at time t, $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the gravitational constant, Λ is the cosmological constant, $c = 299\,792\,458$ m/s is the speed of light, k/a^2 is the space curvature, and $k = \{-1, 0, 1\}$ is the curvature index (normalized curvature). Equation (1) is called the *Friedmann equation*. The value $k = 1$ corresponds to the *hypersphere*

$$\mathbb{S}^3_a = \{(x, y, z, w) \in \mathbb{E}^4 \mid x^2+y^2+z^2+w^2 = a^2\} \qquad (2)$$

with variable radius $a = a(t)$, where $\mathbb{E}^n$ stands for the Euclidean space of dimension n. The case $k = 0$, which was not considered by Friedmann in [19], corresponds to $\mathbb{E}^3$. In 1924, Friedmann also derived Eq. (1) for the pseudosphere $\mathbb{H}^3$ with the negative curvature index $k = -1$ and for a negative density of mass (see [20, p. 2006]). It is not clear how to satisfy such a paradoxical assumption. Fortunately, Eq. (1) may also be formally investigated for $k = -1$ and $\rho \geq 0$. It is very difficult to imagine the manifold $\mathbb{H}^3$ (see [9]), since it cannot be isometrically embedded into $\mathbb{E}^4$ contrary to the hypersphere (2). According to [8], it can be isometrically embedded into $\mathbb{E}^{12}$.

The term "universe" is used in cosmology with various meanings: true spacetime, true space (i.e. spacetime at fixed time), and the observable universe. These are three different objects. Their mathematical models are also three completely different manifolds (see Fig. 1). Thus altogether we have six meanings of the problematic notion "universe." In accordance with Einstein's cosmological principle (that the universe at large scales is homogeneous and isotropic at fixed time), we shall use the term *universe* for a cross-section of spacetime at a fixed time instant t.

In 1922 astronomers had no idea about the real size of our universe because other galaxies were only discovered later by Edwin Hubble [23]. Their typical size is about 10^{10} au (astronomical units), and the size of the whole universe is at least five orders of magnitude higher. In spite of that, Alexander Friedmann applied Einstein's equations to the whole universe (see Section 2), even though these equations are not scale-invariant, and they are "verified" on much smaller scales than is the size of the whole universe (perihelion advance of Mercury's orbit, gravitational redshift, bending of light in the gravitational field of the Sun, slowdown of electromagnetic waves near the Sun—the Shapiro effect, the Lense-Thirring precession effect, and so on).

2. FORBIDDEN EXTRAPOLATIONS

The question of whether the Friedmann equation (1) sufficiently exactly describes the expansion of the real universe is entirely essential. If it is not so,

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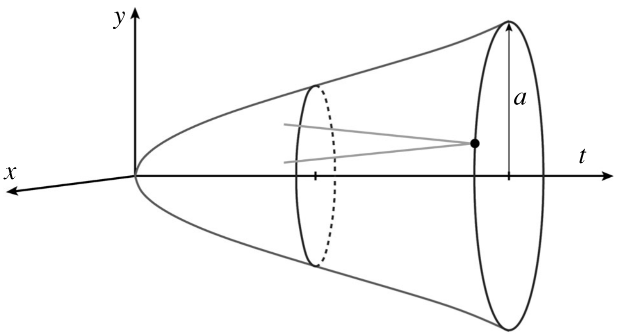

Fig. 1. Three different manifolds corresponding to the curvature index $k = 1$. For simplicity, the hypersphere (2) is replaced by only its great circle $\mathbb{S}^1_{a(t)}$ for $z = w = 0$ and for a fixed time instant t. This is a model of space (universe). A model of spacetime can be obtained by rotation of the graph of the expansion function $a = a(t)$ about the time axis t. The observable universe is marked by a light cone. The space dimensions are reduced by two.

then cosmologists solve by various means the same incorrect equation. We will support our questioning of the unrestricted use of the Friedmann equation by several arguments.

Phenomena in our universe are usually modeled by equations of mathematical physics, such as linear elasticity equations, Maxwell's equations, semiconductor equations, Einstein's equations, and so on. However, no such equation describes reality absolutely exactly. Thus we always get a nonzero modeling error with respect to some criterion (maximum surface temperature, mean velocity, minimum pressure, and so on). Each equation has certain restrictions on the size of investigated objects where reality is modeled well and, on the other hand, where its description fails, i.e., the modeling error essentially depends on the size of these objects. We will demonstrate this by a few examples.

Example 1. Consider a unit homogeneous and isotropic iron cube with edge $e = 1$ m and solve a steady-state heat conduction problem $-\Delta u = f$ with some boundary conditions, where u is the temperature and f is proportional to the density of heat sources. This elliptic problem approximates very well the true temperature in homogeneous isotropic solids that can be verified by direct measurements (see, e.g., [30]). However, in applying the heat equation on the atomic level in a cube with edge $e = 10^{-10}$ m, we get nonsensical numbers, since it is not clear how to define the temperature on such a small scale. We also obtain nonsensical numbers for $e = 10^{10}$ m (see the left part of Fig. 2). Such a large homogeneous and isotropic solid cube would immediately collapse into a black hole, since the diameter of this cube is about ten times larger than the diameter of the Sun. We can, of course, solve the above steady-state heat conduction problem on an arbitrarily large cube. However, the question is for which e do we still get acceptable results, and when the resulting temperatures have nothing to do with reality. The used criterion can be, e.g., $E = E(e) = |U_{\max} - u_{\max}|/U_{\max}$, where $U_{\max}$ (resp., $u_{\max}$) is the maximum real (resp., theoretical) surface temperature on the cube.

Example 2. The use of the Schrödinger equation is justified on scales of the hydrogen atom, i.e., on the scale 10^{-10} m. Nevertheless, we get nonsensical numbers on scales of 10^{-20} m which are less than the cross-section of a quark. Application of the Schrödinger equation to objects of size 1 m will also not produce reliable results.

The validity of other equations of mathematical physics with respect to a given criterion is subject to analogous restrictions, even though we get different graphs of the left part of Fig. 2, in general. Einstein's gravity field equations do not describe well processes on the atomic level, since there act other much stronger fundamental physical interactions that cannot be ignored (see the right-hand part of Fig. 2). Also, the time scales cannot be arbitrary. To see this, we present two more examples.

Example 3. The classical N-body problem yields very good predictions of the positions of the planets in the Solar system after one year. However, it produces nonsensical numbers for the period of 10^{10} years. Also, backward integration to the past has no sense, since the Solar system did not exist 10^{10} years ago. Therefore, long-term extrapolations in time are not reliable as well.

Example 4. Current mathematical models of weather forecast yield quite acceptable results for several days in advance. But we cannot predict the weather forecast for 10^{10} days in advance, and so on.

Thus, in any calculation we have to take care of the modeling error which is small, for example, only in the flat part of the left-hand graph in Fig. 2. For all that, Friedmann when deriving (1) applied Einstein's equations on cosmological scales even though they are "verified" only on much smaller scales than is the size of the whole universe. As mentioned in the Introduction, galaxies have diameters of order 10^{10} au, and the size of our universe is at least five orders of magnitude larger. Let us emphasize that Einstein's equations (see (14) below) are not scale-invariant since they are nonlinear and contain the fundamental constants G and c, and it is assumed that the speed of gravity is just $c < \infty$. Moreover, different structures are observed on different scales of the universe.

All this suggests that Eq. (1) was obtained by incorrect extrapolations from the Solar system scales to cosmological scales (see the right-hand part of

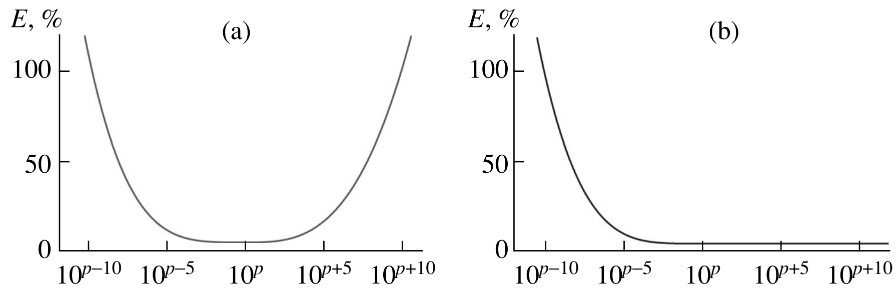

Fig. 2. Left: Schematic illustration of a general behavior of the relative modeling error E for equations of mathematical physics. The horizontal axis (in appropriate units) has a logarithmic scale, and p is the exponent yielding the smallest relative modeling error. Right: Enforced behavior of relative modeling (extrapolation) error in the case of Einstein's equations.

Fig. 2), even though (1) was derived by correct mathematical operations involving Einstein's equations on the maximally symmetric manifolds $\mathbb{S}^3$, $\mathbb{E}^3$, and $\mathbb{H}^3$. The current cosmological model is thus based on a questionable Friedmann equation. Below we present further arguments to support this conjecture.

3. STRANGE BEHAVIOR OF COSMOLOGICAL PARAMETERS

First recall the definition of the *Hubble parameter*

$$H(t) := \frac{\dot{a}(t)}{a(t)} \tag{3}$$

and divide Eq. (1) by the square $H^2 = (\dot{a}/a)^2 \geq 0$. In literature on cosmology this is usually done without any preliminary warning that we may possibly divide by zero that may lead to various paradoxes. Then for all t we get the *normalized Friedmann equation* for three dimensionless parameters

$$1 = \Omega_M(t) + \Omega_\Lambda(t) + \Omega_K(t) \tag{4}$$

which are defined as follows:

$$\Omega_M(t) := \frac{8\pi G\rho(t)}{3H^2(t)} > 0, \qquad \Omega_\Lambda(t) := \frac{\Lambda c^2}{3H^2(t)},$$

$$\Omega_K(t) := -\frac{kc^2}{\dot{a}^2(t)}, \tag{5}$$

Ω_M is called the *mass density of dark and baryonic matter*, Ω_Λ the *vacuum energy density*, and Ω_K is the *curvature parameter*, see [43, 45].

In 1917, Albert Einstein [12] added the positive cosmological constant to his equation of general relativity (see (14) below) to avoid gravitational collapse and save his model $\mathbb{S}^3_a$ ($a = \text{const}$) of the stationary universe (see also [51]). Note that the resulting solution of Eq. (1) is not stable, i.e., any small deviation from constant a will cause either a gravitational collapse or expansion [39, p. 746]. For $a = \text{const}$, we find that $\dot{a}(t) \equiv 0$ for all t (see the top left-hand part of Fig. 3), and by (3) we have $H(t) \equiv 0$. Even though nothing dramatic happens, by (5) the mass density and vacuum density parameters $\Omega_M(t) = \Omega_\Lambda(t) = \infty$ for all t. We should write more precisely that they are not well defined.

For a cyclic (i.e. pulsating or oscillating) universe there exists the time t_2 (see the top right-hand part of Fig. 3) such that $\dot{a}(t_2) = 0$. So we again divide by zero in (5) and get a very strange behavior of the cosmological parameters. In particular, the density of baryonic and dark matter is infinite, even though the density of true baryonic matter is surely finite. The vacuum energy density is also infinite for $t = t_2$, although the universe starts to collapse. Even in a small neighborhood of the point t_2, where we do not divide by zero, the behavior of the cosmological parameters is bizarre, since their values rapidly grow beyond all bounds. We also see that the curvature parameter

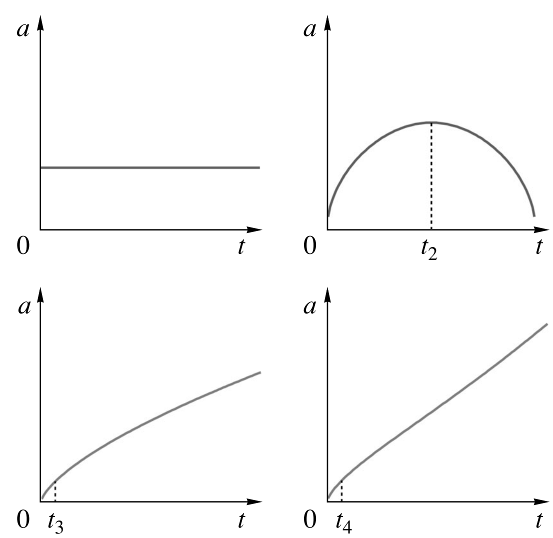

Fig. 3. The expansion function for the stationary universe, the cyclic universe, the universe with zero cosmological constant, and for the currently accepted expansion of the universe with a positive cosmological constant.

$\Omega_K(t_2) = \infty$ provided $k \neq 0$. However, reasonably defined physical quantities should not attain infinite values.

In the model with a zero cosmological constant and $k = -1$ it is derived that the expansion function tends to infinity as $t \to \infty$ and is strictly concave for $t > t_3 > 0$ (see the bottom left-hand part of Fig. 3 and [39, p. 735]). Hence, the derivative $\dot{a}$ and also its square are decreasing functions. By (5) the curvature parameter $\Omega_K > 0$ increases as $t \to \infty$, whereas the spatial curvature k/a^2 tends to zero. From the above paragraphs we observe that all three cosmological density parameters (5) do not have suitable names or they are wrongly defined (cf. also [22, p. 66]).

A somewhat more curious behavior of the parameter Ω_K is obtained for the currently accepted expansion function. Similarly to the previous paragraph, we shall consider only $t > t_4 > 0$, where t_4 denotes the time instant of the origin of the cosmic microwave background radiation. According to [49], the expansion function $a(t)$ is strictly concave over the interval circa $(t_4, 9)$ Gyr and then changes to a strictly convex function on the interval $(9, 14)$ Gyr. In other words, the function $\dot{a}$ is first decreasing and then increasing (see the bottom right-hand part of Fig. 3). From this it follows by (5) that the curvature parameter $\Omega_K(t)$ is not a monotone function, even though the universe expands continually. The absolute value of the curvature parameter $|\Omega_K| > 0$ on the interval $(t_4, 9)$ Gyr increases for $k \neq 0$, but the spatial curvature tends to zero with increasing time. We again see that the name for Ω_K was not appropriately selected.

Let us further note that by the theory of inflation, the universe expanded exponentially during a very short time instant after the Big Bang, i.e., the expansion function $a = a(t)$ was strictly convex. Then it was strictly concave and then surprisingly it was again strictly convex.

According to the data measured by the Planck satellite, the Planck Collaboration concluded that [45–48]

$$\Omega_M \approx 0.3175, \quad \Omega_\Lambda \approx 0.6825, \quad \Omega_K \approx 0. \tag{6}$$

These values were obtained by a combination of the methods of Baryonic Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB), and Supernovae type Ia explosions (SNe), see Fig. 4. Similar pictures can be found, e.g., in [2] and [43]. It is argued that these three methods are independent and that the corresponding sets of admissible cosmological parameters intersect in a small region whose coordinates are close to (6). However, we should keep in mind that all three methods originated from the same normalized Friedmann equation (4) that was derived by inordinate extrapolations, and thus these methods

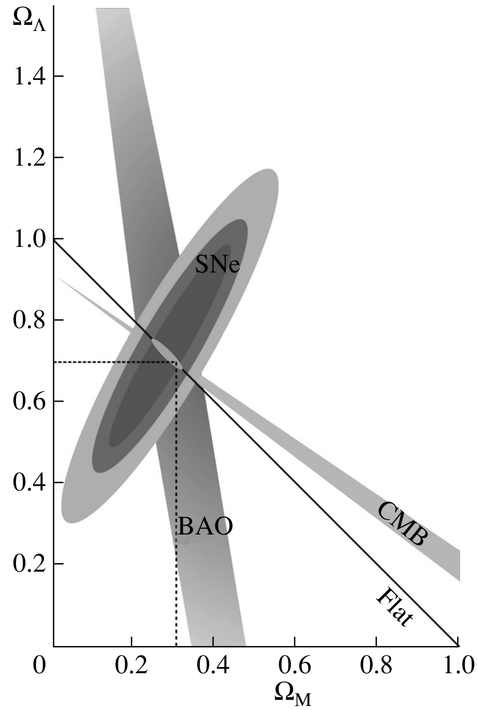

Fig. 4. Admissible values of the cosmological parameters, obtained by three different methods: BAO, CMB, and SNe, intersect in a small region containing the parameters (6). Nevertheless, these methods are not independent since all of them use the Friedmann equation derived by forbidden extrapolations.

are not independent. In other words, the amount of exotic dark matter and dark energy was obtained from an incorrect equation that was solved by three different methods up to four significant digits, see (6).

Concerning SNe, no extinction by the host galaxy was considered. The measured luminosity essentially depends on the position of SNe in the host galaxy, i.e., if it is in the interior or on the edge. Therefore, supernova type Ia explosions are not standard candles as required. Moreover, the measured data were taken from the observable universe which is modeled by a completely different manifold than the universe described by the expansion function as illustrated in Fig. 1.

From the relations (6) we observe that the sum of the measured values of $\Omega_M(t_0)$ and $\Omega_\Lambda(t_0)$ is approximately equal to 1. Nevertheless, this does not allow us to claim that from (4) and (5) it follows with high probability that $k = 0$ and that the true universe is flat (i.e. infinite Euclidean) as is often stated at present. Arguments for a vanishing curvature are also shown to be weak in [17]. Even though the sum would be

$$\Omega_M(t_0) + \Omega_\Lambda(t_0) = 1.000000000000000001,$$

we still have a bounded universe that can be described by the sphere (2) with an incredibly large radius. Moreover, the sphere $\mathbb{S}_a^3$ has an entirely different topology than the proclaimed flat space $\mathbb{E}^3$.

There are several other arguments against $\mathbb{E}^3$ being the correct model of our universe. By Einstein's theory of general relativity, matter curves space. Nevertheless, the curvature of $\mathbb{E}^3$ is independent of the decreasing mean mass density $\rho = \rho(t) > 0$.

The manifolds $\mathbb{E}^3$ and $\mathbb{H}^3$ have an infinite volume. However, the actual space could not first be finite after its origin and then change to infinite. Moreover, we can hardly imagine that the actual infinite universe would have everywhere on large scales the same density, temperature, pressure, and so on, at a given time instant after the Big Bang, as required by the Einstein cosmological principle. In this case, information would have to be transmitted at infinite speed. Therefore, the most probable model of our universe seems to be the sphere $\mathbb{S}_a^3$ defined by (2).

Since the product $\rho(t)a^3(t)$ is constant during the time period when matter dominates over radiation, the Friedmann equation (1) takes the equivalent form

$$\dot{a}^2 = Aa^2 + B + \frac{C}{a} \tag{7}$$

with time-independent constant coefficients $A = \Lambda c^2/3$, $B = -kc^2$, and $C > 0$ which are not exactly known. However, since the ratio (cf. [37, p. 56])

$$\frac{\dot{a}(t)}{a(t)} = H_0 = H(t_0) \approx 70 \text{ km/(s Mpc)}$$

is known, where t_0 is the present time, we may calculate from (7) the terminal condition $a(t_0)$ for $k \neq 0$. Thus we can solve equation (7) backward and also forward in time for given constants A, B, C. From such a simple ordinary differential equation we should not make any categorical conclusions about the deep past and the future of the universe as is often done. For the time period when radiation dominates over matter, the term D/a^2 is added to the right-hand side of Eq. (7).

In applying the standard cosmological model, various "delicate" limits are sometimes performed: $a \to 0$, $t \to 0$, $a \to \infty$, $t \to \infty$, $\cdots$ (see, e.g., [2, 36, 42]). In this way, the age of the universe is derived up to four significant digits as $t_0 = 13.82$ Gyr (see [45]). We should rightly say: According to the Friedmann equation with the parameters (6) the estimated age of the universe could be $t_0 \approx 13.82$ Gyr. The real age might be quite different.

Furthermore, we have to emphasize that the Friedmann equation (1) was derived only for the gravitational interaction. However, shortly after the Big Bang, electromagnetic forces that are many orders of magnitude higher played an important role. Before that, also even stronger nuclear forces surely had an influence on the initial values of the true expansion function. Although non-gravitational forces are investigated at large accelerators, their behavior in an extremely strong gravitational field right after the Big Bang is not known. Moreover, at that time many points of the universe receded from each other by highly superluminar speeds, and thus the application of Einstein's equations is questionable.

For the time being. only two coefficients H_0 and $q_0 = q(t_0) \approx -0.6$ (see [49]) were measured in the Taylor expansion

$$a(t) = a(t_0) + \dot{a}(t_0)(t-t_0) + \tfrac{1}{2}\ddot{a}(t_0)(t-t_0)^2 + \cdots = a(t_0)(1 + H_0(t-t_0) - \tfrac{1}{2}q_0 H_0^2 (t-t_0)^2 + \ldots), \tag{8}$$

where the *deceleration parameter* $q = -\ddot{a}a/(\dot{a})^2$ depends on the second derivative of the expansion function. Let us emphasize that calculation of the second derivatives from biased supernova data is a very ill-conditioned problem. It is evident that the first three terms of the Taylor expansion at the point t_0 cannot well describe the behavior of the expansion function in the far past. Therefore, e.g., the *Hubble time* $1/H_0 = 13.6$ Gyr corresponding to the linear term in (8) need not well approximate the real age of the universe.

4. DARK MATTER CAN BE ONLY A MODELING ERROR

Dark matter is a hypothetical kind of matter that does not interact with electromagnetic waves, and its properties are inferred only from its gravitational effects on visible matter. At present there exist a large number of articles on dark matter and dark energy that categorically state:

The universe consists of 27% dark matter, 5% baryonic matter, and 68% dark energy.

Rightly they should again claim:

According to the standard cosmological model based on the Friedmann equation, the universe could consist of 27% dark matter, 5% baryonic matter, and 68% dark energy.

It is important to understand the difference between the above two assertions. According to the interpretation of measurements of the Planck satellite [45, 46], the parameter of the mass density (cf. (6)) in the standard cosmological model is equal to

$$\Omega_{\mathrm{M}} = \Omega_{\mathrm{DM}} + \Omega_{\mathrm{BM}} \approx 0.32, \quad \Omega_{\mathrm{DM}} \approx 0.27, \quad \Omega_{\mathrm{BM}} \approx 0.05, \tag{9}$$

i.e., 27% consist of dark matter (DM) and 5% consists of baryonic (BM), from which less than 1% is made up of luminous matter. Although the model arose by forbidden extrapolations, and thus is probably not correct, some dark matter may exist. Scientific results should be independently checked. We suspect that the proposed ratio 27 : 5 of dark matter to baryonic matter is highly exaggerated.

Now let us briefly recall other approaches from [28] and [32] that are independent of the Friedmann equation (1). The existence of dark matter was postulated in 1933 by Fritz Zwicky [59] after discovering large velocities of galaxies in the Coma cluster A1656. But his data were not relevant. With the help of classical Newtonian mechanics he derived a very simple relation for the virial mass of the cluster

$$M = \frac{5Rv^2}{3G}. \tag{10}$$

Here $R = 4.58 \times 10^{22}$ m is its radius and $\nu = 1686$ km/s is the weighted root-mean-square speed of all galaxies with respect to the center of mass of the cluster for currently available data [1, 6]. The relation (10) yields ten times more of virial mass,

$$M = 3.25 \times 10^{45}\ \mathrm{kg}, \tag{11}$$

than the luminous mass

$$\mathcal{M} \approx 3.3 \times 10^{44}\ \mathrm{kg} \tag{12}$$

estimated from the Pogson equation, see [28]. Zwicky in [59] and [60] even obtained a more than two orders of magnitude larger value of M than $\mathcal{M}$. However, can we claim on the basis of such a trivial algebraic relation as (10) that dark matter in the Coma cluster really exists? Zwicky became well aware that he needed to make many simplifications; otherwise he could not calculate anything. For instance, he assumed that galaxies are distributed uniformly, that the Virial Theorem holds exactly, and that gravitation has an infinite speed of propagation. He substituted a spacetime curved by more than one thousand galaxies by Euclidean space. He replaced galaxies of diameter about 10^{10} au by mass points. Such approximations do not allow one to consider angular momenta of rotating galaxies that surely contribute to the total angular momentum. Tidal forces among galaxies were not included as well. Further simplifications are listed in [28].

In [28] we found that the virial mass (11) can be considerably reduced by taking into account the gravitational self-lensing effect of the Coma cluster, the relativistic effects of the observed high velocities, gravitational redshift, nonuniformity of mass distribution, gravitational aberration, dark energy, decreasing value of the Hubble parameter (3), and so on. Recently Tutukov and Fedorova [55] found that the intergalactic medium of galaxy clusters contains 30 to 50% of the total number of stars in the cluster. Moreover, clusters of galaxies contain five times more non-luminous baryonic matter in the form of a hot gas producing X-rays than baryonic matter contained in galaxies (see, e.g., [7, 23]). At the end of the last century, astronomers believed that only 3% of all stars are red dwarfs (see [5, p. 93]), at present we know that more than 80% of all stars are red and brown dwarfs of the spectral classes M, L, T, Y that are hardly detectable even in our close neighborhood. Consequently, the large velocities of galaxies in the Coma cluster observed by Zwicky have a natural explanation, and the proposed ratio 27 : 5 of dark matter to baryonic matter is considerably overestimated.

In [32] we give a detailed analysis of the dark matter problem proposed by Vera Rubin [50]. Her greatest discovery was the fact that spiral galaxies have “flat” rotational curves that do not correspond to Keplerian orbits of stars. However, it is important to realize that spiral galaxies do not have a central force field except within a close neighborhood of the central black hole whose mass is usually much less than 0.1% of the total galactic mass. In the Solar system, on the contrary, 99.85% of the mass is concentrated at the Sun. The planets barely interact gravitationally among themselves, and their motion is mainly determined by the central force of the Sun. On the other hand, trajectories of stars in a galactic disk are substantially influenced by neighboring stars because the central bulge usually contains only about 10% of all mass of a spiral galaxy.

Example 5. The radius of the visible part of the disk of our Galaxy is about $r = 16\ \mathrm{kpc} = 4.938 \times 10^{20}$ m. By Hipparcos’ data taken from our neighborhood up to a distance of several hundred parsec, the baryonic mass inside the ball of radius r can be estimated by $M = 3.85 \times 10^{41}$ kg (see [29, p. 128] for details). The Harvard Spectral Classification yields a similar value. Now let us concentrate all baryonic matter inside the ball of radius r to one central point. The Shell theorem indicates that the outside matter (including possible dark matter) has no effect on the motion of stars, provided the mass distribution beyond r is spherically symmetric. Then we find that the orbital velocity of stars on the radius r of the visible disk is

$$v = \sqrt{\frac{GM}{r}} = 228\ \mathrm{km/s}.$$

However, this value is comparable to the measured speed. Although this relation is only approximate, to postulate the existence of 6 times more dark matter than baryonic matter to hold the Milky Way together by gravity seems to be somewhat overestimated.

In [32] we moreover prove the following statement, which supports flat rotational curves without dark matter.

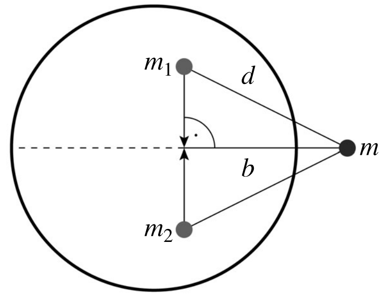

Fig. 5. A ball with symmetrically distributed mass with respect to the horizontal plane acts on a test particle by a smaller force than the mass projected perpendicularly to the horizontal plane of the disk—dashed.

A particle orbiting a central mass point along a circular trajectory of radius R has a smaller speed than if it were to orbit a flat disk with radius R and the same mass with an arbitrary rotationally symmetric density distribution.

To be convinced of this assertion, just consider two arbitrary mass points with masses $m_1 = m_2$ located inside a ball placed symmetrically with respect to the horizontal plane (see Fig. 5). Then the total force F of both mass points acting on the test particle of mass m will be less than the force $\overline{F}$ of both mass points projected perpendicularly to the disk and acting on m. Let d be the distance between m_1 and m. Denoting by b its orthogonal projection onto the horizontal plane, we find that $\overline{F} = (d/b)^3 F \geq F$. This cubic nonlinearity causes a greater attractive gravitational force by the disk than by the ball, and thus also a higher orbital speed of stars around the disk.

In [28] and [32] we have presented several further arguments showing that dark matter can possibly be only a modeling error. Notice that the density parameter Ω_M of dark and baryonic matter exists by definition (5). Furthermore, Pavel Kroupa in [34] and [36] gives several other arguments that point to the absence of dark matter around our Galaxy. For instance, all dwarf galaxies orbit around the Milky Way (and also M31) in almost one plane which would be very unlikely if the amount of halo dark matter would be six times larger than the amount of baryonic matter. A number of other works (see, e.g., [4, 18, 22, 28, 35, 41, 53]) also confirm that it is not necessary to assume the existence of dark matter. Finally note that the famous MOND (Modified Newtonian Dynamics) assumes an infinite speed of gravity, which surely contributes to the modeling error, too.

5. DARK ENERGY AND THE COSMOLOGICAL CONSTANT

Generally speaking, the prevailing conviction says that dark energy is some mysterious substance which is responsible for the accelerated expansion of the universe and this is attributed to the cosmological constant Λ.

Why should a single constant Λ truly model the accelerated expansion of the real universe? Is this not too great a simplification and too rough an approximation? Dark energy was introduced into the standard cosmological model to explain the observed accelerated expansion of the universe based on the Friedmann equation and to eliminate the obvious violation of the energy conservation law. Nevertheless, gravitational aberration (see [26]) also causes a repulsive force and thus it may generate the sought-for energy necessary for the accelerated expansion. Antigravity (sometimes called a *dark force*) acts as a hidden repulsive force between planets, stars, galaxies, and their clusters and thus influences the expansion of the universe [31]. However, the observed local expansion (see [27, 33]) cannot be described by a single constant, since it depends on position, time, mass, and so on. Average values of this expansion are not described by a fundamental constant. Therefore, we should rather consider a time-dependent function $\Lambda = \Lambda(t)$ (like the Hubble parameter $H(t)$ which also depends on time). Fahr and Heyl in [15] present a list of papers where variable Lambda cosmologies have been discussed in detail. There exist many indications that vacuum energy density should decay with the expansion of the universe (see [15, p. 710]) and that it could be proportional to mass density. By Fahr and Sokaliwska [16, p. 382], the cosmological constant excludes to be treated as an equivalent to a vacuum energy density.

The standard cosmological model assumes that the expansion of the universe is manifested only globally and not locally. Nevertheless, according to [10, 11, 33, 38, 58], the universe also expands locally with a speed comparable with the Hubble constant.

There is a fundamental hypothesis (by Pavel Kroupa [35], the so-called null hypothesis) that all present-day matter was created as a relativistic fluid during the hot Big Bang. However, in [29] and [33] we show that the Solar system slightly expands, and it is sufficiently isolated from the influence of other stars. For instance, the gravitational force between the Earth and Alpha Centauri is about one million times smaller than the maximal gravitational force between the Earth and Venus. Since the total energy of the Solar system slightly increases, by the Einstein relation $E = mc^2$ its total mass also slightly increases, i.e., the above null hypothesis need not hold, and the energy conservation law is slightly disturbed. On the other hand, the Friedmann equation assumes that the total mass of all particles remains constant. According to [21, p. 62], the vacuum energy density is nonphysical since it conflicts with the requirements of cosmic thermodynamics (cf. also [14]).

6. FLUCTUATIONS IN COSMIC MICROWAVE BACKGROUND

It would be a mistake to believe that the well-known map of the cosmic background radiation shows the entire universe, how it looked like 380,000 years after the Big Bang. This map shows only a two-dimensional slice of a three-dimensional manifold corresponding to the universe for the redshift $z \approx 1089$ (see [13]). For the normalized curvature $k = 1$ the radius of the universe was $(z + 1) = 1090$ times smaller than it is at present.

Moreover, we observe this radiation only on the projection to the celestial sphere. For example, the relic radiation produced at that time in our neighborhood is not on the map of the cosmic microwave background radiation. There is also no relic radiation from the places where we have found to date all 10^{12} galaxies in the observable universe. At each of these galaxies we would observe at present completely different maps of the cosmic microwave background fluctuations. So on the Earth, an observer may have an idea about how only a tiny part of the early universe looked like.

Since the Solar system is nonsymmetrically placed in the gravitational lens called the Milky Way, the CMB is very slightly deformed. Moreover, the CMB has been biased by gravitational lensing of billions of galaxies and their clusters for more than 13 Gyr. This phenomenon is called *weak lensing*. However, it is weak only for a relatively small z, say $z < 1$. On the other hand, we will illustrate in the next example that it is relatively strong for $z \gg 1$.

A large amount of noise in the CMB was thus produced especially from the distribution of protogalaxies and their clusters when our universe was young. No relic photon traveled along an exactly straight line, since the observable universe contains about 10^{12} galaxies. The bending of a photon trajectory is given by the relativistic relation [40]

$$\phi = \frac{4GM}{c^2 R}, \tag{13}$$

where M is the mass of a given object, and R is the distance of a photon from its center.

Example 6. Assume now that some relic photon was gravitationally bent at the distance $z = 9$ about the angle $\phi = 1'$, which is by (10)–(13) or [28] and [32] quite an acceptable value for galaxy clusters. At that time, the universe was $(z + 1)^3 = 1000$ times more dense than it is at present, which makes bending effects even larger than at present. According to [44], a photon with $z = 9$ has travelled more than 13 Gyr. If

Fig. 6. Bent trajectory of a photon in a nonexpanding universe.

the universe had not expanded, then the distance d of the photon from its original straight trajectory would satisfy

$$d > \tan\phi \cdot c \cdot 13 \times 10^9 \text{ yr} = 3.78 \times 10^6 \text{ ly},$$

as indicated in Fig. 6. However, since the universe expanded by $z + 1 = 10$ times in each direction (see [57, p. 453]), the relic photon will deviate more than 37.8 million light years from its original straight trajectory. For $z > 9$ such a magnification is, of course, larger. Consequently, the larger z is, the larger is the smearing.

Further, recall (see [45]) that the highest peak in the temperature angular power spectrum of the CMB is at about 1°. In other words, the most typical diameter of fluctuations in the CMB is about one angular degree. The whole celestial sphere has 4π steradians which is about 41.253 square degrees. Thus the area of the sky with such a typical circular fluctuation contains, on the average, $(\pi/4) \times 10^{12}/41253 \approx 19 \times 10^6$ galaxies. Many of them have $z > 9$, and thus their gravity produces a really large smearing in the CMB radiation. This fact is not taken into account in [47].

Hence, a natural question arises concerning whether CMB fluctuations are mostly random noise due to gravitational lensing. Relic photons that travelled along more curved trajectories have slightly larger wavelength than those with straighter trajectories. Another a source of bias is the inverse Compton scattering effect of the CMB, called the Sunyaev–Zeldovich effect (see [48]). According to [17, p. 699], fluctuations in CMB can also be understood as an indication of different cosmological expansion dynamics seen in an anisotropically expanding universe in different directions of the sky.

7. "VERIFICATION" OF EINSTEIN'S EQUATIONS

Before explaining why we use the term "verification" in quotation marks, we first recall the famous nonlinear system of Einstein's 10 equations (for details see, e.g., [39, 54])

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{14}$$

$\mu,\ \nu \in \{0, 1, 2, 3\}$, which contains on its left-hand side several thousands of terms (partial derivatives of scalar functions). The reason is that there are

10 components of the unknown symmetric metric tensor $g_{\mu\nu}$, 20 components of the Riemann tensor, 40 components of the so-called Christoffel symbols, and moreover, we have to calculate the inverse matrix of $g_{\mu\nu}$ to evaluate the Christoffel symbols. For comparison, the Poisson equation $-\Delta u = f$ from Example 1 has only three terms $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$, and $\partial^2 u/\partial z^2$ on the left-hand side. Einstein's equations form a hyperbolic set of partial differential equations of the second order. If the curvature of true spacetime were to depend on the third or higher derivatives of the metric tensor, then the right-hand graph in Fig. 2 would be not correct since (14) contains derivatives only up to the second order, and thus a modeling error would appear.

The system (14) is so complicated that its solution for two mutually orbiting bodies is not known. Therefore, many simplifications are made (see, e.g., [39, p. 1076]). The most popular is the parametrized post-Newtonian (PPN) formalism which is also an approximation of several other theories of gravity including general relativity. It assumes low velocities v for which $v \ll c$. After many simplifications we get only algebraic expressions that are tested. But we should evaluate the modeling error between an actual solution of system (14) and the measured quantities. Most of the tests are based only on Solar-system experiments (see Section 1 and, e.g., [39, 52]). For instance, instead of solving the system (14), the true perihelion advance of Mercury's orbit after one revolution is compared with the simple algebraic relation $\varepsilon = 24\pi^3 a^2 T^2 c^{-2}/(1-e^2)$, where T is the orbital period, a is the semi-major axis, and e is the eccentricity of Mercury's orbit [56]. Another simple tested formula is, e.g., (13).

Now we will discuss verification of Einstein's equations in a distant stellar system. Concerning the reduction of orbital periods and periastron shift of very distant binary pulsars, we should keep in mind that the distance of their components is usually about 0.01 au, i.e., it is again on the scale of the Solar system. The detected gravitational redshift $z = 0.4$ of neutron stars arises only in a very close vicinity of the star and is negligible several astronomical units away. On the other hand, distant galaxies represent very large gravitational lenses. However, they are so inhomogeneous that the relation (13) can give only a very rough approximation of the action of these lenses. Moreover, the distribution of mass along the trajectories of observed photons is not known. Therefore, the often-proclaimed statement that Einstein's equations describe reality with precision better than 99% is questionable due to the above-mentioned arguments.

8. CONCLUSIONS

According to the standard cosmological model, it is categorically stated that our universe is flat, 13.82 Gyr old, and that it consists of 68% of dark energy, 27% of dark matter, and only 5% of baryonic matter. The main problem is that cosmological models are often identified with reality. We have shown that dark matter and partly also dark energy can possibly be explained as an extrapolation error of the Friedmann equation.

If the Friedmann equation were to perfectly describe the rate of evolution of our universe, then the standard cosmological model would not possess so many problems and paradoxes, like, e.g., the existence of some mysterious dark matter and dark energy, the horizon problem, the problem of homogeneity and isotropy, the flatness problem, the problem of exact setting of initial conditions, the problem of hierarchical structures, the problem of the existence of young stars orbiting the center of our Galaxy, the problem of the existence of giant black holes in the early universe, and the problem of Big Bang itself. Some black holes (e.g., the one at the center of the M87 galaxy) produce such large giant jets that it seems to be impossible that these jets are supplied only from accretion disks (cf. [15]).

At present it is very difficult to be familiar with the huge amount of information concerning cosmology. When reading literature on cosmology, it is often not clear what is a definition, what is an assumption, what is a statement, what is an experimentally verified fact, what are measured values and which values follow from some model, and what is an attractive numerical simulation or artificially colored picture, and what is a serious estimate. A number of definitions are vague, modeling and extrapolation errors are ignored, confusing notation is used, measured data are wrongly interpreted, "serious" conclusions about the evolution of the universe are made from incorrectly derived equations, and so on. We often do not know in which way some statement was derived and then communicated by papers or lectures without any verification. In this way a cosmological "folklore" arises.

For instance, in the current cosmology, we often meet the following argumentation. Distances between galaxies increase, and thus the entire universe was concentrated at one point in the past (see, e.g., [42, p. 70]). This implication is wrong from a mathematical point of view. As a counterexample, it is enough to take the everywhere increasing non-Big-Bang expansion function,

$$a(t) = C_1 + C_2 e^{C_3 t}, \quad t \in (-\infty, \infty)$$

(where C_1, C_2, C_3 are positive constants), which is not zero—nor arbitrarily close to zero as t approaches $\pm\infty$.

We should not be surprised why the difference between the measured and theoretically derived density of vacuum energy is over 120 orders of magnitude (see [3, pp. 3, 109]), since the standard cosmological model is questionable. From this it is evident that the vacuum energy is not the main reason for the accelerated expansion of the universe, as convincingly explained in [21, p. 71].

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REFERENCES

1. C. Adami et al., "The build-up of the Coma cluster by infalling substructures," Astron. Astrophys. **443**, 17–27 (2005).
2. R. Amanullah et al., "Spectra and HST light curves of six type Ia supernovae at $0.511 < z < 1.12$ and the Union2 compilation," Astrophys. J. **716**, 712–738 (2010).
3. L. Amendola and S. Tsujikawa, *Dark Energy—Theory and Observations* (Cambridge Univ. Press, Cambridge, 2010).
4. D. G. Banhatti, "Newtonian mechanics and gravity fully model disk galaxy rotation curves without dark matter," ArXiv: 0806.1131.
5. J. Binney and M. Merrifield, *Galactic Astronomy* (Princeton, NJ, 1998).
6. A. Biviano et al., "A catalogue of velocities in the central region of the Coma cluster," Astron. Astrophys. Suppl. Ser. **111**, 265–274 (1995).
7. H. Böhringer and N. Werner, "X-ray spectroscopy of galaxy clusters: studying astrophysical processes in the largest celestial laboratories," Astron. Astrophys. Rev. **18**, 127–196 (2010).
8. D. Brander, "Isometric embeddings between space forms," Master Thesis, Univ. of Pennsylvania, 1–48 (2003).
9. J. W. Cannon, W. J. Floyd, R. Kenyon, and W. R. Parry, "Hyperbolic geometry," in *Flavors of Geometry*, Math. Sci. Res. Inst. Publ. Cambridge Univ. Press. **31**, 59–115 (1997).
10. Y. V. Dumin, "A new application of the Lunar laser retroreflectors: Searching for the "local" Hubble expansion," Adv. Space Res. **31**, 2461–2466 (2003).
11. Y. V. Dumin, "Testing the dark-energy-dominated cosmology by the Solar-System experiments", Proc. 11th Marcel Grossmann Meeting on General Relativity, Ed. by H. Kleinert, R. T. Jantzen, and R. Ruffini (World Sci., Singapore, 2008), pp. 1752–1754; arXiv: 0808.1302.
12. A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," Königlich-Preuss. Akad. Wiss. Berlin, 142–152 (1917).
13. D. J. Eisenstein and C. L. Bennett, "Cosmic sound waves rule," Physics Today **61**, 44–50 (2008).
14. H.-J. Fahr, "Cosmological consequences of scale-related comoving masses for cosmic pressure, mass, and vacuum energy density," Found. Phys. Lett. **19**, 423–440 (2006).
15. H.-J. Fahr and M. Heyl, "Cosmic vacuum energy decay and creation of cosmic matter," Naturwissenschaften **94**, 709–724 (2007).
16. H.-J. Fahr and M. Sokaliwska, "The influence of gravitational binding energy on cosmic expansion dynamics: new perspectives for cosmology," Astrophys. Space Sci. **339**, 379–387 (2012).
17. H.-J. Fahr and J. H. Zönnchen, "The 'writing on the cosmic wall': Is there a straightforward explanation of the cosmic microwave background?" Ann. Phys. (Berlin) **18**, 699–721 (2009).
18. J. Q. Feng, C. F. Gallo, "Deficient reasoning for dark matter in galaxies," Phys. Internat. **6**, 1–12 (2015).
19. A. Friedman, "Uber die Krümmung des Raumes," Z. Phys. **10**, 377–386 (1922); ["On the curvature of space," Gen. Rel. Grav. **31** 1991–2000 (1999)].
20. A. Friedmann, "Uber die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes," Z. Phys. **21**, 326–332 (1924); [English translation: "On the possibility of a world with constant negative curvature of space," Gen. Rel. Grav. **31** 2001–2008 (1999)].
21. C. F. Gallo and J. Q. Feng, "Galactic rotation described by a thin-disk gravitational model without dark matter," J. Cosmology **6**, 1373–1380 (2010).
22. M. Heyl and H.-J. Fahr, "The thermodynamics of gravitating vacuum," Phys. Sci. Internat. J. **7**, 65–72 (2015).
23. E. Hubble, "Cepheids in spiral nebulae," The Observatory **48**, 139–142 (1925).
24. J. P. Hughes, "The mass of the Coma cluster: Combined X-ray and optical results," Astrophys. J. **337**, 21–33 (1989).
25. J. Jałocha, Ł. Bratek, and M. Kutschera, "Is dark matter present in NGC 4736? An iterative spectral method for finding mass distribution in spiral galaxies," Astrophys. J. **679**, 373–378 (2008).
26. M. Křížek, "Does a gravitational aberration contribute to the accelerated expansion of the Universe?" Comm. Comput. Phys. **5**, 1030–1044 (2009).
27. M. Křížek, "Dark energy and the anthropic principle," New Astronomy **17**, 1–7 (2012).
28. M. Křížek, F. Křížek, and L. Somer, "Which effects of galactic clusters can reduce the amount of dark matter," Bulg. Astronom. J. **21**, 1–23 (2014).
29. M. Křížek, F. Křížek, and L. Somer, *Antigravity—Its Origin and Manifestations* (Lambert Acad. Publ., Saarbrücken 2015).
30. M. Křížek and P. Neittaanmäki, *Mathematical and Numerical Modeling in Electrical Engineering: Theory and Applications* (Kluwer, Dordrecht, 1996).
31. M. Křížek and L. Somer, "Antigravity—its manifestations and origin," Internat. J. Astron. Astrophys. **3**, 227–235 (2013).

32. M. Křížek and L. Somer, "A critique of the standard cosmological model," Neural Netw. World **24**, 435–461 (2014).
33. M. Křížek, L. Somer, "Manifestations of dark energy in the Solar system," Grav. Cosmol. **21**, 58–71 (2015).
34. P. Kroupa, "Local-group tests of dark-matter concordance cosmology," ArXiv: 1006.1647.
35. P. Kroupa, "The dark matter crisis: Falsification of the current standard model of cosmology," Publ. Astron. Soc. Australia **29**, 395–433 (2012).
36. P. Kroupa, "Galaxies as simple dynamical systems: observational data disfavor dark matter and stochastic star formation," Can. J. Phys. **93**, 169–202 (2015).
37. G. E. Lemaître, "Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extragalactiques," Ann. Soc. Sci. de Bruxelles, 49–59, April 1927.
38. C. G. McVittie, "The mass-particle in expanding universe," Mon. Not. R. Astron. Soc. **93**, 325–339 (1933).
39. C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, 20th ed. (W. H. Freeman, New York, 1997).
40. S. Mollerach and E. Roulet, *Gravitational Lensing and Microlensing* (World Scientific, Singapore, 2002).
41. K. F. Nicholson, "Galactic mass distribution without dark matter or modified Newtonian mechanics," astro-ph/0309762v2 (2007).
42. P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton Univ. Press, New Jersey, 1993).
43. S. Perlmutter, "Supernovae, dark energy, and the accelerating universe," Physics Today **53**, 53–60 (2003).
44. S. V. Pilipenko, "Paper-and-pencil cosmological calculator," arXiv: 1303.5961.
45. Planck Collaboration, "Planck 2013 results, I. Overview of products and scientific results," Astron. Astrophys. **571**, A1, 48 (2014).
46. Planck Collaboration, "Planck 2013 results, XVI. Cosmological parameters," Astron. Astrophys. **571**, A16, 66 pp. (2014).
47. Planck Collaboration, "Planck 2013 results, XVIII. Gravitational lensing—infrared background correlation," Astron. Astrophys. **571**, A18, 24 (2014).
48. Planck Collaboration, "Planck 2013 results, XX. Cosmology from Sunyaev–Zeldovich cluster counts," Astron. Astrophys. **571**, A20, 20 (2014).
49. A. G. Riess et al., "Observational evidence from supernovae for an accelerating universe and a cosmological constant," Astronom. J. **116**, 1009–1038 (1998).
50. V. C. Rubin, "A brief history of dark matter," in *The Dark Universe: Matter, Energy, and Gravity* (Ed. M. Livio, Cambridge Univ. Press, Cambridge, 2003), pp. 1–13 .
51. K. Schwarzschild, "Uber das zulässige Krümmungsmaaß des Raumes," Vierteljahrsschift der Astronomischen Gesellschaft **35**, 337–347 (1900); ["On the permissible numerical value of the curvature of space," Abraham Zelmanov J. **1**, 64–73 (2008)].
52. I. I. Shapiro, "Fourth test of general relativity," Phys. Rev. Lett. **13**, 789–791 (1964).
53. S. Sikora, Ł. Bratek, J. Jałocha, and M. Kutschera, "Gravitational microlensing as a test of a finite-width disk model of the Galaxy," Astron. Astrophys. **546**, A126, 9 (2012).
54. H. Stephani, *General Relativity. An Introduction to the Theory of the Gravitational Field* (2nd edition, Cambridge Univ. Press, Cambridge, 1990).
55. A. V. Tutukov and A. V. Fedorova, "The origin of intergalactic stars in galaxy clusters," Astron. Reports **55**, 383–391 (2011).
56. A. A. Vankov, "General relativity problem of Mercury's perihelion advance revisited," ArXiv: 1008.1811.
57. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley, New York, London, 1972).
58. W. J. Zhang, Z. B. Li, and Y. Lei, "Experimental measurements of growth patterns on fossil corals: Secular variation in ancient Earth-Sun distance," Chinese Sci. Bull. **55**, 4010–4017 (2010).
59. F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln," Helv. Phys. Acta **6**, 110–127 (1933).
60. F. Zwicky, "On the masses of nebulae and of clusters of nebulae," Astrophys. J. **86**, 217–246 (1937).