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Why Masses of Binary Black Hole Mergers Are Overestimated? †

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† Cordially dedicated to Professor Andre Maeder on his 80th birthday.

Abstract: We show that masses of binary black hole mergers are overestimated, since a large gravitational redshift is not taken into account. Such a phenomenon occurs due to time dilation in a close neighborhood of any black hole. This fact allows us to explain a high mass gap between observed binary neutron stars and calculated binary black hole mergers. We also present other reasons why masses of black hole mergers are determined incorrectly.

Keywords: detected frequency; emitted frequency; black hole binary merger; time dilation; mass gap

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Citation: Křížek, M.; Somer, L. Why Masses of Binary Black Hole Mergers Are Overestimated? *Galaxies* **2022**, *10*, 52. https://doi.org/10.3390/galaxies10020052

Academic Editors: Christian Corda and Lorenzo Iorio

Received: 29 January 2022
Accepted: 22 March 2022
Published: 23 March 2022

1. Introduction

In 1916, Albert Einstein [1] (see also [2]) predicted the existence of gravitational waves. From his equations of the General Theory of Relativity for weak gravitational fields, he derived a partial differential equation for plane gravitational waves (see e.g., ([3], p. 24)).

At present there are over one hundred detections of gravitational waves coming from binary black hole mergers. However, their calculated masses seem to be overestimated. To show this, we will concentrate on the first detection of gravitational waves GW150914 that were independently registered by two LIGO detectors on 14 September 2015.

By [4], two black holes with masses

$$m_1 = 36^{+5}_{-4} M_\odot \quad \text{and} \quad m_2 = 29^{+4}_{-4} M_\odot \tag{1}$$

merged. Our criticism concerns especially the methodology that was used to process the measured data and their interpretation. The post-Newtonian model that was used can approximate reality only very roughly, since the gravitational redshift of each component of the above-mentioned binary black holes is infinite. Moreover, this model is situated in Euclidean space which does not allow one to consider a curved spacetime. Hence, from such a heuristic model one cannot establish any trustworthy conclusions about the real masses of these black holes.

In Section 2, we present main drawbacks of the formula that was used to calculate the so-called chirp mass. In Section 3, we show that the infinite gravitational redshift of each black hole was neglected which leads to an overestimation of the masses (1) with their associated error bars. In Section 4, we collect further arguments to illustrate that masses (1) do not correspond to reality. Finally, Section 5 contains some important concluding remarks.

2. Basic Relationship for Detected Frequencies

The redshift of frequencies of detected gravitational waves consists of 3 basic components:

(i) a Doppler component caused by the movement of the source or the observer with respect to its neighborhood,
(ii) a cosmological component caused by the expansion of the universe,

(iii) a gravitational component caused by the change of frequency of waves in a gravitational field.

In this paper, we show that the third component associated with the gravitational redshift was not taken into account. By ([4], p. 3) the total chirp mass $\mathcal{M}$ in the detector frame is given by the formula

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}, \tag{2}$$

where m_1 and m_2 denote the masses of the particular black holes in the binary system, $f = f(t)$ stands for the detected frequency of gravitational waves in time t, and $\dot{f} = \dot{f}(t)$ is its time derivative. As usual, c = 299,792,458 m/s is the speed of light in a vacuum and $G = 6.674 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$ is the gravitational constant. Relation (2) is, in fact, the only formula from [4] which is presented on a single line and, therefore, our critical analysis below will concern just this relation.

From (2) we immediately see that f cannot be the detected frequency, since f obviously depends on the total redshift z, while the masses m_1 and m_2 have to be independent of z. Consequently, the initial relationship (2) can only be true if $z = 0$. This formula was taken from the paper ([5], p. 3516) by Blanchet et al. which was published already in 1995. Nevertheless, Blanchet et al. rightly consider the emitted frequency f_{em} in the source frame, see also ([6], p. 2663). Hence, the detected frequency f in the detector frame appearing in Formula (2) has to be replaced by the emitted frequency, namely,

$$f_{\mathrm{em}} = (z+1)f, \tag{3}$$

where z is the total redshift of gravitational waves.

The paper [7] contains a number of various numerical simulations of merging black hole binaries with variable input data m_1 and m_2. The corresponding approximation methods for constructing these simulations are also given in [8,9]. Let us emphasize that no gravitational redshift is taken into account in the cited papers [7–9] on numerical simulations.

Note also that the corresponding modeling error cannot be established, since the true solution of Einstein's equation for two bodies is not known. Moreover, from [4] one cannot find the method by which the redshift z was estimated and how the derivative $\dot{f}$ from (2) was calculated.

Now we shall investigate the crucial relationship (2) in detail. This formula was derived from the relation (3) of paper [5] so that various higher order terms were omitted. Nonetheless, the equality sign in (2) is kept, see [4]. Moreover, we observe that the left-hand side of (2) is independent of time t, whereas its right-hand side is time dependent. Hence, the equality (2) can hold if and only if $f^{-11/3}\dot{f}$ is an appropriate constant C (for example $C = 0.00015142\ldots\ \mathrm{s}^{5/3}$ for the case (1)). From this we get the following first order ordinary differential equation

$$\dot{f} = C f^{11/3}. \tag{4}$$

It is easy to verify that its general solution reads

$$f(t) = \left(\frac{3}{8}\right)^{3/8} \frac{1}{(K - Ct)^{3/8}} \tag{5}$$

with an arbitrary integration constant $K \in (-\infty, \infty)$, cf. e.g., ([6], p. 2663). According to ([10], p. 14), the only solution of the differential Equation (4) is given by (5) for $t < K/C$. However, we can set $K = 0$, since (4) is an autonomous equation with coefficients independent of time. In this way, the formula (5) expresses the dependence of f on time.

The paper [4] analyzes only approximations of detected frequencies f from the interval 35–250 Hz for about eight orbital periods. It is well known that a numerical calculation of the first derivatives $\dot{f}$ from smoothed data is a very ill-conditioned problem which may produce incorrect results (see e.g., [11,12]). The mathematical and numerical analysis of the smoothed signal GW150914 can be found in [13,14].

3. Neglected Gravitational Redshift in Detections of Gravitational Waves

Papers [4,5,15–18] do not mention anything about gravitational redshift of detected gravitational waves. By ([4], p. 7), the luminosity distance of the source of GW150914 is 410^{+160}_{-180} Mpc. This value corresponds to the cosmological redshift (ii),

$$z = 0.09^{+0.03}_{-0.04}, \tag{6}$$

that is stated by the authors (see e.g., [19]). Thus, for the Doppler component (i) and the gravitational redshift (iii) we only get

$$z \approx 0. \tag{7}$$

The corresponding Doppler redshift cannot be reliably determined, because the orientation of the orbital plane and the local movement of the binary merger are not known.

By [20], the gravitational redshift for the surface of a neutron star is $z \approx 0.3$. This value is even larger than that in (6) and it can be derived from Formula (10) given later. For the horizon of an isolated black hole with mass m and Schwarzschild radius

$$r = \frac{2Gm}{c^2} \tag{8}$$

we obtain

$$z = \infty. \tag{9}$$

Hence, we see that there is a huge difference between relations (7) and (9) which was not taken into account in [4].

By the General Theory of Relativity the gravitational redshift is a direct consequence of time dilation. The reason is that time flows more slowly near massive objects than at greater distances. Hence, each photon spends some energy to leave a gravitational potential hole corresponding to a given mass object. The reciprocal value of the corresponding frequency is proportional to the speed of flowing of time. Consequently, electromagnetic waves leaving a binary black hole merger have to reduce their frequency and the same property must hold also for gravitational waves that carry away energy.

The following formula expresses the change of frequency of any photon leaving the gravitational field of an isolated black hole at the distance $R > r$ from its center

$$f = f_{\rm em}\sqrt{1 - \frac{r}{R}}. \tag{10}$$

Here r is given by (8), $f_{\rm em}$ stands for the emitted frequency of a photon, and f denotes the detected frequency by a distant observer. From this and the Formula (3) we find the limiting relationship (9) for $R \to r$.

Relations (8) and (10) can also be used to derive that the detected frequency is negligibly changed by the gravitational field at the measurement site, such as Earth, since $r = 9$ mm.

Example 1. *If e.g., $R = 2r$ in Formula (10) (compare with ([4], p. 3)), we observe that*

$$f = \frac{1}{\sqrt{2}} f_{\rm em}.$$

Using the relation for the emitted frequency (3), we obtain the following associated gravitational redshift

$$z = \sqrt{2} - 1 = 0.414. \tag{11}$$

This number is at least four times larger than the observed cosmological redshift from (6). In an analogous manner, we obtain further values given in Table 1 that are larger than (6) as well.

Table 1. Gravitational redshift z of a black hole at the distance $R = nr$ from its center, where $n = 2, 3, 4, 5$ and r is the Schwarzschild radius.

R	$2r$	$3r$	$4r$	$5r$
z	0.414	0.225	0.155	0.118

According to [4], the spacetime between two colliding black holes exhibits very large deformations. It produces gravitational waves with increasing frequency in the interval 35–250 Hz. The distance between these black holes is only a few Schwarzschild radii ([4], p. 3). Applying (8)–(11), we can conclude that the gravitational redshift of the emitted gravitational waves is quite essential and probably greater than that in (6). The exact analytical solution of Einstein's equations for two orbiting black holes is not known. Nevertheless, a common gravitational potential hole of two black holes is deeper than that of each of its components (see Figure 1).

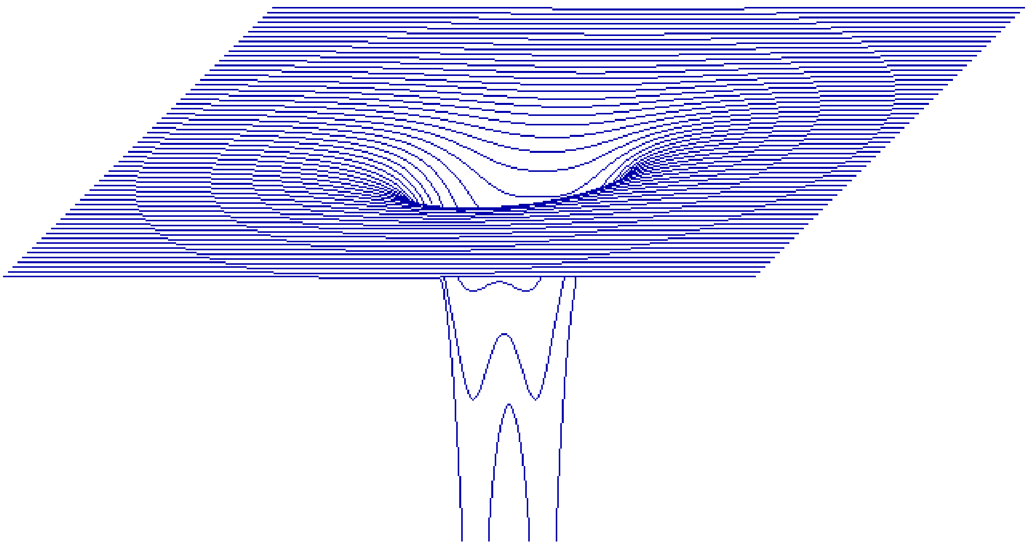

Figure 1. Schematic illustration of the gravitational potential of a binary black hole merger. By ([4], p. 3) the highest amplitude of the detected gravitational waves was reached for the separation $R = \frac{3}{2}r$.

By (2) we see that the sought masses of the black holes and also the constant C appearing in (4) depend nonlinearly on the emitted frequency $f_{\text{em}} = (z+1)f$. Consequently, an exact determination of the total redshift z is important. Using (3), we obtain $\dot{f}_{\text{em}} = (z+1)^2\dot{f}$, where the additional factor $(z+1)$ is due to time dilation as described above. Substituting this and (3) into relation (2), we get the missing factor

$$(z+1), \tag{12}$$

since

$$\left(f^{-11/3}\dot{f}\right)^{3/5} = (z+1)\left(f_{\text{em}}^{-11/3}\dot{f}_{\text{em}}\right)^{3/5} \tag{13}$$

and since the remaining factors in (2) are constants. In the paper [4], only the cosmological redshift (6) was considered, but the total redshift is larger (cf. e.g., (11)).

4. Other Arguments

The mechanism of the origin of two very close stellar black holes with such high masses as in (1) is not known. By the survey paper ([21], Figure 8), all X-ray binaries observed in our Galaxy have components with masses not exceeding $10M_\odot$. Moreover, masses of all known observed (not calculated) single stellar mass black holes are in the interval 5–20 $M_\odot$ (cf. e.g., [22–24]). Therefore, from a statistical point of view the existence of black holes satisfying (1) seems to be quite exceptional, although some selection effects could be present, because larger masses yield stronger signals.

Remark 1. *By [25] the mass of the larger component of the binary black hole corresponding to the event GW190521 was* $85\ M_\odot$ *and the total mass after coalescence was* $150\ M_\odot$*. The evolution path of such a large black hole binary is unknown. It is clear that many of its parameters have to be tuned very finely.*

Remark 2. *There is a large statistically significant mass gap between all known black hole mergers and binary neutron stars, see [26]. This also indicates that the gravitational redshift was ignored.*

Chen et al. [27] have also noticed that the large gravitational redshift of GW150914 should be taken into account. They suppose that this binary black hole merger was located in a close neighborhood of a supermassive black hole producing a high gravitational redshift. Hence, they conclude that the mass of each component is less than $10M_\odot$.

Remark 3. *For the highest detected frequency* $f = 250$ *Hz the associated wavelength* $\lambda = c/f$ *is obviously equal to* $\lambda = 1200$ *km. We see that this is a much larger size than the diameter of the wave zone determined by the corresponding Schwarzschild radii* ≈ 100 *km of the particular black holes (1). However, we see that the emitted frequency* $f_{\rm em} > f$ *would produce a more reliable size of the wave zone. This fact also indicates that the gravitational redshift of GW150914 was neglected.*

Theoretically, a distant observer cannot see the plunging of any mass object into a single black hole due to (9). On the other hand, a collision of one black hole with another takes only a few milliseconds. Is this not strange? Anyway, we should never identify any mathematical model with reality.

The signal from LIGO and VIRGO detectors of gravitational waves is continually corrupted by white noise due to thermal movement of particles, seismic waves, quantum noise, etc. It is remarkable that such noise does not seem to be present in Figure 1 of [4] in a 40 ms long time interval around the maximum amplitude of the detected signal. The white noise can be partially suppressed from the detected signal by means of a wavelet transform which removes components with a small amplitude and high frequencies at the same time (see e.g., [28,29]).

The smoothed signals presented from both the detectors show relatively small third and fourth amplitudes with high frequency appearing after the maximal amplitude, see ([4], p. 3). However, in principle wavelet analysis does not allow one to detect such small amplitudes with high frequencies in noisy data. The question is whether the noise removed from the signals (the so-called residual) from both the detectors is correlated or not. It appeared that it is uncorrelated except for that particular 40 ms long interval, where no noise seems to be present—see [30–32]. In this time interval, the removed noise from both the detectors is correlated. It was enough to shift the noise from the detector in Livingston about 7 ms similarly as in the detected signal. The corresponding figure can be found in [33]. In [4] such an important comparison is missing. The removed noise from the other two detected gravitational waves GW151226 and GW170104 is correlated as well, see [30]. Therefore, such correlated noise could cause a bias in orbital frequencies of the black holes (cf. e.g., (1)).

5. Concluding Remarks

A comprehensive and detailed study of gravitational waves and their role in astrophysics and cosmology is given in [34]. Our hypothesis that masses of binary hole mergers are overestimated is based on the following arguments:

(a) The key formula (2) possesses several essential drawbacks which are described in Section 2.
(b) In the literature on GW, it is not taken into account that the redshift on horizon of each black hole is infinity, compare (7) with (9).
(c) No mechanism is known which would produce binary stellar black holes with masses greater than 50 solar masses, see Remark 1.
(d) There is a large statistically significant mass gap between all known black hole mergers and binary neutron stars, see Remark 2.
(e) Our hypothesis yields a more reliable size of the wave zone than in [4], see Remark 3.

Therefore, something has to be wrong in the current analysis of GW signals. The claim that masses (1) correspond to the collision of two black holes that have merged thus seems to be somewhat too strong. As shown in Section 2, the main reason is that the detected frequency f appearing in (2) has to be replaced by the emitted frequency f_{em}. In other words, the chirp mass given by (2) has to be divided by the missing factor (12).

Remark 4. *From Section 3 one may deduce that the total redshift z should be larger than $\frac{1}{2}$ due to (11) and (6). Therefore, the masses (1) were incorrectly established. To see this, assume for simplicity that*

$$m_1 = m_2. \tag{14}$$

Then from (2) we find that that the associated chirp mass

$$\mathcal{M} = \frac{m_1^{6/5}}{(m_1 + m_1)^{1/5}} = 2^{-1/5} m_1$$

depends linearly on m_1. Therefore, the masses (14) are correspondingly affected by the same redshift. In this special case with $z + 1 \geq \frac{3}{2}$, the masses m_1 and m_2 would be at least 33% smaller.

Remark 5. *The famous Figure 2, which is often shown to demonstrate that General Relativity holds, in fact, contradicts General Relativity. To see this, denote by T the orbital period of the two black holes and by d their coordinate distance. Multiply the trivial inequality*

$$\pi > 2$$

by d/T. Then we immediately get a contradiction

$$v = \frac{\pi d}{T} > \frac{2d}{T} = \frac{|AB|}{T} = c, \tag{15}$$

where v is the orbital velocity, c the speed of gravitational waves (equal to the speed of light), and $|AB|$ is the distance of two consecutive maximum amplitudes of the right black hole as indicated in Figure 2. However, $v \lesssim \frac{1}{3}c$ by [4].

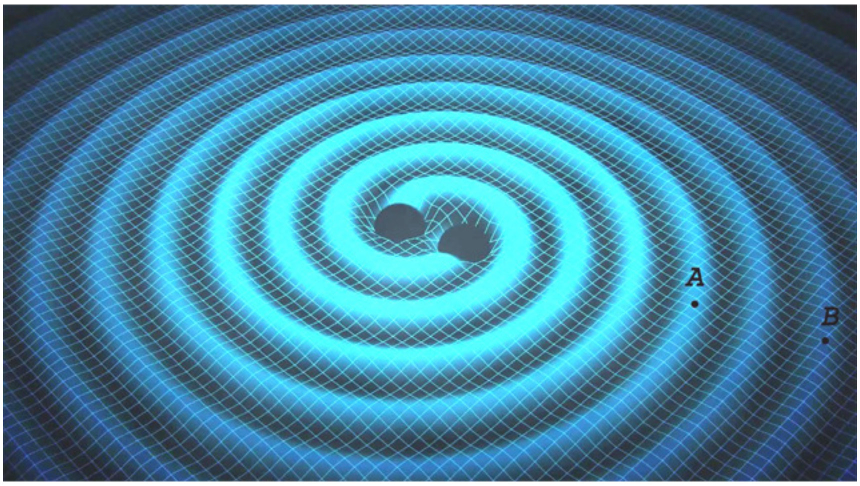

Figure 2. This naive illustration implies that the orbital velocity v of binary black holes is larger than the speed of light c, see (15).

Remark 6. *Another objection to Figure 2 is that the gravitational waves should approximately form a double spiral of Archimedes (see ([35], p. 136)). Such a spiral should leave each black hole in an almost radial direction and not in the tangential direction as illustrated in the center of Figure 2. Moreover, it is easy to see that Figure 2 shows only a dipole and not quadrupole character of gravitational waves. Also the exposed spacetime near the center that produced the largest amplitudes of gravitational waves, should be much more largely deformed. A formula for the amplitude of deformation of the spacetime in the direction orthogonal to the orbital plane is given in ([36], Formula (26)).*

Author Contributions: Conceptualization, M.K. and L.S.; writing—original draft preparation, M.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by RVO 67985840 of the Czech Republic.

Acknowledgments: The authors are indebted to Jan Brandts, Caroline Griffis, and Filip Křížek, for fruitful discussions.

Conflicts of Interest: The authors declare no conflict of interest.

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