

# Differentiability and structure in Banach spaces. Interaction with Marián Fabian

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In the framework of Banach spaces, questions concerning their structure, with emphasis in the nonseparable case, geometry and analysis, differentiability of the norm, and more generally of convex functions on them, renormings, interaction with the topology (weak topologies, compactness,...), optimization, variational principles, and many other topics conform Marian Fabian's mathematical universe. Without the pretension of completeness, I shall review some of the work in which I cooperated with Marian. I shall focuss mainly in the classification of weakly compact sets in Banach spaces and its use in the analysis of the structure of the space of continuous functions on those compacta and of the spaces having special weakly compact sets as generators. Due to the fact that this study is intrinsically blended with differentiability questions, I shall mention some of the results in this direction that we obtained or reproved. Since, in particular, differentiation of the norm, weak compactness and topological and geometrical structure are intimately related, I shall present some of our joint work regarding those relationships. Our favorite tools are projectional resolutions of the identity —acting as the Occam's razor—, biorthogonal systems —a good substitution for a (maybe missing) coordinate system—, and topology. Another satisfactory cooperation has been writing several books together, that shall be mentioned in passing. The talk will be based on the papers in which I had the privilege to participate as a coauthor, included in the list below.

## References

- [1] Fabian, Marián; Habala, Petr; Hájek, Petr; Montesinos, Vicente; Zizler, Václav, Banach space theory. The basis for linear and nonlinear analysis. CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC. Springer, New York, 2011. xiv+820 pp. ISBN: 978-1-4419-7514-0
- [2] Fabian, M.; Montesinos, V.; Zizler, V., Gulko, descriptive, and Gruenhage compact spaces. Rev. R. Acad. Cienc. Exactas Fs. Nat. Ser. A Math. RACSAM 104 (2010), no. 2, 201–220.
- [3] Fabian, M.; Montesinos, V.; Zizler, V., On weak compactness in  $L_1$  spaces. Rocky Mountain J. Math. 39 (2009), no. 6, 1885–1893.

- [4] Fabian, Marián; González, Alma; Montesinos, Vicente, A note on biorthogonal systems in weakly compactly generated Banach spaces. *Ann. Acad. Sci. Fenn. Math.* 34 (2009), no. 2, 555–564.
- [5] Fabian, M.; Montesinos, V.; Zizler, V., Sigma-locally uniformly rotund and sigma-weak\* Kadets dual norms. *Topology Appl.* 156 (2009), no. 7, 1374–1380.
- [6] Fabian, M.; Montesinos, V.; Zizler, V., Sigma-finite dual dentability indices. *J. Math. Anal. Appl.* 350 (2009), no. 2, 498–507.
- [7] Hájek, Petr; Montesinos Santaluca, Vicente; Vanderwerff, Jon; Zizler, Václav, Biorthogonal systems in Banach spaces. *CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC*, 26. Springer, New York, 2008. xviii+339 pp. ISBN: 978-0-387-68914-2
- [8] Fabian, M.; Montesinos, V.; Zizler, V., Weak compactness and -Asplund generated Banach spaces. *Studia Math.* 181 (2007), no. 2, 125–152.
- [9] Fabian, M.; Hájek, P.; Montesinos, V.; Zizler, V., Weakly compact generating and shrinking Markuevi bases. *Serdica Math. J.* 32 (2006), no. 4, 277–288.
- [10] Fabian, Marian; Montesinos, Vicente; Zizler, Václav, Smoothness in Banach spaces. Selected problems. *RACSAM. Rev. R. Acad. Cienc. Exactas Fs. Nat. Ser. A Mat.* 100 (2006), no. 1-2, 101–125.
- [11] Fabian, M.; Hájek, P.; Montesinos, V.; Zizler, V., A quantitative version of Krein’s theorem. *Rev. Mat. Iberoamericana* 21 (2005), no. 1, 237–248.
- [12] Fabian, M.; Montesinos, V.; Zizler, V., Biorthogonal systems in weakly Lindelf spaces. *Canad. Math. Bull.* 48 (2005), no. 1, 69–79.
- [13] Fabian, M.; Godefroy, G.; Montesinos, V.; Zizler, V., Inner characterizations of weakly compactly generated Banach spaces and their relatives. Special issue dedicated to John Horváth. *J. Math. Anal. Appl.* 297 (2004), no. 2, 419–455.
- [14] Fabian, M.; Montesinos, V.; Zizler, V., The Day norm and Gruenhagen compacta. *Bull. Austral. Math. Soc.* 69 (2004), no. 3, 451–456.
- [15] Fabian, M.; Montesinos, V.; Zizler, V., A characterization of subspaces of weakly compactly generated Banach spaces. *J. London Math. Soc.* (2) 69 (2004), no. 2, 457–464
- [16] Fabian, Marián; Montesinos, Vicente; Zizler, Václav, Pointwise semicontinuous smooth norms. *Arch. Math. (Basel)* 78 (2002), no. 6, 459–464.
- [17] Fabian, Marián; Montesinos, Vicente; Zizler, Václav, Weakly compact sets and smooth norms in Banach spaces. *Bull. Austral. Math. Soc.* 65 (2002), no. 2, 223–230.

- [18] Fabian, Marián; Habala, Petr; Hájek, Petr; Montesinos Santalucía, Vicente; Pelant, Jan; Zizler, Václav, Functional analysis and infinite-dimensional geometry. CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, 8. Springer-Verlag, New York, 2001. x+451 pp. ISBN: 0-387-95219-5