Time delay observable in classical and quantum geometries

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Outline

Motivation

If I had a theory of Quantum Gravity, what would I do with it?

- Answer should be independent of QG model.
- My answer: Compute qualitative and quantitative QG corrections to experiments and observations.
- Unfortunately, what is easiest to compute in QG is model dependent may not have a direct experimental interpretation.
- Idea: Work backwards! Start with a potential experiment (even if only in principle possible), described operationally. Construct a mathematical model of it and obtain an observable quantity with an unambiguous interpretation.
- Bonus: Direct comparison of various QG models.

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Observables from (Thought) Experiments

- Which experiments are sensitive to QG effects? All of them!
- However, we do not know which are most sensitive.
- A safe bet is to learn to model all experiments.
- Only when reliable methods for computing QG corrections are available, would it make sense to look for where the largest corrections occur.
- So, let's start with something easy!
  ... And see how it could lead to something interesting.

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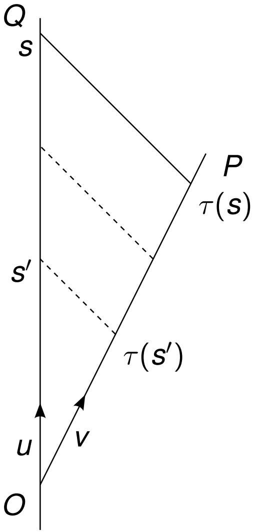
Time Delay Observable

operational definition



- Consider two inertially moving, localized systems: the lab and the probe. Probe is launched from the lab at event O.
- Each carries a proper-time clock. The clocks are synchronized at O.
- The probe broadcasts signals time stamped with the emission time, τ at P.
- The lab records the reception time, s at Q, together with the time stamp $\tau(s)$.
- The time delay

$$\delta\tau(s) = s - \tau(s)$$

is the observable we seek.

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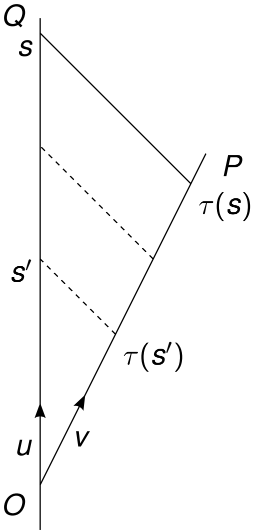

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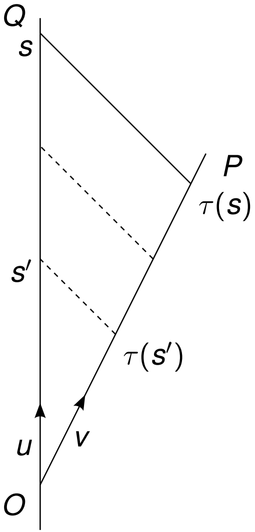

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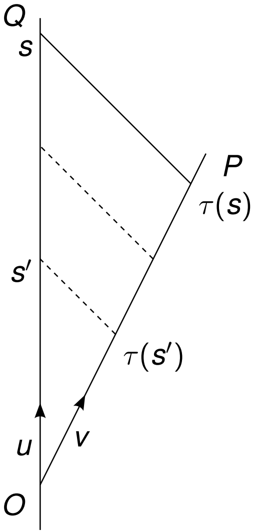

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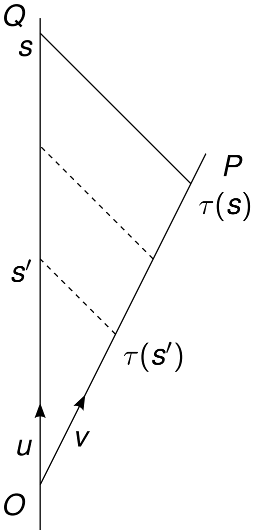

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mathematical model

Definition (Spacetime + Apparatus)

A *lab-equipped spacetime* (M, g, O, e^α) is a (time oriented, globally hyperbolic) Lorentzian manifold (M, g) together with a point $O \in M$ and an orthonormal tetrad $e_i^\alpha \in T_O M$, with e_0^α timelike and future directed.

Definition (Gauge Equivalence)

Two lab-equipped spacetimes (M, g, O, e^α) and (M', g', O', e'^α) are *gauge equivalent* if there exists a diffeomorphism $\phi \colon M \to M'$ such that $\phi_* g = g'$, $\phi(O) = O'$ and $\phi_* e_i^\alpha = e'^\alpha_i$.

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Definition (Observers)

The *lab worldline* is the geodesic tangent to $u^\alpha = e_0^\alpha$ at O. The *probe worldline* is the geodesic tangent to $v^\alpha = v^i e_i^\alpha$ at O, for a timelike, unit, future directed $v^i \in \mathbb{R}^{1,3}$.

Definition (Signal)

For $t > 0$ and $V^\alpha = t v^\alpha$, let $P = \exp_O(V^\alpha)$ and let Q be the intersection of the lab worldline with $E^+(P)$ (future null cone of P).

Definition (Observables)

For a *reception time* $s > 0$ and $U^\alpha = s u^\alpha$, let *emission time* $\tau(s)$ be the largest t such that $Q = \exp_O(U^\alpha)$. The *time delay* is $\delta\tau(s) = s - \tau(s)$.

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diffeomorphism invariance and causal bounds

Theorem (Gauge Invariance)

Given two gauge equivalent, lab-equipped spacetimes (M, g, O, e^α) *and* (M', g', O', e'^α)*, the respective time delays* $\delta\tau$ *and* $\delta\tau'$ *(keeping* s *and* v^i *fixed) are equal.*

Proof.
By construction. □

Remark:

- The time delay obeys interesting inequalities, which probe the causal structure of classical Lorentzian manifolds.

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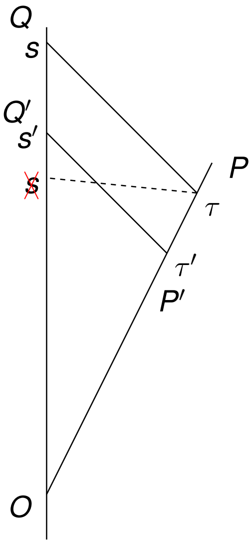
causal bounds: speed of light



Theorem (Maximality of Light Speed)

In a Lorentzian geometry

$$\tau' < \tau \implies s' < s.$$

In particular, if $\tau(s)$ is smooth, then $\dot\tau(s) > 0$.

Proof.

$P \in I^+(P')$, $Q \in J^+(P) \implies Q \in \operatorname{int} J^+(P')$.

[Hawking & Ellis, Proposition 4.5.10] □

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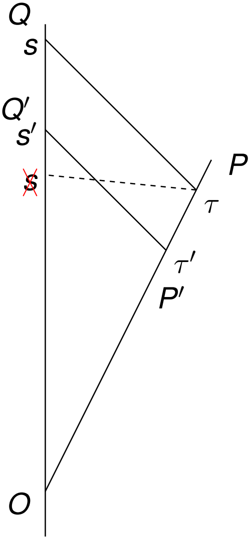

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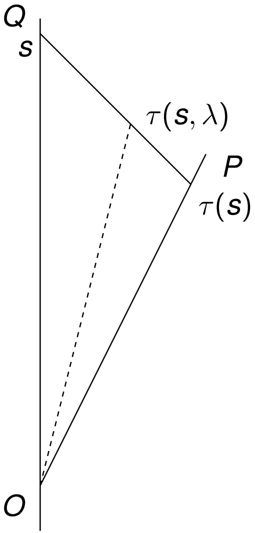

Theorem (Local Geodesic Extremality)

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$$\tau(s) \leq s \quad (\textit{or} \;\; \delta\tau(s) \geq 0).$$

Proof.

Adapt the formula for the first variation of the length of piecewise geodesics to show $\frac{\partial}{\partial\lambda}\tau(s,\lambda) < 0$ and write

$$\tau(s) = s + \int_0^1 \mathrm{d}\lambda \, \frac{\partial}{\partial\lambda}\tau(s,\lambda),$$

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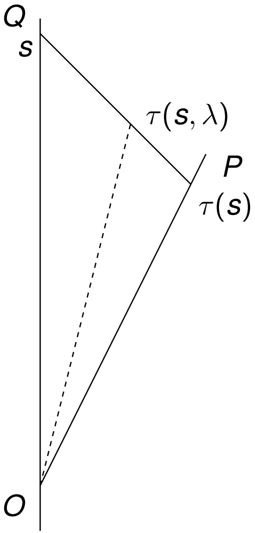

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Time Delay in Linearized Gravity

- Linearization about Minkowski space: $g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$.
- Explicit expression for $\tau(s)$ at $O(h)$ is available:

$$\begin{aligned}\tau(s) &= \tau[\eta](s) + \tau_1[h](s) + \cdots \\ &= se^{-\theta}(1 + r[h] + \cdots) \\ r[h] &= r^x h_x = H + J\end{aligned}$$

  - θ—rapidity, $v_{\mathrm{rel}} = \tanh(\theta)$
  - r^x—integro-differential operator
  - H, J—separately invariant under linearized diffeomorphisms that fix O and e^α
- Note: H, J, $\ldots$ may have been found by brute force, but it would not have been obvious how these invariants would combine into an observable with direct phenomenological interpretation

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- Quantization: $g_{\mu\nu} \to \eta_{\mu\nu} + \hat{h}_{\mu\nu}$, fix gauge, Fock space.
- Quantum model of experiment: add apparatus physical degrees freedom, take limit where irrelevant internal dynamics and back reaction on the geometry become negligible.
  Use ideas of [Page & Wootters (1983)] and [Gambini & Pullin (2009)].
- Time delay observable: $\tau(s) \to \hat{\tau}(s) = se^{-\theta}(1 + r[\hat{h}] + \cdots)$.
- The correction $r[\hat{h}]$ is invariant under linearized diffeomorphisms fixing O and e^α.
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- Expectation values $\langle 0|F(r[\hat{h}])|0\rangle$ are independent of gauge and of choice of O and e^α.

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Causal Bounds in Quantum Linearized Gravity

Lorentzian signature

- For a classical, everywhere small h, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ is Lorentzian.

- For h arbitrary, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ may be not Lorentzian:
  causal bounds may not hold!

- Spectral Density of state ψ with respect to observable $\hat{A}$

$$P_\psi(a) = \langle\psi|\delta(\hat{A} - a)|\psi\rangle$$

  - $\hat{A}$—observable, operator on a Hilbert space
  - $|\psi\rangle$—state, element of a Hilbert space
  - $\delta(\hat{A} - a)$—spectral projection

- For linear perturbations, P_ψ is Gaussian with respect to $\hat{h}_{\mu\nu}(x)$ for Fock vacuum and thermal states.

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- For linear perturbations, P_ψ is Gaussian with respect to $\hat{h}_{\mu\nu}(x)$ for Fock vacuum and thermal states.

Causal Bounds in Quantum Linearized Gravity

Gaussian states

- The correction $r[\hat{h}]$ is linear in $\hat{h}$. Therefore, the spectral density of the Fock vacuum $|0\rangle$ with respect to $r[\hat{h}]$ is a Gaussian.

$$\hat{\tau}(s) = se^{-\theta}(1 + r[\hat{h}] + \cdots), \quad P(\tau) = \langle 0|\delta(\hat{\tau}(s) - \tau)|0\rangle$$

- Causal bounds on the spectrum of $\hat{\tau}(s)$ are generically violated!
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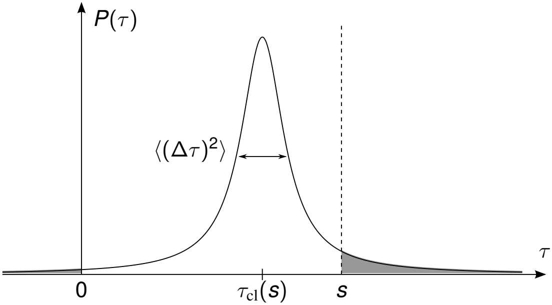

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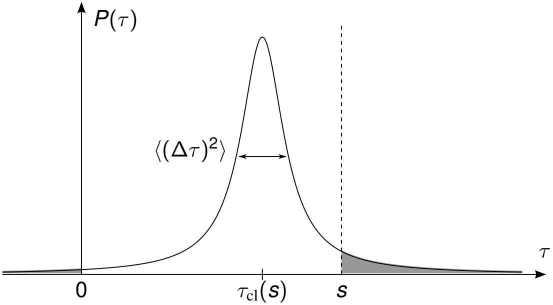

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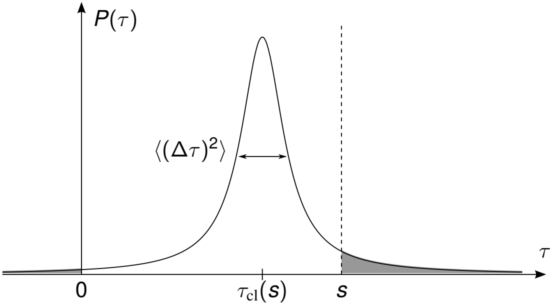

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"Causality Violation" in Quantum Gravity

- Speculation, proposals, phenomenology over the years:
  - space-time foam
  - generalized uncertainty principles
  - non-locality
  - discreteness
- Almost no direct investigation without presupposing the answer.
- Notable exception: light cone fluctuations in quantum linearized gravity [Ford *et al* (1995,1999,2005,2006)] [Roura & Arteaga (priv. comm.)]
- Ford's work has many loose ends: gauge invariance, regularization, linearization artifacts.
  This work and independent work by Roura & Arteaga aims to clarify these issues.
- Lorentz signature violation in naïve perturbation theory obscures possible quantum causality violation, or lack thereof.
- What other QG model + approximation method is available?

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Explicit Calculation

basic idea

- What to calculate?

$$\langle 0|\hat{\tau}(s)|0\rangle = \langle 0|se^{-\theta}(1 + r[\hat{h}] + \cdots)|0\rangle = se^{-\theta}(1 + 0 + \cdots)$$

- $P(\tau)$ is Gaussian. Need only compute variance to know full spectral density.

$$\langle(\Delta\tau)^2\rangle = s^2 e^{-2\theta}(\langle 0|r[\hat{h}]^2|0\rangle + \cdots)$$

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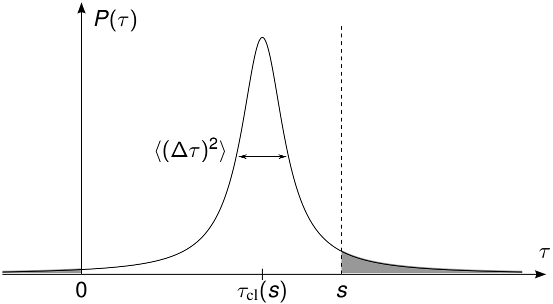

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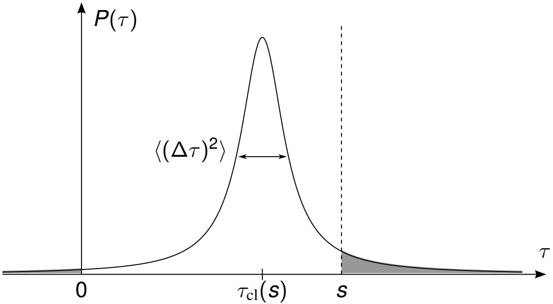

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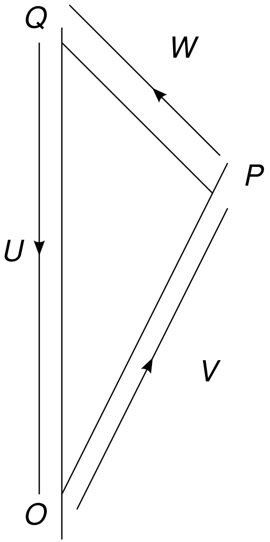
Time Delay in Quantum Linearized Gravity

explicit expressions



$$r^x h_x = H + J$$

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$$H \sim \sum_{X=V,U,W} (\longrightarrow)_X,$$

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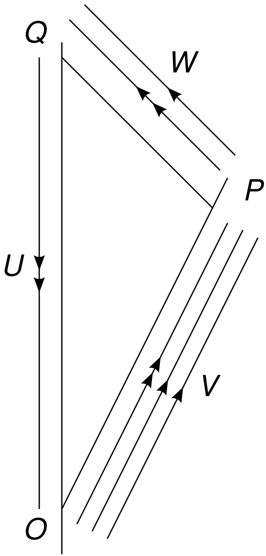

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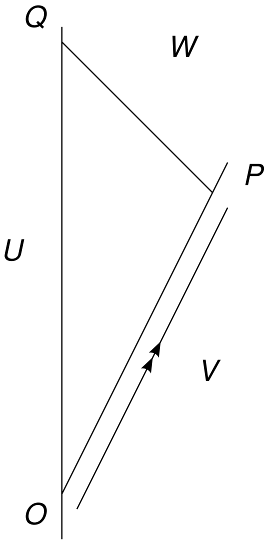

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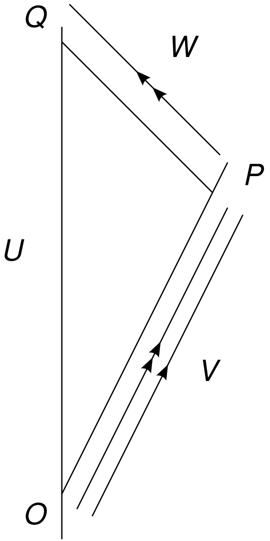

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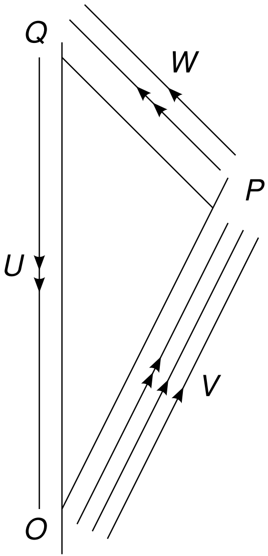

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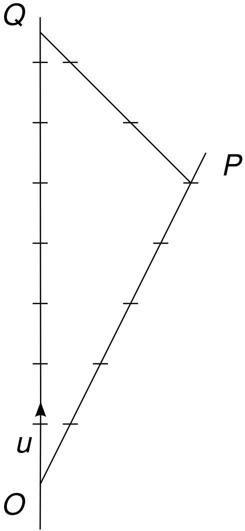
Smearing and Detector Resolution



- In QED, $\langle E(x)^2 \rangle$ diverges, but $\langle \tilde{E}(x)^2 \rangle$ is finite and represents the vacuum noise in a detector of sensitivity profile $\tilde{g}(x)$.
- Physically speaking, μ, the spread of $\tilde{g}(x)$, is the spatial resolution of the detector.
- We can back-of-envelope estimate μ as the wavelength of the light/radio signals exchanged between lab and probe.
- A more detailed detector model should unambiguously fix $\tilde{g}(x)$ for each leg of $\triangle OPQ$.
- Provisionally, set $\tilde{g}(x) \sim \delta(u \cdot x) g(x_\perp^2)$ everywhere.

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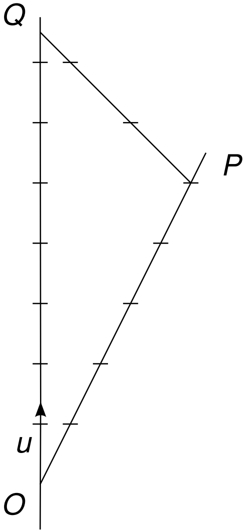

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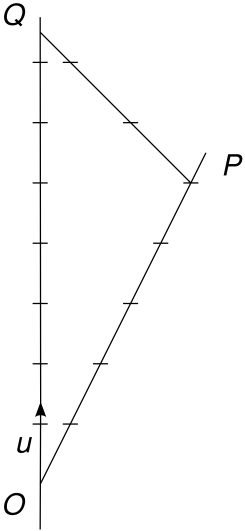

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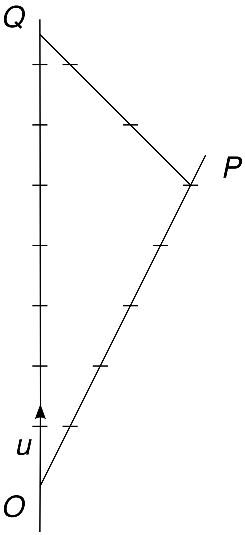

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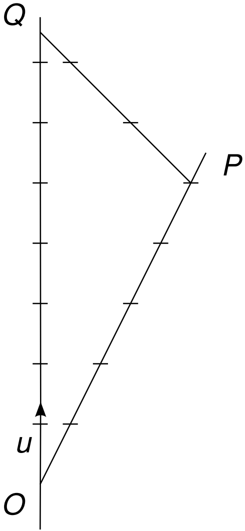

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Explicit Calculation

computational strategy

- Start with $r^x = \sum_{|K|mX} r^K_{mX} \int_X^{(m)} \nabla_K$ and conclude that

$$\langle \tilde{r}^2 \rangle = \sum_{|K|mX} \sum_{|L|nY} r^K_{mX} r^L_{nY} \tilde{I}^{mn}_{K\cup L}(X;Y),$$

$$\tilde{I}^{mn}_K(X;Y) = \langle\!\langle \nabla_K I^{mn}(X;Y+z) \rangle\!\rangle$$

$$I^{mn}(X;Y) = \int_X^{(m)} \mathrm{d}\sigma \int_Y^{(n)} \mathrm{d}\tau\, G(x(\sigma), y(\tau))$$

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- What is the leading order in μ and $\ln(s/\mu)$?

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Explicit Calculation

dimensional analysis

- Dimensional analysis: $[\mu] = [\ell_p] = [z] = 1$, $[\nabla] = -1$, $G \sim \ell_p^2/z^2$.

$$\langle \tilde{r}^2 \rangle \sim s^2 \langle\langle \int_X \int_Y \nabla^2 G(z) \rangle\rangle \sim s^2 \langle\langle \int_X \int_Y \frac{\ell_p^2}{z^4} \rangle\rangle$$

$$\sim s^2 \langle\langle \frac{\ell_p^2}{s^2 z^2} \rangle\rangle \sim \frac{\ell_p^2}{\mu^2}$$

- Detailed calculations confirm the structure

$$\langle \tilde{r}^2 \rangle = \frac{\ell_p^2}{\mu^2} \left(\rho_0 + \rho_1 \frac{\mu}{s} + \rho_2 \frac{\mu^2}{s^2} + O\left(\frac{\mu}{s}\right)^3 \right) + O\left(\frac{\ell_p^2}{\mu^2}\right)$$

- rms fluctuation in $\hat{\tau}(s) \sim s\sqrt{\langle \tilde{r}^2 \rangle} \sim s\frac{\ell_p}{\mu}$, $\mu \sim 1$nm (X-rays)

laboratory: $s \sim 1\text{m} \sim 10^{-9}\text{s}$, $\frac{\ell_p}{\mu} \sim 10^{-26}$: 10^{-35}s

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cosmology: $s \sim 1\text{Mpc} \sim 10^{14}\text{s}$, $\frac{\ell_p}{\mu} \sim 10^{-26}$: 10^{-12}s

Explicit Calculation

dimensional analysis

- Dimensional analysis: $[\mu] = [\ell_p] = [z] = 1$, $[\nabla] = -1$, $G \sim \ell_p^2/z^2$.

$$\langle \tilde{r}^2 \rangle \sim s^2 \langle\!\langle \int_X \int_Y \nabla^2 G(z) \rangle\!\rangle \sim s^2 \langle\!\langle \int_X \int_Y \frac{\ell_p^2}{z^4} \rangle\!\rangle$$
$$\sim s^2 \langle\!\langle \frac{\ell_p^2}{s^2 z^2} \rangle\!\rangle \sim \frac{\ell_p^2}{\mu^2}$$

- Detailed calculations confirm the structure

$$\langle \tilde{r}^2 \rangle = \frac{\ell_p^2}{\mu^2} \left(\rho_0 + \rho_1 \frac{\mu}{s} + \rho_2 \frac{\mu^2}{s^2} + O\left(\frac{\mu}{s}\right)^3 \right) + O\left(\frac{\ell_p^2}{\mu^2}\right)$$

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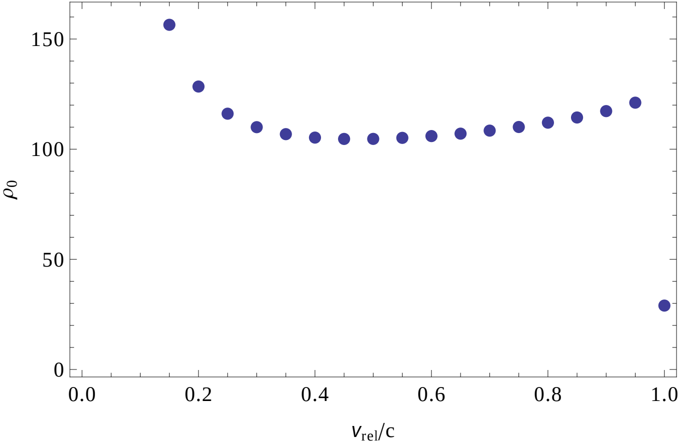
Explicit Calculation

result [B. Bonga, MSc thesis]

All dependence on geometry (relative lab-probe speed v_{rel}/c) is in the coefficients $\rho_0, \rho_1, \ldots$



The second coefficient is just $\rho_1 = -2\pi^2$.

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- Observables in Quantum Gravity can be constructed by carefully modeling (thought) experiments.
- The time delay is an interesting example, especially sensitive to the causal structure of spacetime.
- The time delay is only one member of a larger class: *Quantum Astrometric Observables*.
- Question: Are non-perturbative calculations possible? (improved perturbation theory, $2+1$, CDT, ...)
- Question: Is minimal matter content necessary? (astrometry + Komar-Bergmann observables)
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Thank you for your attention!