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Distributed computation of supremal conditionally-controllable sublanguages

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In this paper, we further develop the coordination control framework for discrete-event systems with both complete and partial observations. First, a weaker sufficient condition for the computation of the supremal conditionally-controllable sublanguage and conditionally-normal is presented. Then we show that this condition can be imposed by synthesizing a posteriori supervisors. The paper further generalizes the previous study by considering general, non-prefix-closed languages. Moreover, we prove that for prefix-closed languages the supremal conditionally-controllable sublanguage and conditionally-normal sublanguage can always be computed in the distributed way without any restrictive conditions we have used in the past.

Keywords: discrete-event systems; supervisory control; coordination control

1. Introduction

Large scale discrete-event systems (DES) are often formed in a compositional way as a synchronous or asynchronous composition of smaller components, typically automata (or 1-safe Petri nets that can be viewed as products of automata). Supervisory control theory was proposed in Ramadge and Wonham (1989) for automata as a formal approach that aims to solve the safety issue and nonblockingness.

A major issue is the computational complexity of the centralized supervisory control design, because the global system has an exponential number of states in the number of components. Therefore, a modular supervisory control of DES based on a compositional (local) control synthesis has been introduced and developed by many authors. Structural conditions have been derived for the local control synthesis to equal the global control synthesis in the case of both local and global specification languages.

Specifications are mostly defined over the global alphabet, which means that the global specifications are more relevant than the local specifications. However, several restrictive conditions have to be imposed on the modular plant such as mutual controllability (and normality) of local plant languages for maximal permissiveness of modular control, and other conditions are required for nonblockingness.

For that reason, a coordination control approach was proposed for modular DES in Komenda and van Schuppen (2008) and further developed in Komenda, Masopust, and van Schuppen (2012b). Coordination control can be seen as a reasonable trade-off between a purely modular control synthesis, which is in some cases unrealistic, and a global control synthesis, which is naturally prohibitive for high complexity reasons. The concept of a coordinator is useful for both safety and nonblockingness. The complete supervisor then consists of the coordinator, its supervisor, and the local supervisors for the subsystems. In Komenda and van Schuppen (2008), necessary and sufficient conditions are formulated for nonblockingness and safety, and a sufficient condition is formulated for the maximally permissive control synthesis satisfying a global specification using a coordinator. Later, in Komenda et al. (2012b), a procedure for a distributive computa-

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tion of the supremal conditionally controllable sublanguage of a given specification has been proposed. We have extended coordination control for non-prefix-closed specification languages in Komenda, Masopust, and van Schuppen (2014) and for partial observations in Komenda, Masopust, and van Schuppen (2011b).

In this paper, we first propose a new sufficient condition for a distributed computation of the supremal conditionally controllable sublanguage. We prove that it generalizes (is weaker than) both conditions we have introduced earlier in Komenda et al. (2014) and Komenda et al. (2012b). Then we revise (simplify) the concepts of conditional observability and conditional normality and present new sufficient conditions for a distributive computation of the supremal conditionally controllable and conditionally normal sublanguage. Finally, we propose and we prove that surprisingly .

The paper is organized as follows. The next section recalls the basic concepts from the algebraic language theory that are needed in this paper. Our coordination control framework is briefly recalled in Section 3. In Section 3.1, new results in coordination control with complete observations are presented: a new, weaker, sufficient condition for distributed computation of supremal conditionally controllable sublanguages. Section 3.2 is dedicated to coordination control with partial observations, where the main concepts are first simplified and then weaker sufficient conditions are presented for distributed computation of supremal conditionally controllable and conditionally normal sublanguages. In section 4, we propose a posteriori supervisors that impose these sufficient conditions, and we prove that this is without altering the maximal permissiveness in the prefix-closed case, i.e. supremal conditionally controllable and conditionally normal sublanguages can always be computed in a distributed way in the prefix-closed case. Concluding remarks are given in Section 5.

2. Preliminaries

We now briefly recall the elements of supervisory control theory. The reader is referred to Cassandras and Lafortune (2008) for more details. Let Σ be a finite nonempty set of *events*, and let Σ^* denote the set of all finite words (strings) over Σ. The *empty word* is denoted by ε. Let $|\Sigma|$ denote the cardinality of Σ.

A *generator* is a quintuple $G = (Q,\Sigma, f, q_0, Q_m)$, where Q is a finite nonempty set of *states*, Σ is an *event set*, $f : Q\times\Sigma \to Q$ is a *partial transition function*, $q_0 \in Q$ is the *initial state*, and $Q_m \subseteq Q$ is the set of *marked states*. In the usual way, the transition function f can be extended to the domain $Q\times\Sigma^*$ by induction. The behavior of G is described in terms of languages. The language *generated* by G is the set $L(G) = \{s \in \Sigma^* \mid f(q_0,s) \in Q\}$ and the language *marked* by G is the set $L_m(G) = \{s \in \Sigma^* \mid f(q_0,s) \in Q_m\} \subseteq L(G)$.

A *(regular) language* L over an event set Σ is a set $L \subseteq \Sigma^*$ such that there exists a generator G with $L_m(G) = L$. The prefix closure of L is the set $\overline{L} = \{w \in \Sigma^* \mid \text{there exists } u \in \Sigma^* \text{ such that } wu \in L\}$; L is *prefix-closed* if $L = \overline{L}$.

A *(natural) projection* $P : \Sigma^* \to \Sigma_o^*$, for some $\Sigma_o \subseteq \Sigma$, is a homomorphism defined so that $P(a) = \varepsilon$, for $a \in \Sigma\setminus\Sigma_o$, and $P(a) = a$, for $a \in \Sigma_o$. The *inverse image* of P, denoted by $P^{-1} : \Sigma_o^* \to 2^{\Sigma^*}$, is defined as $P^{-1}(s) = \{w \in \Sigma^* \mid P(w) = s\}$. The definitions can naturally be extended to languages. The projection of a generator G is a generator $P(G)$ whose behavior satisfies $L(P(G)) = P(L(G))$ and $L_m(P(G)) = P(L_m(G))$.

A *controlled generator* is a structure (G,Σ_c,P,Γ), where G is a generator over Σ, $\Sigma_c \subseteq \Sigma$ is the set of *controllable events*, $\Sigma_u = \Sigma\setminus\Sigma_c$ is the set of *uncontrollable events*, $P : \Sigma^* \to \Sigma_o^*$ is the projection, and $\Gamma = \{\gamma \subseteq \Sigma \mid \Sigma_u \subseteq \gamma\}$ is the *set of control patterns*. A *supervisor* for the controlled generator (G,Σ_c,P,Γ) is a map $S : P(L(G)) \to \Gamma$. A *closed-loop system* associated with the controlled generator (G,Σ_c,P,Γ) and the supervisor S is defined as the smallest language $L(S/G) \subseteq \Sigma^*$ such that (i) $\varepsilon \in L(S/G)$ and (ii) if $s \in L(S/G)$, $sa \in L(G)$, and $a \in S(P(s))$, then $sa \in L(S/G)$. The marked behavior of the closed-loop system is defined as $L_m(S/G) = L(S/G)\cap L_m(G)$.

Let G be a generator over Σ, and let $K \subseteq L_m(G)$ be a specification. The aim of supervisory control theory is to find a nonblocking supervisor S such that $L_m(S/G) = K$. The nonblockingness means that $\overline{L_m(S/G)} = L(S/G)$, hence $L(S/G) = \overline{K}$. It is known that such a supervisor exists if and only if K is (i) *controllable* with respect to $L(G)$ and Σ_u, that is $\overline{K}\Sigma_u \cap L \subseteq \overline{K}$, (ii) $L_m(G)$-*closed*, that is $K = \overline{K}\cap L_m(G)$, and (iii) *observable* with respect to $L(G)$, Σ_o, and Σ_c, that is for all $s \in \overline{K}$ and $\sigma \in \Sigma_c$, ($s\sigma \notin \overline{K}$) and ($s\sigma \in L(G)$) imply that $P^{-1}[P(s)]\sigma \cap \overline{K} = \emptyset$, where $P : \Sigma^* \to \Sigma_o^*$, cf. Cassandras and Lafortune (2008).

The synchronous product (parallel composition) of languages $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$ is defined by $L_1 \| L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2) \subseteq \Sigma^*$, where $P_i : \Sigma^* \to \Sigma_i^*$, for $i = 1,2$, are projections to local event sets. In terms of generators, see Cassandras and Lafortune (2008) for more details, it is known that $L(G_1 \| G_2) = L(G_1) \| L(G_2)$ and $L_m(G_1 \| G_2) = L_m(G_1) \| L_m(G_2)$.

3. Coordination Control Framework

A language $K \subseteq (\Sigma_1 \cup \Sigma_2)^*$ is *conditionally decomposable* with respect to event sets Σ_1, Σ_2, and Σ_k, where $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$, if $K = P_{1+k}(K) \parallel P_{2+k}(K)$, where $P_{i+k} : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_i \cup \Sigma_k)^*$ is a projection, for $i = 1,2$. Note that Σ_k can always be extended so that the language K becomes conditionally decomposable. A polynomial algorithm how to compute an extension can be found in Komenda, Masopust, and van Schuppen (2012a). However, to find the minimal extension is NP-hard Komenda et al. (2014).

Now we recall the coordination control problem that is further developed in this paper.

Problem 1 (Coordination control problem)**:** Consider generators G_1 and G_2 over Σ_1 and Σ_2, respectively, and a generator G_k (called a *coordinator*) over Σ_k with $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$. Assume that a specification $K \subseteq L_m(G_1 \| G_2 \| G_k)$ and its prefix-closure $\overline{K}$ are conditionally decomposable with respect to event sets Σ_1, Σ_2, and Σ_k. The aim of coordination control is to determine nonblocking supervisors S_1, S_2, and S_k for respective generators such that $L_m(S_k/G_k) \subseteq P_k(K)$ and $L_m(S_i/[G_i \| (S_k/G_k)]) \subseteq P_{i+k}(K)$, for $i = 1,2$, and the closed-loop satisfies $L_m(S_1/[G_1 \parallel (S_k/G_k)]) \parallel L_m(S_2/[G_2 \parallel (S_k/G_k)]) = K$.

Recall that coordinator for safety can be defined as a projected plant to the coordinator alphabet and can be computed in the distributed way as $G_k = P_k(G_1) \parallel P_k(G_2)$, cf. Komenda et al. (2012b, 2014). This choice of coordinator implies that $G_1 \| G_2 \| G_k = G_1 \| G_2$, because for any language $L \subseteq \Sigma^*$ we have that $L \parallel P_k(L) = L$. However, other choices of a coordinator are also possible, notably a coordinator for nonblockingness of Komenda et al. (2014).

3.1 *Coordination Control with Complete Observations*

Conditional controllability introduced in Komenda and van Schuppen (2008) and further developed and studied in Komenda, Masopust, and van Schuppen (2011a); Komenda et al. (2011b, 2012b, 2014) plays the central role in coordination control. In what follows, we use the notation $\Sigma_{i,u} = \Sigma_i \cap \Sigma_u$ to denote the set of uncontrollable events of the event set Σ_i for $i = 1,2$.

We point out that although only two subsystems are considered, all concepts and results can be extended to an arbitrary number of components. For instance, coordinator alphabet Σ_k then should contain all events common to two or more subsystems. Since there might be too many events in the coordinator alphabet for systems with a large number of local automata, it is better for large systems to organize local automata into groups and apply coordination control separately in smaller groups, which might require additional coordination of groups at a higher level as we have proposed in Komenda, Masopust, and van Schuppen (2013).

Definition 1 (Conditional controllability)**:** Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1 \| G_2 \| G_k)$ is *conditionally controllable* with respect to generators G_1, G_2, G_k and uncontrollable event sets $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$ if

(1) $P_k(K)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$,
(2) $P_{1+k}(K)$ is controllable with respect to $L(G_1) \parallel \overline{P_k(K)}$ and $\Sigma_{1+k,u}$,
(3) $P_{2+k}(K)$ is controllable with respect to $L(G_2) \parallel \overline{P_k(K)}$ and $\Sigma_{2+k,u}$,

where $\Sigma_{i+k,u} = (\Sigma_i \cup \Sigma_k) \cap \Sigma_u$, for $i = 1,2$.

The supremal conditionally controllable sublanguage always exists and equals to the union of all conditionally controllable sublanguages Komenda et al. (2014). Let

$$\sup \mathrm{cC} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$$

denote the supremal conditionally controllable sublanguage of K with respect to $L = L(G_1 \| G_2 \| G_k)$ and sets of uncontrollable events $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$. The problem is now reduced to determining how to compute the supremal conditionally-controllable sublanguage.

Consider the setting of Problem 1 and define the languages

$$\boxed{\begin{aligned} \sup \mathrm{C}_k &= \sup \mathrm{C}(P_k(K), L(G_k), \Sigma_{k,u}) \\ \sup \mathrm{C}_{1+k} &= \sup \mathrm{C}(P_{1+k}(K), L(G_1) \| \overline{\sup \mathrm{C}_k}, \Sigma_{1+k,u}) \\ \sup \mathrm{C}_{2+k} &= \sup \mathrm{C}(P_{2+k}(K), L(G_2) \| \overline{\sup \mathrm{C}_k}, \Sigma_{2+k,u}) \end{aligned}} \tag{1}$$

where $\sup \mathrm{C}(K, L, \Sigma_u)$ denotes the supremal controllable sublanguage of K with respect to L and Σ_u, see Cassandras and Lafortune (2008) for more details and algorithms. We have shown that $P_k(\sup \mathrm{C}_{i+k}) \subseteq \sup \mathrm{C}_k$ always holds, for $i = 1,2$, and that if the converse inclusion holds, we can compute the supremal conditionally-controllable sublanguage in a distributed way.

Theorem 1: *Komenda et al. (2014) Consider the setting of Problem 1 and languages defined in (1). If* $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$*, for* $i = 1,2$*, then* $\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$*, where* $L = L(G_1 \| G_2 \| G_k)$*.* □

We can now further improve this result by introducing a weaker condition for nonconflicting supervisors. Recall that two languages L_1 and L_2 are *nonconflicting* if $\overline{L_1 \| L_2} = \overline{L_1} \| \overline{L_2}$.

Theorem 2: *Consider the setting of Problem 1 and languages defined in (1). Assume that* $\sup \mathrm{C}_{1+k}$ *and* $\sup \mathrm{C}_{2+k}$ *are nonconflicting. If* $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ *is controllable with respect to* $L(G_k)$ *and* $\Sigma_{k,u}$*, then* $\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$*, where* $L = L(G_1 \| G_2 \| G_k)$*.*

Proof. Let $\sup \mathrm{cC} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$ and $M = \sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k}$. To prove $M \subseteq \sup \mathrm{cC}$, we show that $M \subseteq P_{1+k}(K) \parallel P_{2+k}(K) = K$ (by conditional decomposability) is conditionally controllable with respect to G_1, G_2, G_k and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$. However, $P_k(M) = P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ (by Lemma 5) is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$ by the assumption. Furthermore, $P_{1+k}(M) = \sup \mathrm{C}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})$ implies that $\sup \mathrm{C}_{1+k} \parallel P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k}) = \sup \mathrm{C}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k}) = P_{1+k}(M)$. Thus, $P_{1+k}(M) = \sup \mathrm{C}_{1+k} \parallel [P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})]$ is controllable with respect to $[L(G_1) \| \overline{\sup \mathrm{C}_k}] \parallel \overline{P_k(M)} = L(G_1) \parallel \overline{P_k(M)}$ by Lemma 2 (because nonconflictingness of $\sup \mathrm{C}_{1+k}$ and $\sup \mathrm{C}_{2+k}$ implies nonconflictingness of $\sup \mathrm{C}_{1+k}$ and $P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})$) and by the fact that $P_k^{i+k}(\sup \mathrm{C}_{i+k}) \subseteq \sup \mathrm{C}_k$, for $i = 1,2$, cf. Komenda et al. (2014). Similarly for $P_{2+k}(M)$, hence $M \subseteq \sup \mathrm{cC}$.

To prove the opposite inclusion, it is sufficient to show by Lemma 6 that $P_{i+k}(\sup \mathrm{cC}) \subseteq \sup \mathrm{C}_{i+k}$, for $i = 1,2$. To prove this note that $P_{1+k}(\sup \mathrm{cC})$ is controllable with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cC})}$ and $\Sigma_{1+k,u}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cC})}$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{C}_k}$ and $\Sigma_{1+k,u}$ (by Lemma 2) because $P_k(\sup \mathrm{cC})$ being controllable with respect to $L(G_k)$ is also controllable with respect to $\overline{\sup \mathrm{C}_k} \subseteq L(G_k)$. By the transitivity of controllability (Lemma 4), $P_{1+k}(\sup \mathrm{cC})$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{C}_k}$ and $\Sigma_{1+k,u}$, which implies that $P_{1+k}(\sup \mathrm{cC}) \subseteq \sup \mathrm{C}_{1+k}$. The other case is analogous, hence $\sup \mathrm{cC} \subseteq M$ and the proof is complete. □

It should be stated here that checking nonconflictingness is computationally demanding as we discuss at the end of Section 4. However, we have shown how to construct a coordinator for nonblockingness in Komenda et al. (2014) based on abstraction satisfying the observer property. The coordinator for non-blockingness imposes the nonconflictingness on an appropriate subalphabet and, by the observer property, the nonconflictingness also holds on the whole alphabet. This coordinator can also be used here, but the goal of the paper is to leave out all restrictive conditions in the prefix-closed case, which is done at Section 4. On the way to this goal we first notice that the controllability condition of Theorem 2 is weaker than to require that $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$, for $i = 1,2$.

Proposition 1: *If* $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$*, for* $i = 1,2$*, then* $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ *is controllable with respect to* $L(G_k)$ *and* $\Sigma_{k,u}$*.*

Proof. This is obvious, because due to the converse inclusion being always true we have that $P_k(\sup \mathrm{C}_{i+k}) = \sup \mathrm{C}_k$, for $i = 1,2$. Hence, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \sup \mathrm{C}_k$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$ by definition of $\sup \mathrm{C}_k$. □

Using the example from Komenda et al. (2014) we can now show that there are languages such that $\sup \mathrm{C}_k \not\subseteq P_k(\sup \mathrm{C}_{i+k})$, but such that $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Example 1: Let G_1 and G_2 be generators and K be the language of the generator shown in Figure 1. Let $\Sigma_c = \{a_1, a_2, c\}$ and $\Sigma_k = \{a_1, a_2, c, u\}$. Let the coordinator $G_k = P_k(G_1) \parallel P_k(G_2)$. Then K is conditionally decomposable, $\sup \mathrm{C}_k = \overline{\{a_1a_2, a_2a_1\}}$, $\sup \mathrm{C}_{1+k} = \overline{\{a_2a_1u_1\}}$, $\sup \mathrm{C}_{2+k} = \overline{\{a_1a_2u_2\}}$, and $\sup \mathrm{C}_k \not\subseteq P_k(\sup \mathrm{C}_{i+k})$. However, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \{\varepsilon\}$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$. ◁

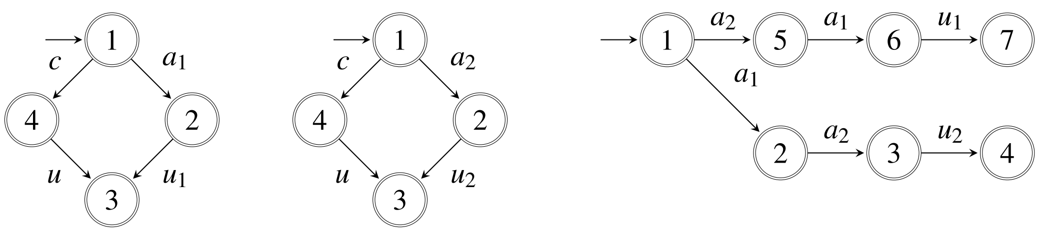

Figure 1. Generators G_1 and G_2 and specification K.

On the other hand, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is not always controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Example 2: Let G_1 and G_2 be generators and K be the language of the generator shown in Figure 2. Let $\Sigma_c = \{a, c_1, c_2\}$ and $\Sigma_k = \{a, b\}$. Let the coordinator $G_k = P_k(G_1) \parallel P_k(G_2)$. Then the language K is conditionally decomposable, $\sup \mathrm{C}_k = \overline{\{b\}}$, $\sup \mathrm{C}_{1+k} = \overline{\{c_1 b\}}$, $\sup \mathrm{C}_{2+k} = \{\varepsilon\}$, and $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \{\varepsilon\}$ is not controllable with respect to $L(G_k) = \overline{\{ab, b\}}$ and $\Sigma_{k,u} = \{b\}$. ◁

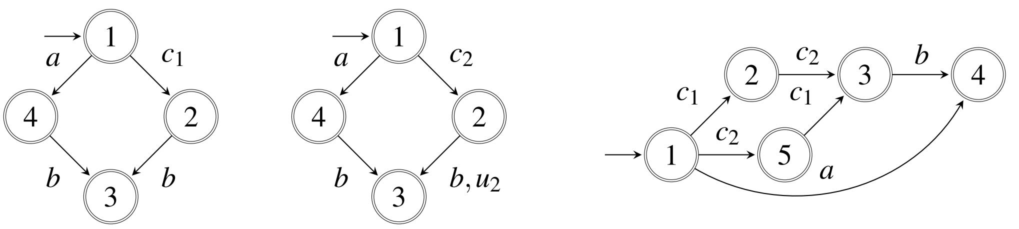

Figure 2. Generators G_1 and G_2 and specification K.

Recall that it is still an open problem how to compute the supremal conditionally-controllable sublanguage for a general, non-prefix-closed language.

The following conditions were required in Komenda et al. (2012b) to prove the main result for prefix-closed languages. We recall the result here and show that the previous condition is a weaker condition than the one required in Komenda et al. (2012b).

The projection $P : \Sigma^* \to \Sigma_0^*$, where $\Sigma_0 \subseteq \Sigma$, is an *L-observer* for $L \subseteq \Sigma^*$ if, for all $t \in P(L)$ and $s \in \overline{L}$, $P(s)$ is a prefix of t implies that there exists $u \in \Sigma^*$ such that $su \in L$ and $P(su) = t$.

The projection $P : \Sigma^* \to \Sigma_0^*$ is *output control consistent* (OCC) for $L \subseteq \Sigma^*$ if for every $s \in \overline{L}$ of the form $s = \sigma_1 \ldots \sigma_\ell$ or $s = s'\sigma_0\sigma_1 \ldots \sigma_\ell$, $\ell \geq 1$, where $s' \in \Sigma^*$, $\sigma_0, \sigma_\ell \in \Sigma_k$, and $\sigma_i \in \Sigma \setminus \Sigma_k$, for $i = 1, 2, \ldots, \ell - 1$, if $\sigma_\ell \in \Sigma_u$, then $\sigma_i \in \Sigma_u$, for all $i = 1, 2, \ldots, \ell - 1$.

The OCC condition can be replaced by a weaker condition called local control consistency (LCC) discussed in Schmidt and Breindl (2008, 2011), see Komenda et al. (2014). Let L be a prefix-closed language over Σ, and let Σ_0 be a subset of Σ. The projection $P : \Sigma^* \to \Sigma_0^*$ is *locally control consistent* (LCC) with respect to a word $s \in L$ if for all events $\sigma_u \in \Sigma_0 \cap \Sigma_u$ such that $P(s)\sigma_u \in P(L)$, it holds that either there does not exist any word $u \in (\Sigma \setminus \Sigma_0)^*$ such that $su\sigma_u \in L$, or there exists a word $u \in (\Sigma_u \setminus \Sigma_0)^*$ such that $su\sigma_u \in L$. The projection P is LCC with respect to L if P is LCC for all words of L.

Theorem 3: *Komenda et al. (2014) Consider the setting of Problem 1 with a prefix-closed specification K. Consider the languages defined in (1) and assume that* $\sup \mathrm{C}_{1+k}$ *and* $\sup \mathrm{C}_{2+k}$ *are nonconflicting. Let P_k^{i+k} be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1, 2$. Then the parallel composition* $\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$*, where $L = L(G_1 \| G_2 \| G_k)$.* □

We can now prove that the assumptions of the previous theorem are stronger than the assumptions of Theorem 2. This is shown in the following lemma and corollary, and summarized in Theorem 4.

Lemma 1: *Consider the setting of Problem 1 and the languages defined in (1). Assume that* $\sup \mathrm{C}_{1+k}$ *and* $\sup \mathrm{C}_{2+k}$ *are nonconflicting, and let the projection $P_k^{i+k} : (\Sigma_i \cup \Sigma_k)^* \to \Sigma_k^*$ be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1, 2$. Then $P_k^{1+k}(\sup \mathrm{C}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{C}_{2+k})$ is controllable with respect to $P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k)$ and $\Sigma_{k,u}$.*

Proof. Since $\Sigma_{1+k} \cap \Sigma_{2+k} = \Sigma_k$, Lemma 5 implies that $P_k^{1+k}(\sup \mathrm{C}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{C}_{2+k}) = P_k(\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k})$. By Lemma 7, because $P_k^k = id$ is an $L(G_k)$-observer, P_k is an $L := L(G_1 \| G_2 \| G_k)$-observer. Assume that $t \in \overline{P_k(\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k})}$, $u \in \Sigma_{k,u}$, and $tu \in P_k(L) = P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k)$. Then there exists $s \in \overline{\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k}} \subseteq L$ such that $P_k(s) = t$. By the observer property, there exists v such that $sv \in L$ and $P_k(sv) = tu$, that is, $v = v_1 u$ with $P_k(v_1 u) = u$. By the OCC property, $v_1 \in \Sigma_u^*$, and by controllability of $\sup \mathrm{C}_{i+k}$, $i = 1, 2$, $sv_1u \in \overline{\sup \mathrm{C}_{1+k}} \parallel \overline{\sup \mathrm{C}_{2+k}} = \overline{\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k}}$, hence $tu \in \overline{P_k(\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k})}$.

Similarly for LCC: from $sv = sv_1u \in L$, by the LCC property, there exists $v_2 \in (\Sigma_u \setminus \Sigma_k)^*$ such that $sv_2u \in L$, and by controllability of $\sup \mathrm{C}_{i+k}$, $i = 1, 2$, $sv_2u \in \overline{\sup \mathrm{C}_{1+k}} \parallel \overline{\sup \mathrm{C}_{2+k}} = \overline{\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k}}$, hence $tu \in \overline{P_k(\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k})}$. □

Note that if $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$, which is actually the way we usually define the coordinator (since we usually define $G_k = P_k(G_1) \parallel P_k(G_2)$), we get the following corollary.

Corollary 1: *Consider the setting of Problem 1 with $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ and the languages defined in (1). Assume that* $\sup \mathrm{C}_{1+k}$ *and* $\sup \mathrm{C}_{2+k}$ *are nonconflicting. Let $P_k^{i+k} : (\Sigma_i \cup \Sigma_k)^* \to \Sigma_k^*$ be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1, 2$. Then $P_k^{1+k}(\sup \mathrm{C}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{C}_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.*

Proof. The assumption $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ implies that $P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k) = L(G_k)$. □

Finally, as a consequence of Lemma 1 and Theorem 2, we obtain the following result.

Theorem 4: *Consider the setting of Problem 1 with the inclusion* $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ *and the languages defined in (1). Assume that* $\sup C_{1+k}$ *and* $\sup C_{2+k}$ *are nonconflicting. Let* P_k^{i+k} *be an* $(P_i^{i+k})^{-1}L(G_i)$*-observer and OCC (resp. LCC) for* $(P_i^{i+k})^{-1}L(G_i)$*, for* $i = 1,2$*. Then the parallel composition* $\sup C_{1+k} \parallel \sup C_{2+k} = \sup cC(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$ *, where* $L = L(G_1 \| G_2 \| G_k)$*.* □

The following simple example demonstrates the approach.

Example 3: Database transactions are examples of discrete-event systems that should be controlled to avoid incorrect behaviors. Transactions are modeled by a sequence of request (r), access (a), and exit (e) operations. Often, several users access the database, which can lead to inconsistencies when executed concurrently, because not all interleavings of operations give a correct behavior.

Consider two users with events r_i, a_i, e_i, for $i = 1,2$. All possible schedules are described by the behavior of the plant $G_1 \| G_2$, where G_1, G_2 are nonblocking generators with $L_m(G_i) = \{(r_i a_i e_i)^j \mid j \geq 0\}$, which is also denoted as $(r_i a_i e_i)^*$, defined as in Figure 3. Naturally, we can control the access event, but not the other events, hence the set of controllable events is $\Sigma_c = \{a_1, a_2\}$. The specification K (Figure 4) describes

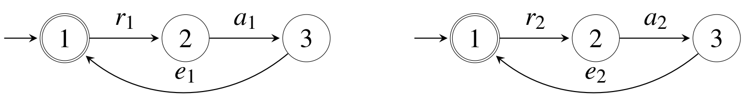

Figure 3. Generators G_i, $i = 1,2$.

the correct behavior consisting in finishing the transaction in the exit stage before another transaction can proceed to the exit phase.

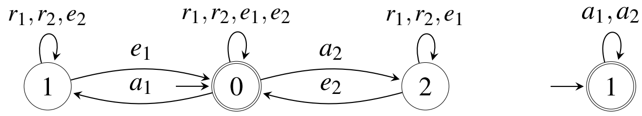

Figure 4. Specification K and the coordinator G_k, where $\sup C_k = G_k$

We can verify that for $\Sigma_k = \{a_1, a_2\}$, the specification K and its prefix closure $\overline{K}$ are conditionally decomposable with respect to Σ_1, Σ_2, and Σ_k. The coordinator is then computed as $G_k = P_k(G_1) \| P_k(G_2)$. The projection $P_k : \Sigma^* \to \Sigma_k^*$ is a K-observer, but it is not an $L_m(G_i)$-observer for $i = 1,2$. However, the projected generators $P_k(G_i)$, $i = 1,2$, have only one state (Figure 4). Furthermore, it can be verified that $P_k(K) = \{a_1, a_2\}^*$ is controllable with respect to $L(G_k) = P_k(K)$ and $\Sigma_{k,u} = \emptyset$. This does not hold for $P_{i+k}(K)$ because the language is not included in $L(G_i) \| \overline{P_k(K)}$, for $i = 1,2$. Moreover, $P_k(K)$ is $L_m(G_k)$-closed, but $P_{i+k}(K)$ is not $L_m(G_i) \| P_k(K)$-closed, for $i = 1,2$. Thus, there do not exist supervisors that would reach the specification K, cf. Komenda et al. (2014).

Thus, we compute supremal controllable sublanguages $\sup C_k$ (Figure 4) and $\sup C_{1+k}$ and $\sup C_{2+k}$ depicted in Figure 5, which correspond to the supervisor for the coordinator, and local supervisors, respectively. Then the assumptions of Theorem 2 are satisfied. As the language $\sup C_k$ is $L_m(G_k)$-closed and

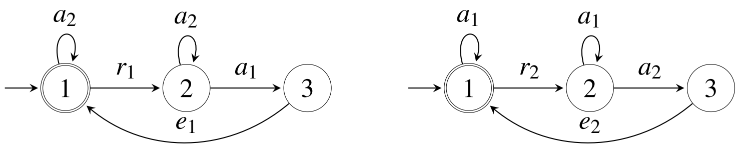

Figure 5. Supervisors $\sup C_{1+k}$ and $\sup C_{2+k}$.

$\sup C_{i+k}$ are $L_m(G_i) \| \sup C_k$-closed, for $i = 1,2$, they form a solution for the database problem by Theorem 2. Moreover, the language $\sup C_{1+k} \| \sup C_{2+k}$ is nonblocking, hence we do not need a coordinator for nonblockingness in this example. The solution is actually optimal, measured with respect to the monolithic solution, that is, the supremal conditionally-controllable sublanguage of K coincides with the supremal controllable sublanguage of K. Independently on the size of the global plant, the local supervisors have only three states. $\triangleleft$

3.2 *Coordination Control with Partial Observations*

In this section, we study coordination control of modular DES, where both the coordinator supervisor and the local supervisors have incomplete (partial) information about occurrences of their events and, hence, they do not know the exact state of the coordinator and the local plants.

The contribution of this section is twofold. First, basic concepts of conditional observability and conditional normality are simplified in a similar way as it has been done in Komenda et al. (2014). Then, we propose new sufficient conditions for a distributed computation of the supremal conditionally normal and conditionally controllable sublanguage. In particular, a weaker condition is presented that combines the weaker condition for distributed computation of the supremal conditionally controllable sublanguage presented in Section 3.1 with a similar condition for computation of the supremal conditionally normal sublanguage. Furthermore, a stronger condition is presented that is easy to check and that works also for non-prefix-closed specifications.

3.2.1 Conditional Observability

For coordination control with partial observations, the notion of conditional observability is of the same importance as observability for monolithic supervisory control theory with partial observations.

Definition 2: Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1 \| G_2 \| G_k)$ is *conditionally observable* with respect to generators G_1, G_2, G_k, controllable sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

(1) $P_k(K)$ is observable with respect to $L(G_k)$, $\Sigma_{k,c}$, Q_k,
(2) $P_{1+k}(K)$ is observable with respect to $L(G_1) \parallel \overline{P_k(K)}$, $\Sigma_{1+k,c}$, Q_{1+k},
(3) $P_{2+k}(K)$ is observable with respect to $L(G_2) \parallel \overline{P_k(K)}$, $\Sigma_{2+k,c}$, Q_{2+k},

where $\Sigma_{i+k,c} = \Sigma_c \cap (\Sigma_i \cup \Sigma_k)$, for $i = 1, 2$.

Analogously to the notion of $L_m(G)$-closed languages, we recall the notion of conditionally-closed languages defined in Komenda et al. (2011a). A nonempty language K over Σ is *conditionally closed* with respect to generators G_1, G_2, G_k if

(1) $P_k(K)$ is $L_m(G_k)$-closed,
(2) $P_{1+k}(K)$ is $L_m(G_1) \parallel P_k(K)$-closed,
(3) $P_{2+k}(K)$ is $L_m(G_2) \parallel P_k(K)$-closed.

We can now formulate the main result for coordination control with partial observation. This is a generalization of a similar result for prefix-closed languages given in Komenda et al. (2011b) stated moreover with the above defined simplified (but equivalent) form of conditional observability.

Theorem 5: *Consider the setting of Problem 1. There exist nonblocking supervisors S_1, S_2, S_k such that*

$$L_m(S_1/[G_1 \| (S_k/G_k)]) \parallel L_m(S_2/[G_2 \| (S_k/G_k)]) = K \tag{1}$$

if and only if K is (i) conditionally controllable with respect generators G_1, G_2, G_k and $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, (ii) conditionally closed with respect to generators G_1, G_2, G_k, and (iii) conditionally observable with respect to G_1, G_2, G_k, event sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections Q_{1+k}, Q_{2+k}, Q_k from Σ_i^* to $\Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$.

Proof. (If) Since the language $K \subseteq L_m(G_1\|G_2\|G_k)$, we have $P_k(K) \subseteq L_m(G_k)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $L_m(G_k)$-closed, and observable with respect to $L(G_k)$, $\Sigma_{k,c}$, and Q_k. It follows, see Cassandras and Lafortune (2008), that there exists a nonblocking supervisor S_k such that $L_m(S_k/G_k) = P_k(K)$. Similarly, we have $P_{1+k}(K) \subseteq L_m(G_1) \parallel L_m(G_k)$ and $P_{1+k}(K) \subseteq (P_k^{1+k})^{-1}P_k(K)$, hence $P_{1+k}(K) \subseteq L_m(G_1) \parallel L_m(G_k) \parallel P_k(K) = L_m(G_1) \parallel P_k(K) = L_m(G_1) \parallel L_m(S_k/G_k)$. This, together with the assumption that K is conditionally controllable, conditionally closed, and conditionally observable imply, see Cassandras and Lafortune (2008), that there exists a nonblocking supervisor S_1 such that $L_m(S_1/[G_1\|(S_k/G_k)]) = P_{1+k}(K)$. A similar argument shows that there exists a nonblocking supervisor S_2 such that $L_m(S_2/[G_2\|(S_k/G_k)]) = P_{2+k}(K)$. Since K is conditionally decomposable, $L_m(S_1/[G_1\|(S_k/G_k)]) \parallel L_m(S_2/[G_2\|(S_k/G_k)]) = P_{1+k}(K) \parallel P_{2+k}(K) = K$.

(Only if) To prove this direction, projections P_k, P_{1+k}, P_{2+k} are applied to (1). The closed-loop languages can be written as synchronous products, thus (1) can be written as $K = L_m(S_1) \parallel L_m(G_1) \parallel L_m(S_k) \parallel L_m(G_k) \parallel L_m(S_2) \parallel L_m(G_2) \parallel L_m(S_k) \parallel L_m(G_k)$, which gives $P_k(K) \subseteq L_m(S_k) \parallel L_m(G_k) = L_m(S_k/G_k)$. On the other hand, $L_m(S_k/G_k) \subseteq P_k(K)$, see Problem 1, hence $L_m(S_k/G_k) = P_k(K)$, which means, according to the basic theorem of supervisory control Cassandras and Lafortune (2008), that $P_k(K)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $L_m(G_k)$-closed, and observable with respect to $L(G_k)$, $\Sigma_{k,c}$, and Q_k. Now, the application of P_{1+k} to (1) gives $P_{1+k}(K) \subseteq L_m(S_1/[G_1\|(S_k/G_k)]) \subseteq P_{1+k}(K)$. According to the basic theorem of supervisory control, $P_{1+k}(K)$ is controllable with respect to $L(G_1\|(S_k/G_k))$ and $\Sigma_{1+k,u}$, $L_m(G_1\|(S_k/G_k))$-closed, and observable with respect to $L(G_1\|(S_k/G_k))$, $\Sigma_{1+k,c}$, and Q_{1+k}. Similarly, $P_{2+k}(K)$ is controllable with respect to $L(G_2\|(S_k/G_k))$ and $\Sigma_{2+k,u}$, $L_m(G_2\|(S_k/G_k))$-closed, and observable with respect to $L(G_2\|(S_k/G_k))$, $\Sigma_{2+k,c}$, and Q_{2+k}, which was to be shown. □

3.2.2 Conditional normality

It is well known that supremal observable sublanguages do not exist in general and it is also the case of conditionally observable sublanguages. Therefore, a stronger concept of language normality has been introduced.

Let G be a generator over Σ, and let $P : \Sigma^* \to \Sigma_o^*$ be a projection. A language $K \subseteq L_m(G)$ is *normal* with respect to $L(G)$ and P if $\overline{K} = P^{-1}P(\overline{K}) \cap L(G)$. It is known that normality implies observability Cassandras and Lafortune (2008).

Definition 3: Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1\|G_2\|G_k)$ is *conditionally normal* with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

(1) $P_k(K)$ is normal with respect to $L(G_k)$ and Q_k,
(2) $P_{1+k}(K)$ is normal with respect to $L(G_1) \parallel \overline{P_k(K)}$ and Q_{1+k},
(3) $P_{2+k}(K)$ is normal with respect to $L(G_2) \parallel \overline{P_k(K)}$ and Q_{2+k}.

The following result is an immediate application of conditional normality in coordination control.

Theorem 6: *Consider the setting of Problem 1. If the specification K is conditionally controllable with respect to G_1, G_2, G_k and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$, conditionally closed with respect to G_1, G_2, G_k, and conditionally normal with respect to G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k from Σ_i^* to $\Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, then there exist nonblocking supervisors S_1, S_2, S_k such that $L_m(S_1/[G_1\|(S_k/G_k)]) \parallel L_m(S_2/[G_2\|(S_k/G_k)]) = K$.*

Proof. As normality implies observability, the proof follows immediately from Theorem 5. □

The following result was proved for prefix-closed languages in Komenda et al. (2011b). Here we generalize it for not necessarily prefix-closed languages.

Theorem 7: *The supremal conditionally normal sublanguage always exists and equals to the union of all conditionally normal sublanguages.*

Proof. We show that conditional normality is preserved under union. Let I be an index set, and let K_i be conditionally normal sublanguages of $K \subseteq L_m(G_1 \| G_2 \| G_k)$ with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k to local observable event sets, for $i \in I$. We prove that $\bigcup_{i\in I} K_i$ is conditionally normal with respect to those generators and natural projections.

i) The language $P_k(\bigcup_{i\in I} K_i)$ is normal with respect to $L(G_k)$ and Q_k because $Q_k^{-1}Q_kP_k(\overline{\bigcup_{i\in I} K_i}) \cap L(G_k) = \bigcup_{i\in I}(Q_k^{-1}Q_kP_k(\overline{K_i}) \cap L(G_k)) = \bigcup_{i\in I} P_k(\overline{K_i}) = P_k(\overline{\bigcup_{i\in I} K_i}) = P_k(\bigcup_{i\in I} \overline{K_i})$, where the second equality is by normality of $P_k(K_i)$ with respect to $L(G_k)$ and Q_k, for $i \in I$.

ii) Note that it holds that $Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{\cup_{i\in I}K_i}) \cap L(G_1)\|P_k(\overline{\cup_{i\in I}K_i}) = \cup_{i\in I}(Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i})) \cap \cup_{i\in I}(L(G_1)\|P_k(\overline{K_i})) = \cup_{i\in I}\cup_{j\in I}(Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j}))$ and that the language $P_{1+k}(\overline{\cup_{i\in I}K_i}) \subseteq Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{\cup_{i\in I}K_i}) \cap L(G_1)\|P_k(\overline{\cup_{i\in I}K_i})$. For the sake of contradiction, assume that there exist indexes $i \neq j$ in I such that $Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j}) \not\subseteq P_{1+k}(\overline{\cup_{i\in I}K_i})$. Then the left-hand side must be nonempty, which implies that there exists $x \in Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j})$ and $x \notin P_{1+k}(\overline{\cup_{i\in I}K_i})$. As $x \in Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i})$, there exists $w \in \overline{K_i}$ such that $Q_{1+k}(x) = Q_{1+k}P_{1+k}(w)$. Applying the projection $P'_k : \Sigma_{1+k,o}^* \to \Sigma_{k,o}^*$, we get that $P'_kQ_{1+k}(x) = P'_kQ_{1+k}P_{1+k}(w)$. As $Q_kP_k^{1+k} = P'_kQ_{1+k}$ and $Q_kP_k = P'_kQ_{1+k}P_{1+k}$ (see Figure 6), we have $Q_kP_k^{1+k}(x) = Q_kP_k(w)$, that is, $P_k^{1+k}(x) \in Q_k^{-1}Q_kP_k(\overline{K_i})$. Since $P_k^{1+k}(x) \in P_k(\overline{K_j}) \subseteq$

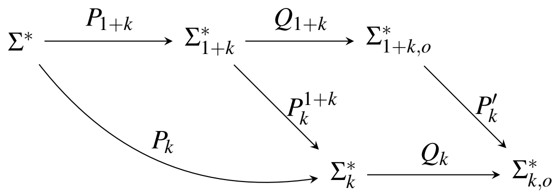

Figure 6. A commutative diagram of the natural projections.

$L(G_k)$, the normality of $P_k(K_i)$ with respect to $L(G_k)$ and Q_k gives that $P_k^{1+k}(x) \in P_k(\overline{K_i})$. But then $x \in L(G_1)\|P_k(\overline{K_i})$, and normality of $P_{1+k}(K_i)$ implies that $x \in P_{1+k}(\overline{K_i}) \subseteq P_{1+k}(\overline{\cup_{i\in I}K_i})$, which is a contradiction.

iii) As the last item of the definition is proven in the same way, the theorem holds. □

Given generators G_1, G_2, and G_k, let

$$\sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$$

denote the supremal conditionally controllable and conditionally normal sublanguage of the specification language K with respect to the plant language $L = L(G_1\|G_2\|G_k)$, the sets of uncontrollable events $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$.

In the sequel, the computation of the supremal conditionally controllable and conditionally normal sublanguage is investigated. In the same way as in Komenda et al. (2011b), the following notation is adopted.

Consider the setting of Problem 1 and define the languages

$$\boxed{\begin{aligned} \sup \mathrm{CN}_k &= \sup \mathrm{CN}(P_k(K), L(G_k), \Sigma_{k,u}, Q_k) \\ \sup \mathrm{CN}_{1+k} &= \sup \mathrm{CN}(P_{1+k}(K), L(G_1)\|\overline{\sup \mathrm{CN}_k}, \Sigma_{1+k,u}, Q_{1+k}) \\ \sup \mathrm{CN}_{2+k} &= \sup \mathrm{CN}(P_{2+k}(K), L(G_2)\|\overline{\sup \mathrm{CN}_k}, \Sigma_{2+k,u}, Q_{2+k}) \end{aligned}} \quad (2)$$

where the notation $\sup \mathrm{CN}(K, L, \Sigma_u, Q)$ denotes the supremal controllable and normal sublanguage of K with respect to L, Σ_u, and Q. We recall that the supremal controllable and normal sublanguage always exists and equals the union of all controllable and normal sublanguages of K, cf. Cassandras and Lafortune (2008).

Theorem 8: *Komenda et al. (2011b) Consider the setting of Problem 1 with a prefix-closed specification K and the languages defined in (2). Let P_k^{i+k} be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1,2$. Assume that the language $P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is normal with respect to $L(G_k)$ and Q_k. Then $\sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k} = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$, where $L = L(G_1 \| G_2 \| G_k)$.* □

We can now further improve the above result as follows.

Theorem 9: *Consider the setting of Problem 1 and the languages defined in (2). Assume that the languages $\sup \mathrm{CN}_{1+k}$ and $\sup \mathrm{CN}_{2+k}$ are nonconflicting and that $P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k. Then the parallel composition $\sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k} = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$, where $L = L(G_1 \| G_2 \| G_k)$.*

Proof. To simplify the notation, let $\sup \mathrm{cCN} = \sup \mathrm{cCN}(K, L, (E_{1+k,u}, E_{2+k,u}, E_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$ and $M = \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$.

To prove $M \subseteq \sup \mathrm{cCN}$, we show that $M \subseteq P_{1+k}(K) \parallel P_{2+k}(K) = K$ (by conditional decomposability) is conditionally controllable with respect to L and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$ (which follows from Theorem 2), and conditionally normal with respect to L and Q_{1+k}, Q_{2+k}, Q_k (which needs to be shown). However, $P_k(M) = P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is normal with respect to $L(G_k)$ and Q_k by the assumption. Furthermore, $P_{1+k}(M) = \sup \mathrm{CN}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{CN}_{2+k})$. Since $P_{1+k}(M) \subseteq \sup \mathrm{CN}_{1+k}$ and $P_k(M) \subseteq \sup \mathrm{CN}_k$ (by the assumption), $x \in Q_{1+k}^{-1} Q_{1+k}(\overline{P_{1+k}(M)}) \cap L(G_1) \parallel \overline{P_k(M)} \subseteq Q_{1+k}^{-1} Q_{1+k}(\overline{\sup \mathrm{CN}_{1+k}}) \cap L(G_1) \parallel \overline{\sup \mathrm{CN}_k} = \overline{\sup \mathrm{CN}_{1+k}}$ (by normality of $\sup \mathrm{CN}_{1+k}$). In addition, $P_k^{1+k}(x) \in \overline{P_k(M)} \subseteq P_k^{2+k}(\overline{\sup \mathrm{CN}_{2+k}})$. Thus, $x \in \overline{\sup \mathrm{CN}_{1+k}} \parallel P_k^{2+k}(\overline{\sup \mathrm{CN}_{2+k}}) = \overline{P_{1+k}(M)}$ by the nonconflictingness of the supervisors. The case for $P_{2+k}(M)$ is analogous, hence $M \subseteq \sup \mathrm{cCN}$.

To prove $\sup \mathrm{cCN} \subseteq M$, it is sufficient by Lemma 6 to show that $P_{i+k}(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_{i+k}$, for $i = 1,2$. To do this, note that $P_{1+k}(\sup \mathrm{cCN}) \subseteq P_{1+k}(K)$ is controllable and normal with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, $\Sigma_{1+k,u}$, and Q_{1+k} by definition. Since $P_k(\sup \mathrm{cCN})$ is controllable and normal with respect to $L(G_k)$, $E_{k,u}$, and Q_k, it is also controllable and normal with respect to $\overline{\sup \mathrm{CN}_k} \subseteq L(G_k)$ because $P_k(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_k$. As $P_{1+k}(\sup \mathrm{cCN})$ is controllable with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ by Lemma 2, the transitivity of controllability (Lemma 4) implies that $P_{1+k}(\sup \mathrm{cCN})$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ and $\Sigma_{1+k,u}$. Similarly, as $P_{1+k}(\sup \mathrm{cCN})$ is normal with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$ is normal with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ by Lemma 9, transitivity of normality (Lemma 8) implies that $P_{1+k}(\sup \mathrm{cCN})$ is normal with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ and Q_{1+k}. Thus, we have shown that $P_{1+k}(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_{1+k}$. The case of $P_{2+k}(M)$ is analogous, hence $\sup \mathrm{cCN} \subseteq M$ and the proof is complete. □

Note that the sufficient condition in Theorem 9 is not practical for verification, although the intersection is only over the coordinator alphabet that is hopefully small. Unlike controllability, normality is not preserved by natural projections under observer and OCC assumptions. This would require results on hierarchical control under partial observations that are not known so far. Therefore, we propose a condition that is (similarly as in the case of complete observations) stronger than the one of Theorem 9, but is easy to check and, moreover, is sufficient for a distributed computation of the supremal conditionally controllable and conditionally normal sublanguage even in the case of non-prefix-closed specifications. Namely, we observe that controllability and normality conditions of Theorem 9 are weaker than to require that $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$. The intuition behind the condition $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, is that local supervisors (given by $\sup \mathrm{CN}_{i+k}$) do not need to improve the action by the supervisor for the coordinator on the coordinator alphabet. In this case, the intuition is the same as if the three supervisors (the supervisor for the coordinator and local supervisors) would operate on disjoint alphabets (namely Σ_k, $\Sigma_1 \setminus \Sigma_k$ and $\Sigma_2 \setminus \Sigma_k$) and it is well known that there is no problem with blocking and maximal permissiveness in this case, because nonconflictness and mutual controllability of modular control are then trivially satisfied.

Proposition 2: *Consider the setting of Problem 1 and the languages defined in (2). If* $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$*, for* $i = 1,2$*, then* $P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k})$ *is controllable and normal with respect to* $L(G_k)$*,* $\Sigma_{k,u}$*, and* Q_k*.*

Proof. First of all, we have shown that the inclusion $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, always holds true. From its definition, $P_k(\sup \mathrm{CN}_{i+k}) \subseteq P_k(L(G_i) \| \overline{\sup \mathrm{CN}_k}) \subseteq \overline{\sup \mathrm{CN}_k}$ and, clearly, $P_k(\sup \mathrm{CN}_{i+k}) \subseteq P_k(K)$ as well. In order to show that $P_k(\sup \mathrm{CN}_{i+k}) \subseteq \sup \mathrm{CN}_k$, it suffices to show that $\overline{\sup \mathrm{CN}_k} \cap P_k(K) \subseteq \sup \mathrm{CN}_k$. This can be proven by showing that $\overline{\sup \mathrm{CN}_k} \cap P_k(K)$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k.

For controllability, let $s \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$, $u \in \Sigma_{k,u}$ with $su \in L(G_k)$. Since there exists $t \in \Sigma_k^*$ such that $st \in \overline{\sup \mathrm{CN}_k} \cap P_k(K) \subseteq \overline{\sup \mathrm{CN}_k}$, we have that $s \in \overline{\sup \mathrm{CN}_k}$ as well. Since $\sup \mathrm{CN}_k$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $su \in \overline{\sup \mathrm{CN}_k} \subseteq \overline{P_k(K)}$. Hence, there exists $v \in \Sigma_k^*$ such that $suv \in \sup \mathrm{CN}_k \subseteq P_k(K)$. Altogether, $suv \in \overline{\sup \mathrm{CN}_k} \cap P_k(K)$, i.e., $su \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$.

For normality, let $s \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$ and $s' \in L(G_k)$ with $Q_k(s) = Q_k(s')$. Recall that $s \in \overline{\sup \mathrm{CN}_k}$ as well. Again, normality of $\sup \mathrm{CN}_k$ with respect to $L(G_k)$ and Q_k implies that $s' \in \overline{\sup \mathrm{CN}_k}$. Thus, there exists $v \in \Sigma_k^*$ such that $s'v \in \sup \mathrm{CN}_k \subseteq P_k(K)$. This implies that $s'v \in \overline{\sup \mathrm{CN}_k} \cap P_k(K)$, i.e., $s' \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$, which completes the proof of the inclusion $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$.

According to the assumption that the other inclusions also hold, we have the equalities $\sup \mathrm{CN}_k = P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$. Therefore, $P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k}) = \sup \mathrm{CN}_k$, which is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k by definition of $\sup \mathrm{CN}_k$. □

Now, combining Proposition 2 and Theorem 9 we obtain the corollary below.

Corollary 2: *Consider the setting of Problem 1 and the languages defined in (2). If the inclusions* $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$ *hold true for* $i = 1,2$*, then we obtain that the parallel composition* $\sup \mathrm{CN}_{1+k} \| \sup \mathrm{CN}_{2+k} = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$*, where* $L = L(G_1 \| G_2 \| G_k)$*.*

Proof. Let $\sup \mathrm{cCN} = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$ and $M = \sup \mathrm{CN}_{1+k} \| \sup \mathrm{CN}_{2+k}$. To prove that M is a subset of $\sup \mathrm{cCN}$, we show that (i) M is a subset of K, (ii) M is conditionally controllable with respect to generators G_1, G_2, G_k and uncontrollable event sets $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and (iii) M is conditionally normal with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k. To this aim, notice that M is a subset of $P_{1+k}(K) \| P_{2+k}(K) = K$, because K is conditionally decomposable. Moreover, by Lemma 5 and the fact shown in the proof of Proposition 2 that $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, the language $P_k(M) = P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k}) = \sup \mathrm{CN}_k$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k. Similarly, $P_{i+k}(M) = \sup \mathrm{CN}_{i+k} \| P_k(\sup \mathrm{CN}_{j+k}) = \sup \mathrm{CN}_{i+k} \| \sup \mathrm{CN}_k = \sup \mathrm{CN}_{i+k}$, for $j \neq i$, which is controllable and normal with respect to $L(G_i) \| \overline{P_k(M)}$. Hence, M is a subset of $\sup \mathrm{cCN}$.

The opposite inclusion is shown in Theorem 9, because nonconflictingness is not needed in this direction of the proof. □

4. A posteriori supervisor for the coordinator

In this section we will show that the controllability and normality assumptions of the previous section can be left out. It is actually natural to impose these properties by a new supervisor that we call an a posteriori supervisor. We will prove that the use of these supervisors do not alter maximal permissiveness for nonconflicting supervisors (e.g. in the prefix-closed case), which means that the supremal conditionally controllable and conditionally normal sublanguage can always be computed in a distributed way.

Consider the setting of Problem 1 and the languages defined in (2). To simplify the notation of this section, we denote

$$\sup \mathrm{cCN} = \sup \mathrm{cCN}(K, L, (\Sigma_{1+k,u}, \Sigma_{2+k,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}))$$

for $L = L(G_1 \parallel G_2 \parallel G_k)$, and

$$\sup \mathrm{CN}'_k = \sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k}), L(G_k), \Sigma_{k,u}, Q_k)\,.$$

Note that $\sup \mathrm{CN}'_k$, the a posteriori supervisor for the coordinator, can be computed in parallel, that is, we can compute $\sup \mathrm{CN}'_k = \sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{1+k}), L_k, \Sigma_{k,u}) \cap \sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{2+k}), L_k, \Sigma_{k,u})$ or, in the full observation case, $\sup \mathrm{C}'_k = \sup \mathrm{C}(P_k(\sup \mathrm{C}_{1+k}), L(G_k), \Sigma_{k,u}) \cap \sup \mathrm{C}(P_k(\sup \mathrm{C}_{1+k}), L(G_k), \Sigma_{k,u})$. This parallel computation is possible because the plant language, $L(G_k)$, is the same in both components of the intersection, and $L(G_k)$ is trivially mutually controllable with respect to itself, cf. Komenda, van Schuppen, Gaudin, and Marchand (2008). Such a distributed computation is also important from the complexity viewpoint, because instead of computing the supervisor for the projection of composition of local supervisors, separate supervisors are computed for projection of individual local supervisors. Similarly as in modular control, their composition (here intersection) is never computed and they operate locally in conjunction with local supervisors $\sup \mathrm{CN}_{i+k}$, for $i = 1,2$, provided the languages on the right-hand sides are nonconflicting (e.g., prefix-closed). Thus, in the following, we assume that

$$\begin{aligned}\overline{\sup \mathrm{CN}'_k} &= \overline{\sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{1+k}), L_k, \Sigma_{k,u}) \cap \sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{2+k}), L_k, \Sigma_{k,u})} \\ &= \overline{\sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{1+k}), L_k, \Sigma_{k,u})} \cap \overline{\sup \mathrm{CN}(P_k(\sup \mathrm{CN}_{2+k}), L_k, \Sigma_{k,u})}\,.\end{aligned}$$

and similarly for $\sup \mathrm{C}'_k$.

Theorem 10: *Consider the setting of Problem 1 and the languages defined in (2). Let* $\sup \mathrm{CN}'_k$ *be defined as above. If the languages* $\sup \mathrm{CN}_{i+k}$ *and* $\sup \mathrm{CN}'_k$ *are synchronously nonconflicting (e.g., prefix-closed) for* $i = 1,2$*, then* $\sup \mathrm{CN}'_k \parallel \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k} = \sup \mathrm{cCN}$ *is the supremal conditionally controllable and conditionally normal sublanguage of* K*.*

Proof. Let $M' = \sup \mathrm{CN}'_k \parallel \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$ and $M = \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$. Then the projection $P_k(M') = \sup \mathrm{CN}'_k \parallel P_k(M) = \sup \mathrm{CN}'_k$ is controllable and normal with respect to $L(G_k)$. Similarly, the projection $P_{i+k}(M') = \sup \mathrm{CN}_{i+k} \parallel P_k(\sup \mathrm{CN}_{j+k}) \parallel \sup \mathrm{CN}'_k = \sup \mathrm{CN}_{i+k} \parallel \sup \mathrm{CN}'_k$, for $i = 1,2$ and $j \neq i$. Combining Lemmas 2 and 4, we obtain that $P_{i+k}(M)$ is controllable with respect to $L(G_i) \parallel \overline{\sup \mathrm{CN}_k} \parallel L(G_k) = L(G_i) \parallel \overline{\sup \mathrm{CN}_k}$, hence it is also controllable with respect to $L(G_i) \parallel \overline{\sup \mathrm{CN}'_k}$, for $i = 1,2$, because $\sup \mathrm{CN}'_k \subseteq P_k(\sup \mathrm{CN}_{i+k}) \subseteq \sup \mathrm{CN}_k$. Similarly, using Lemmas 3 and 8, the language is normal with respect to the same languages and Q_{i+k}. Therefore, $M \subseteq \sup \mathrm{cCN}$.

To prove the other direction, we need to show that $P_{i+k}(\sup \mathrm{cCN}) \subseteq (P^{i+k}_k)^{-1}(\sup \mathrm{CN}'_k)$, that is, to show that $P_k(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}'_k$. Since $P_k(\sup \mathrm{cCN})$ is, by definition, controllable and normal with respect to $L(G_k)$, it means to show that $P_k(\sup \mathrm{cCN}) \subseteq P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k})$. However, note that $\sup \mathrm{cCN} \subseteq \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$, since $P_{i+k}(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_{i+k}$, for $i = 1,2$, which completes the proof. □

An immediate consequence for systems with full observations follows.

Corollary 3: *Consider the setting of Problem 1 and the languages defined in (1). Let* $\sup \mathrm{C}'_k$ *be defined as above. Then* $\sup \mathrm{C}'_k \parallel \sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}$ *is the supremal conditionally controllable sublanguage of* K*.*

In the following example, we illustrate the above results, namely that of Corollary 3, on a simple example. This example also demonstrates the case where a solution exists, but the sufficient conditions of Theorem 2

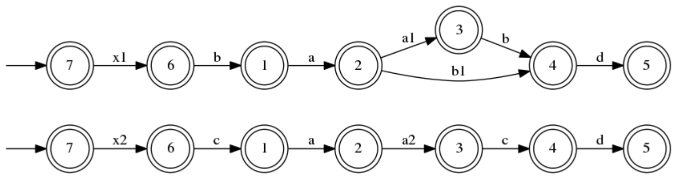

Figure 7. Generators G_1 and G_2

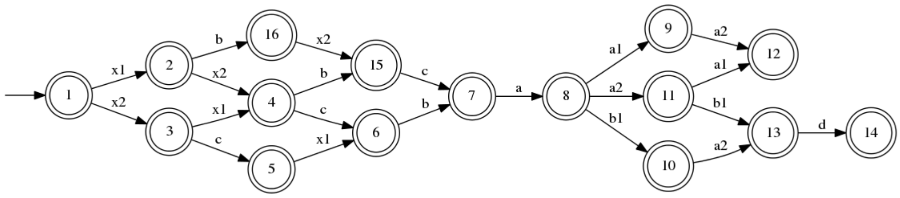

Figure 8. The specification K

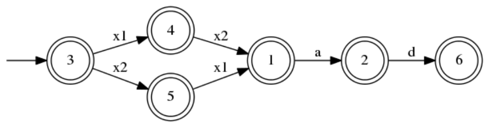

Figure 9. The coordinator G_k

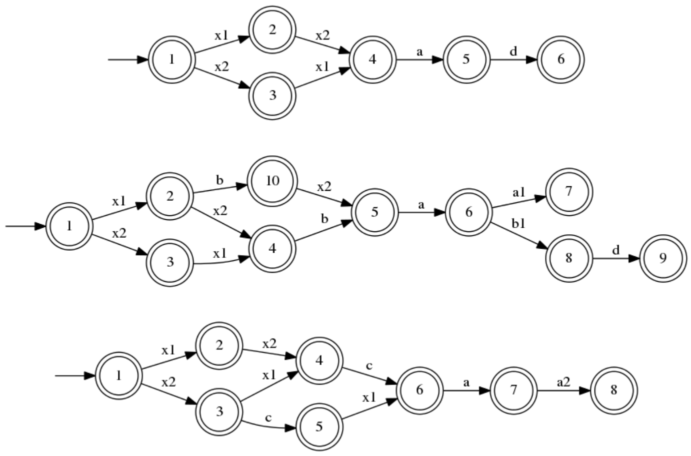

Figure 10. Supervisors $\sup \mathrm{C}_k$, $\sup \mathrm{C}_{1+k}$ and $\sup \mathrm{C}_{2+k}$

are not satisfied. A deeper discussion on this topic can be found in Komenda, Masopust, and van Schuppen (2015).

Example 4: Consider the generators G_1 and G_2 shown in Figure 7, where the set of controllable events is $\Sigma_c = \{a,b,c\}$. The shared (synchronization) events are a and d. It can be verified that for the choice of $\Sigma_k = \{x_1,x_2,a,d\}$, the specification K shown in Figure 8 is conditionally decomposable. Using the method from the paper, we compute a coordinator G_k depicted in Figure 9 and three supervisors $\sup \mathrm{C}_k$, $\sup \mathrm{C}_{1+k}$ and $\sup \mathrm{C}_{2+k}$ depicted in Figure 10. Then the language $\sup \mathrm{C}_{1+k} \| \sup \mathrm{C}_{2+k}$ coincides with the optimal solution of the monolithic case, even though the parallel composition is not conditionally controllable. Moreover, it does not satisfy the conditions of Theorem 2, because $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ coincides with G_k without the last uncontrollable transition d. The missing event d is the reason why the intersection of projected local supervisors is not controllable with respect to $L(G_k)$.

Thus, we use Corollary 3 and compute the a-posteriori supervisor $\sup \mathrm{C}'_k$ depicted in Figure 11. By

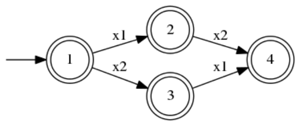

Figure 11. The a-posteriori supervisor $\sup \mathrm{C}'_k$

Corollary 3, the parallel composition $\sup \mathrm{C}_{1+k} \| \sup \mathrm{C}_{2+k} \| \sup \mathrm{C}'_k$ coincides with the supremal conditionally controllable sublanguage of the specification K. However, one can notice that this solution does not coincide with the optimal solution from the centralized (monolithic) case. ◁

Note that in the non-prefix-closed case, the assumptions that the languages are synchronously nonconflicting are required. For instance, these assumptions are trivially satisfied for prefix-closed languages. If general, non-prefix-closed languages are considered, it is known that to verify whether a synchronous product (of an unspecified number) of generators is synchronously nonconflicting is a PSPACE-complete problem Rohloff and Lafortune (2005). However, it is the worst case and some optimization techniques could be found in the literature, see e.g. Flordal and Malik (2006), or a maximal nonconflicting sublanguage can be computed, cf. Chen and Lafortune (1991). Moreover, the good news of the PSPACE-completeness is that it is computable in polynomial space. For the general case with non-prefix-closed languages we have proposed in Komenda et al. (2014)a procedure based on abstraction for computing coordinators for nonblockingness, which are needed if local supervisors $\sup \mathrm{CN}_{i+k}$ are conflicting.

5. Conclusion

In this paper, we have further generalized several results of coordination control of concurrent automata with both complete and partial observations. We have presented weaker sufficient conditions for the computation of supremal conditionally controllable sublanguages and supremal conditionally controllable and conditionally normal sublanguages with simplified concepts of conditional observability and conditional normality. It has been proven that for prefix-closed languages supremal conditionally controllable and conditionally normal sublanguages can always be computed in a distributed way. It follows that supremal conditionally controllable and conditionally normal sublanguages are always conditionally decomposable unlike what we have believed so far.

Our plan is to generalize all these new results into a multi-level coordination control framework in order to make them applicable for large scale systems consisting of a large number of components. A single (centralized) coordinator would then include too many events for such large systems, hence it should be replaced by a multi-level hierarchical structure of coordinators for different groups of systems at all levels of the hierarchy. In another future work we would like to extend our results to timed systems based on approximate projections of (max,+) and interval automata and apply our coordination control framework to DES models of engineering systems.

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Appendix A. Auxiliary results

In this section, we list the auxiliary results.

Lemma 2 (Proposition 4.6 in Feng (2007))**:** *Let $L_i \subseteq \Sigma_i^*$, for $i = 1,2$, be prefix-closed languages, and let $K_i \subseteq L_i$ be controllable with respect to L_i and $\Sigma_{i,u}$. Let $\Sigma = \Sigma_1 \cup \Sigma_2$. If K_1 and K_2 are synchronously nonconflicting, then $K_1 \parallel K_2$ is controllable with respect to $L_1 \parallel L_2$ and Σ_u.* □

Lemma 3: *Let $K_1 \subseteq L_1$ over Σ_1 and $K_2 \subseteq L_2$ over Σ_2 be languages such that K_1 is normal with respect to L_1 and $Q_1 : \Sigma_1^* \to \Sigma_{1,o}^*$ and K_2 is normal with respect to L_2 and $Q_2 : \Sigma_2^* \to \Sigma_{2,o}^*$. If K_1 and K_2 are synchronously nonconflicting, then $K_1 \parallel K_2$ is normal with respect to $L_1 \parallel L_2$ and $Q : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_{1,o} \cup \Sigma_{2,o})^*$.*

Proof. $Q^{-1}Q(\overline{K_1 \parallel K_2}) \cap L_1 \parallel L_2 \subseteq Q_1^{-1}Q_1(\overline{K_1}) \parallel Q_2^{-1}Q_2(\overline{K_2}) \parallel L_1 \parallel L_2 = \overline{K_1} \parallel \overline{K_2} = \overline{K_1 \parallel K_2}$. As the other inclusion always holds, the proof is complete. ☐

Lemma 4: *Komenda et al. (2012b) Let $K \subseteq L \subseteq M$ be languages over Σ such that K is controllable with respect to $\overline{L}$ and Σ_u, and L is controllable with respect to $\overline{M}$ and Σ_u. Then K is controllable with respect to $\overline{M}$ and Σ_u.* ☐

Lemma 5: *Wonham (2009) Let $P_k : \Sigma^* \to \Sigma_k^*$ be a projection, and let $L_i \subseteq \Sigma_i^*$, where $\Sigma_i \subseteq \Sigma$, for $i = 1,2$, and $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$. Then $P_k(L_1 \parallel L_2) = P_k(L_1) \parallel P_k(L_2)$.* ☐

Lemma 6: *Komenda et al. (2012b) Let $L_i \subseteq \Sigma_i^*$, for $i = 1,2$, and let $P_i : (\Sigma_1 \cup \Sigma_2)^* \to \Sigma_i^*$ be a projection. Let $A \subseteq (\Sigma_1 \cup \Sigma_2)^*$ such that $P_1(A) \subseteq L_1$ and $P_2(A) \subseteq L_2$. Then $A \subseteq L_1 \parallel L_2$.* ☐

Lemma 7: *Pena, Cury, and Lafortune (2009) Let $L_i \subseteq \Sigma_i^*$, for $i \in J$, be languages, and let $\cup_{k,\ell \in J}^{k \neq \ell} (\Sigma_k \cap \Sigma_\ell) \subseteq \Sigma_0 \subseteq (\cup_{i \in J} \Sigma_i)^*$. If $P_{i,0} : \Sigma_i^* \to (\Sigma_i \cap \Sigma_0)^*$ is an L_i-observer, for $i \in J$, then $P_0 : (\cup_{i \in J} \Sigma_i)^* \to \Sigma_0^*$ is an $(\parallel_{i \in J} L_i)$-observer.* ☐

Lemma 8: *Let $K \subseteq L \subseteq M$ be languages such that K is normal with respect to L and Q, and L is normal with respect to M and Q. Then, K is normal with respect to M and Q.*

Proof. $Q^{-1}Q(\overline{K}) \cap \overline{L} = \overline{K}$ and $Q^{-1}Q(\overline{L}) \cap \overline{M} = \overline{L}$, hence $Q^{-1}Q(\overline{K}) \cap \overline{M} \subseteq Q^{-1}Q(\overline{L}) \cap \overline{M} = \overline{L}$. It implies $Q^{-1}Q(\overline{K}) \cap \overline{M} = Q^{-1}Q(\overline{K}) \cap \overline{M} \cap \overline{L} = \overline{K} \cap \overline{M} = \overline{K}$. ☐

Lemma 9: *Let $K_1 \subseteq L_1$ over Σ_1 and $K_2 \subseteq L_2$ over Σ_2 be nonconflicting languages such that K_1 is normal with respect to L_1 and $Q_1 : \Sigma_1^* \to \Sigma_{1,o}^*$ and K_2 is normal with respect to L_2 and $Q_2 : \Sigma_2^* \to \Sigma_{2,o}^*$. Then $K_1 \| K_2$ is normal with respect to $L_1 \| L_2$ and $Q : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_{1,o} \cup \Sigma_{2,o})^*$.*

Proof. $Q^{-1}Q(\overline{K_1 \parallel K_2}) \cap L_1 \parallel L_2 \subseteq Q_1^{-1}Q_1(\overline{K_1}) \parallel Q_2^{-1}Q_2(\overline{K_2}) \parallel L_1 \parallel L_2 = \overline{K_1} \parallel \overline{K_2} = \overline{K_1 \parallel K_2}$. As the other inclusion always holds, the proof is complete. ☐