The Kurzweil integral in financial markets

Joint work with H. Lamba, S. Melnik, G. A. Monteiro, and D. Rachinskii

Dedicated to Jaroslav Kurzweil on the occasion of his 90th birthday

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May 12, 2016

Empirical observation:

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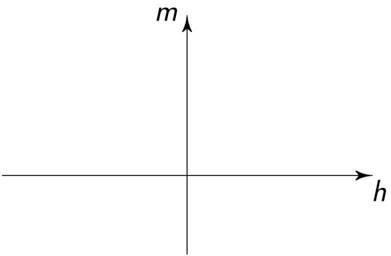
Difference between the two:

Solids remember their shape, fluids have no memory.

Madelung's memory laws

- The shape of a monotone magnetization curve does not depend on the rate of change.
- The starting slope of a curve at a turning point does not depend on the previous history.
- After second turn the curve returns back to its starting point.
- As soon as the minor loop is closed, the process continues as if no turn had taken place.



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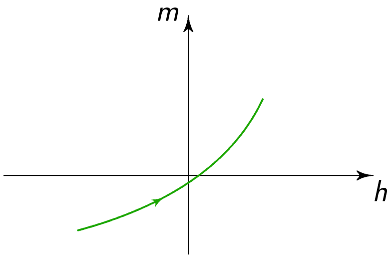

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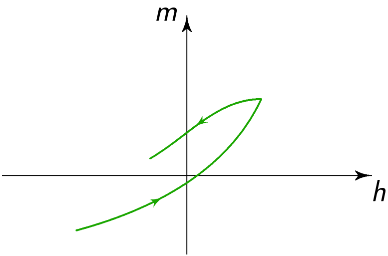

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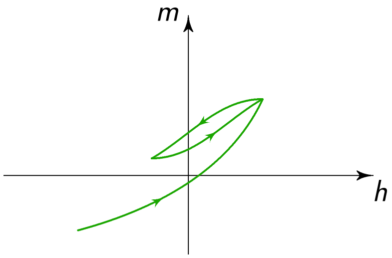

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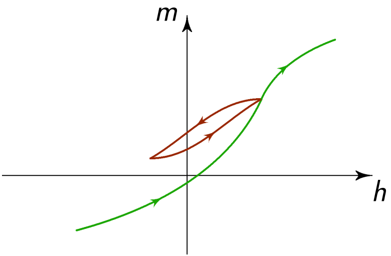

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Mathematical description of memory

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Theorem (Brokate, Sprekels 1996). *Every memory relation satisfying Madelung's memory rules can be represented by the system* $\{\mathfrak{p}_r ; r > 0\}$ *of operators* $\mathfrak{p}_r : u \mapsto \xi_r$ *, which with a given function* $u : [0, T] \to \mathbb{R}$ *associate the solution* $\xi_r : [0, T] \to \mathbb{R}$ *of the variational inequality*

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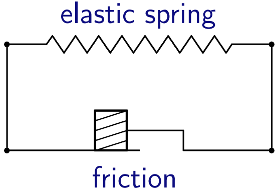
The parameter r characterizes the memory depth.

The Prandtl mechanical model (1928)

The Prandtl operator $\mathfrak{p}_r$ describes the relation between the stress u and strain (deformation) $\mathfrak{p}_r[u]$ in a parallel elastoplasticity model with unit elasticity modulus and yield point $r > 0$.

Rheological model



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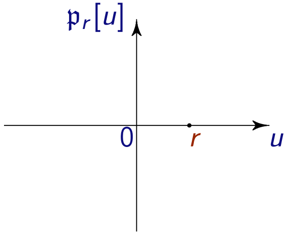

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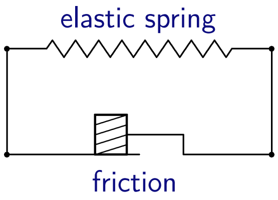

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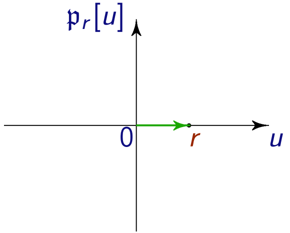

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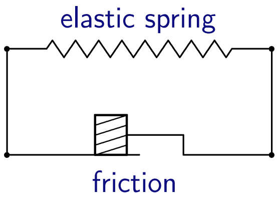

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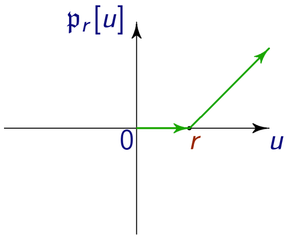

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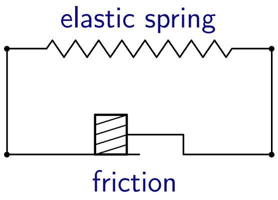

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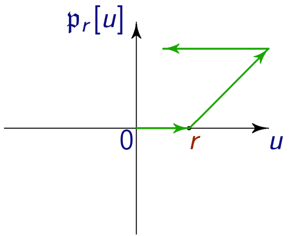

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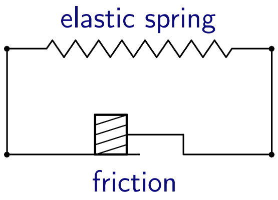

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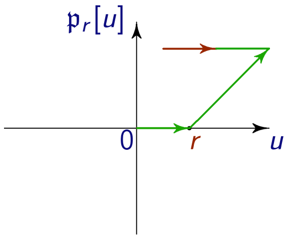

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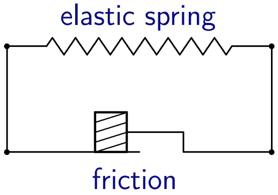

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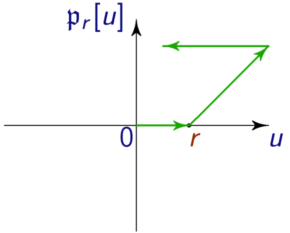

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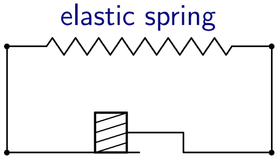

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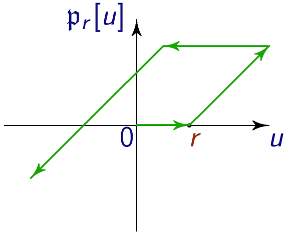

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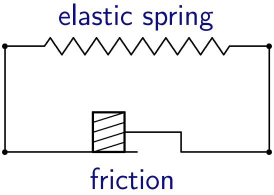

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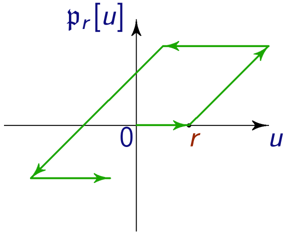

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Prandtl operator in the space of continuous functions

For each $r > 0$ and each $u \in C[0,T]$, the functions $\xi_r = \mathfrak{p}_r[u]$ have bounded variation, and the Riemann-Stieltjes variational inequality holds:

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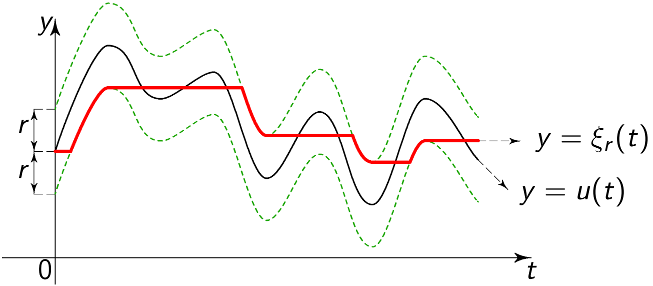
The function $\xi_r = \mathfrak{p}_r[u]$ is precisely the function in the r-neighborhood of u which has minimal total variation! (Tronel, Vladimirov 2000)



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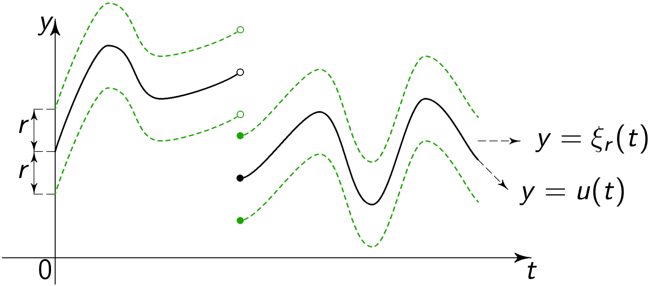

What happens if u is discontinuous?

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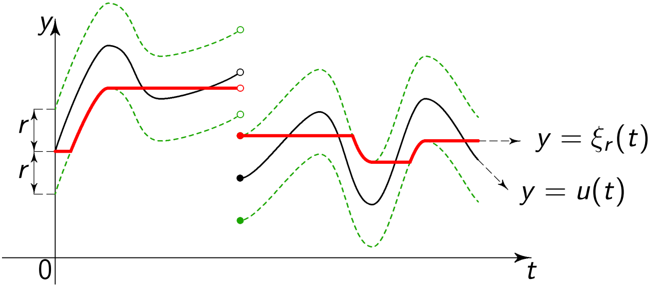

What happens if u is discontinuous?

Prandtl operator in the space of regulated functions

For each $r > 0$ and each $u \in G_R[0,T]$, the functions $\xi_r = \mathfrak{p}_r[u]$ have bounded variation, and the Kurzweil-Stieltjes variational inequality holds:

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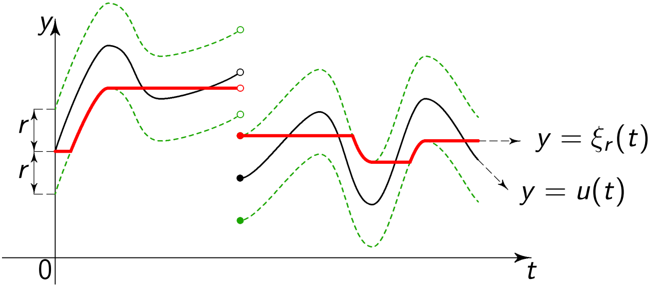
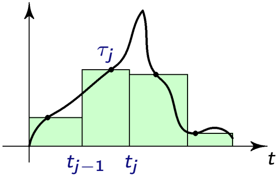


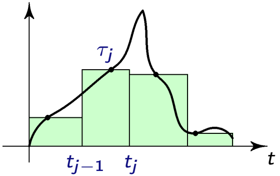
What happens if u is discontinuous? Kurzweil integral!

Comparison between the Riemann-Stieltjes and the Kurzweil concept of $\int_a^b f(t)\,\mathrm{d}g(t)$



Riemann-Stieltjes

$$\sum_{j=1}^{m} f(\tau_j)\,(g(t_j) - g(t_{j-1}))$$



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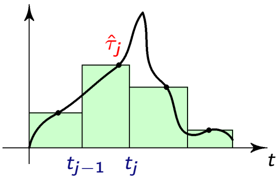

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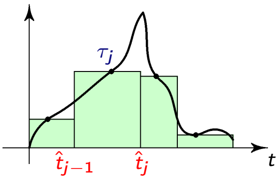
Small change of the integral sum if the tags stay close



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Small change of the integral sum if the division points stay close

General properties

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(i) Let $G(a,b)$ denote the space of regulated functions on $[a,b]$, that is, functions $f:[a,b]\to\mathbb{R}$ which admit at each point $t\in[a,b]$ both limits $f(t+), f(t-)$,

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(ii) The following substitution formula holds:
Let $f:[0,b]\to\mathbb{R}$ be bounded and such that $f\big|_{[a,b]}\in G(a,b)$ for all $a\in(0,b)$. Let $\varphi:[0,b]\to[0,B]$ be nondecreasing, $\varphi(0)=0$, $\varphi(b)=B$, and let $\psi\in BV(0,B)$ be a right continuous function. For $s\in[0,B]$ put

$$\varphi^{-1}(s)=\inf\{t\in[0,b]:s\le\varphi(t)\}.$$

Then for all $a\in[0,b)$ we have

$$\int_a^b f(t)\,\mathrm{d}(\psi\circ\varphi)(t)=\int_{\varphi(a)}^{\varphi(b)} f(\varphi^{-1}(s))\,\mathrm{d}\psi(s)\,.$$

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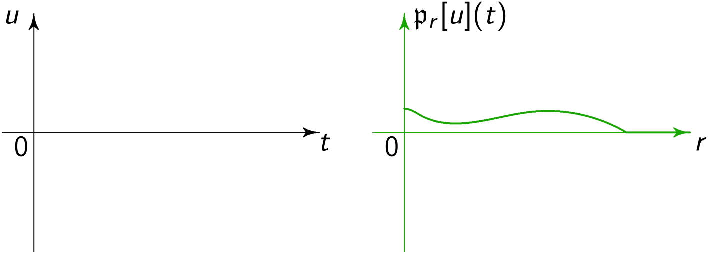
Theorem (Krejčí, Laurençot 2001). *For every right continuous* $u \in G(0,T)$ *there exists a unique right continuous solution* $\xi_r \in BV(0,T)$ *of the Kurzweil integral variational inequality, it minimizes the total variation in the* r*-neighborhood of* u*, and the solution mapping* $\mathfrak{p}_r : G_R(0,T) \to G_R(0,T)$ *is Lipschitz continuous.*



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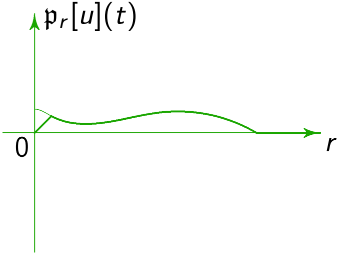

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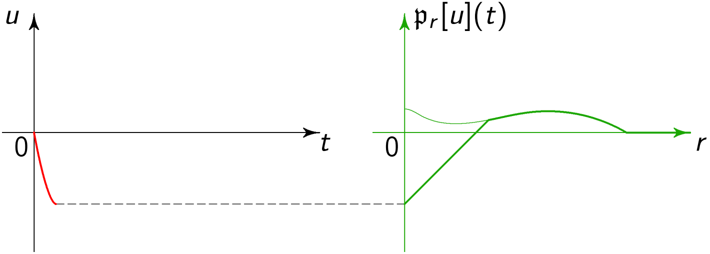

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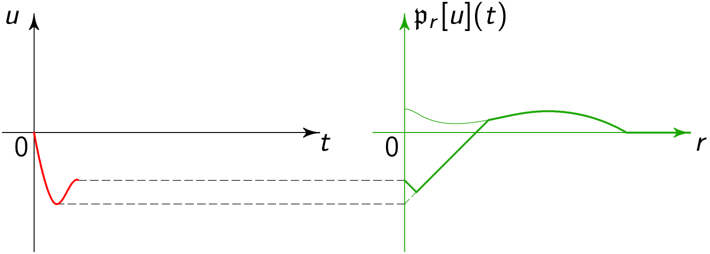

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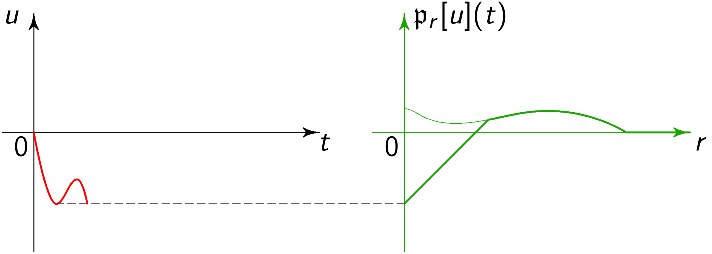

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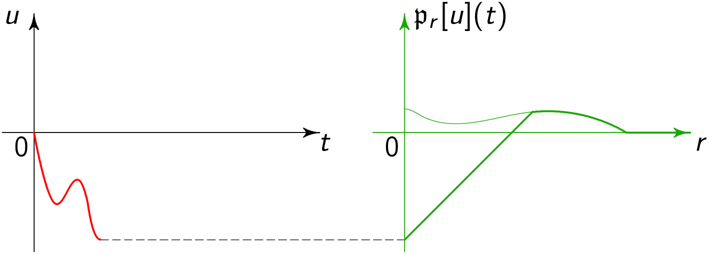

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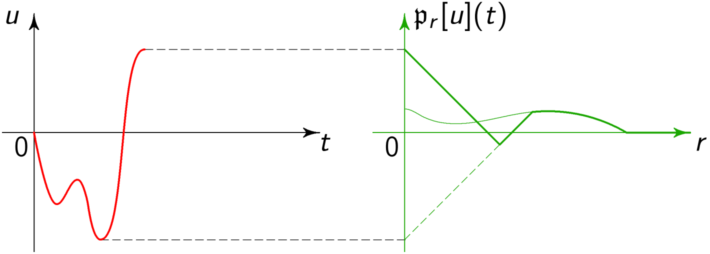

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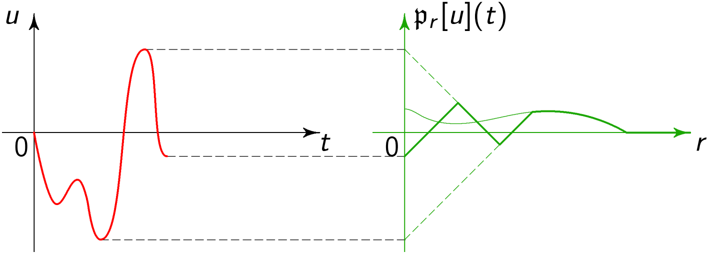

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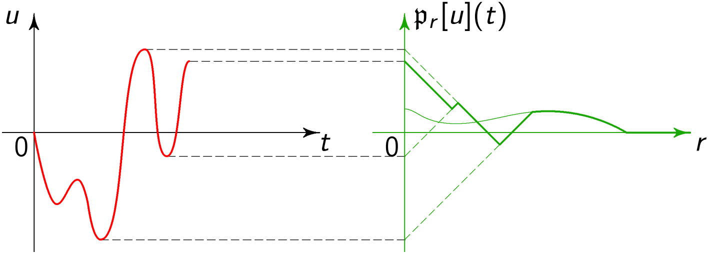

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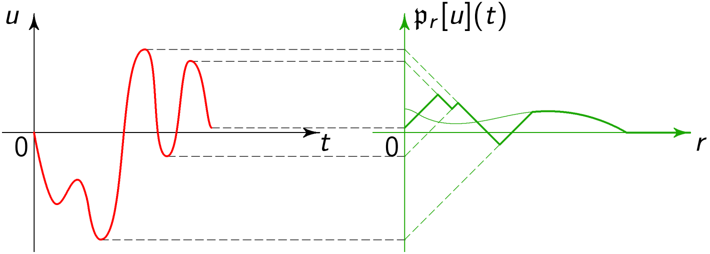

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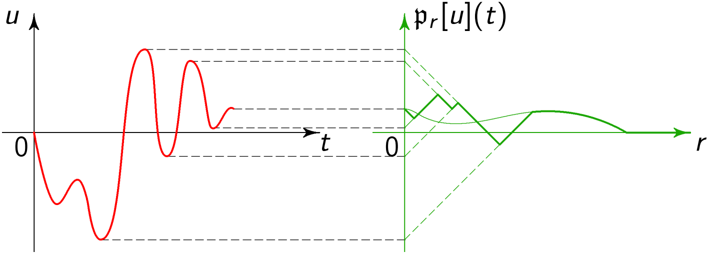

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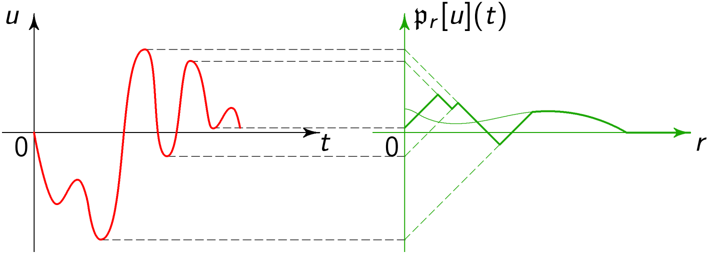

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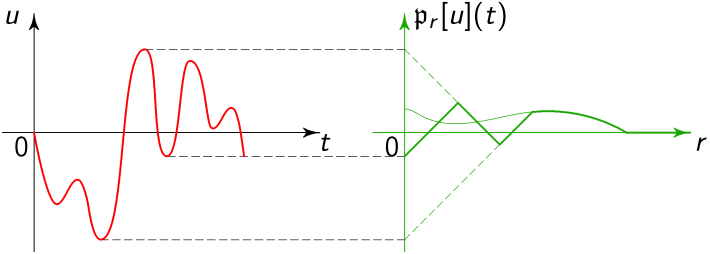

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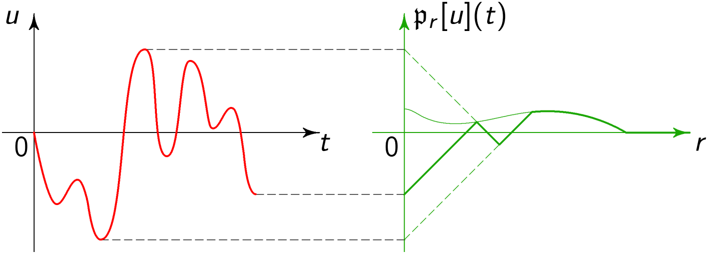

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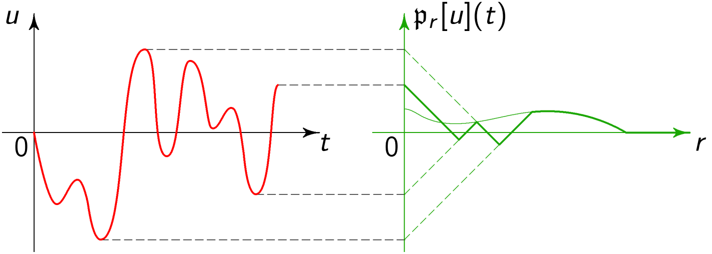
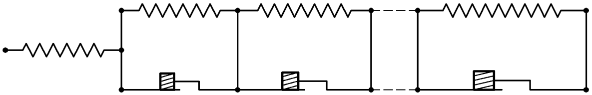


Prandtl-Ishlinskii model



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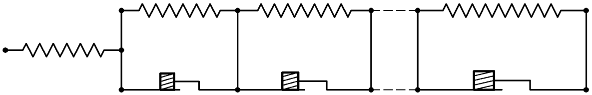

Consider a system of Prandtl models of elastic springs with friction combined in series with different values of the yield points $r > 0$ and different elasticity moduli $1/\gamma(r) > 0$, under given stress $u(t)$. The total strain $\xi(t)$ is given by the sum (integral) of partial strains

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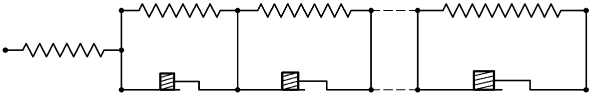

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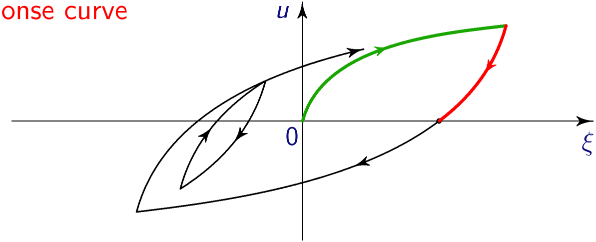
that is,

$$\xi(t) = \gamma(0)u(t) + \int_0^\infty \gamma(r)\mathfrak{p}_r[u](t)\,\mathrm{d}r\,.$$

Primary response curve



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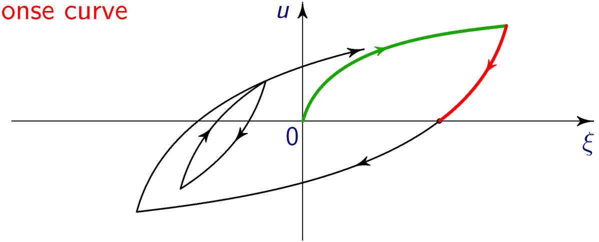

We define primary response curve

$$\xi = g(u) := \gamma(0)u + \int_0^u (u-r)\gamma(r)\,\mathrm{d}r.$$

Then $g'(u) = \gamma(0) + \int_0^u \gamma(r)\,\mathrm{d}r$, $g'' = \gamma$, and we can rewrite the Prandtl-Ishlinskii formula as

$$\boxed{\xi(t) = P_g[u] := -\int_0^\infty \frac{\partial^-}{\partial r}\mathfrak{p}_r[u](t)\,\mathrm{d}g(r)\,.}$$

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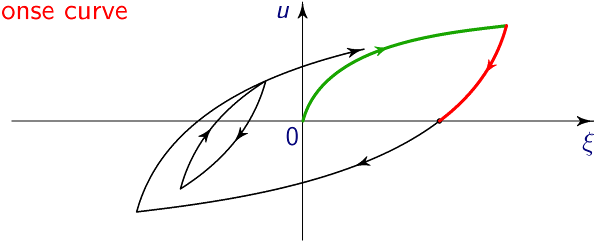

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All Prandtl-Ishlinskii hysteresis branches are images of the primary response curve scaled by a factor 2, shifted, and possibly rotated by 180°.

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Theorem (Krejčí, Lamba, Melnik, Rachinskii 2015). *Let g_1, g_2 be right continuous nondecreasing functions, and let P_{g_1}, P_{g_2} be Prandtl-Ishlinskii operators with primary response curves g_1, g_2. Then the superposed operator $P_{g_1} \circ P_{g_2}$ is the Prandtl-Ishlinskii operator with primary response curve $g_1 \circ g_2$.*

Financial markets

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Consider trading in a time interval $t \in [0, T]$ with one given commodity, the basic price of one unit (determined e.g. by changing production costs, transportation costs, natural disasters, etc.) at time t is given in terms of a piecewise constant function $p(t) > 0$. Its market price is

$$q(t) = \varrho^{\kappa}(t)p(t)\,,$$

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Let A be the set of *traders* who buy or sell the assets. The traders are divided into classes $A_{d,a} \subset A$ according to their *trading strategy* characterized by numbers $d, a > 0$ describing their *degree of risk-taking*.

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(a) If α buys the asset at time t_0 for the price $q(t_0)$, he keeps it until the relative decrease with respect to the maximal value for $t > t_0$ is larger or equal to d, that is, the selling time is given as

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$$t_2 = \min \left\{ t > t_1 : \frac{q(t)}{\min\{q(\tau) : t_1 \le \tau \le t\}} \ge 1 + a \right\}.$$

Logarithmic prices

We further introduce the relative *log-prices* $v(t) = \log \frac{p(t)}{\bar{p}}$, $w(t) = \log \frac{q(t)}{\bar{p}}$ with respect to a fixed currency unit $\bar{p}$, and the *logarithmic market sentiment* $\sigma(t) = \log \varrho(t)$, so that we have

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Logarithmic prices II

For simplicity, we reduce the problem to a single parameter $r > 0$ by choosing $a = a(r)$, $d = d(r)$ such that

$$\log(1 - d(r)) = -r\,, \quad \log(1 + a(r)) = r\,,$$

and we denote $A_r := A_{d(r),a(r)}$.

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Logarithmic prices II

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$$\log(1 - d(r)) = -r\,, \quad \log(1 + a(r)) = r\,,$$

and we denote $A_r := A_{d(r),a(r)}$. The trading strategy in A_r then becomes

(a'') If α buys the asset at time t_0 for the log-price $w(t_0)$, the next selling time t_1 is defined as the minimal time $t > t_0$ such that

$$w(t) - \max\{w(\tau) : t_0 \le \tau \le t\} \le -r.$$

(b'') If α sells the asset at time t_1 for the log-price $w(t_1)$, the next buying time t_2 is defined as the minimal time $t > t_1$ such that

$$w(t) - \min\{w(\tau) : t_1 \le \tau \le t\} \ge r.$$

The fact of possession or non-possession of the asset by traders from A_r is described by a function $S_r(t)$ taking only the values 1 (traders from A_r possess the asset at time t) or -1 (do not possess the asset).

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Theorem. *Let the log-price $w : [0, T] \to [-w_0, w_0]$ be a right continuous step function. Then for each $t \in (0, T]$ and $r \in [0, 2w_0]$ the "ownership" function $S_r(t)$ corresponding to trading strategies (a"), (b") is represented by the formula*

$$S_r(t) = -\frac{\partial^-}{\partial r}\mathfrak{p}_r[2w](t).$$

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More specifically, we assume that there exists a non-negative function $\mu(r)$ characterizing the *relative weight of the opinion* of the traders in A_r, and that

$$\sigma(t) = \int_0^{2w_0} \mu(r) S_r(t)\, \mathrm{d}r.$$

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Recall that

$$w(t) = \kappa\sigma(t) + v(t)\,, \quad S_r(t) = -\frac{\partial^-}{\partial r}\mathfrak{p}_r[2w](t).$$

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Theorem. *The logarithmic market sentiment* $\sigma(t)$ *can be represented by the Prandtl-Ishlinskii operator* $\mathcal{P}_\phi$ *with primary response curve*

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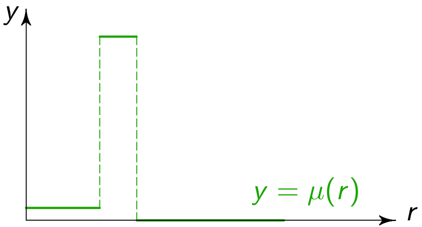

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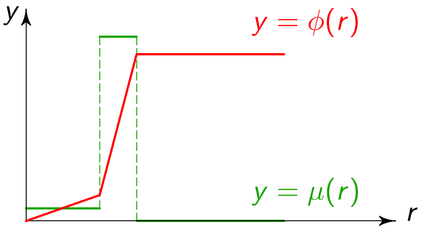

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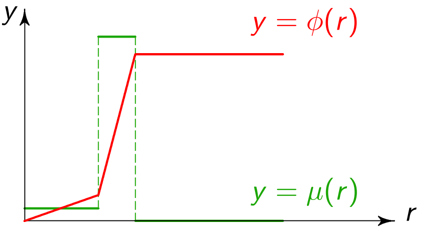

The market log-price $w(t)$ is thus obtained as the solution of the equation

$$w(t) = \kappa\mathcal{P}_\phi[2w](t)) + v(t).$$

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has a continuous solution $w(t) = \frac{1}{2}(I - 2\kappa\mathcal{P})^{-1}[2v](t)$, if and only if the primary response curve $r - 2\kappa\phi(r)$ of the operator $I - 2\kappa\mathcal{P}$ is increasing, that is,

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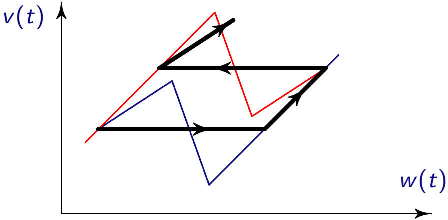

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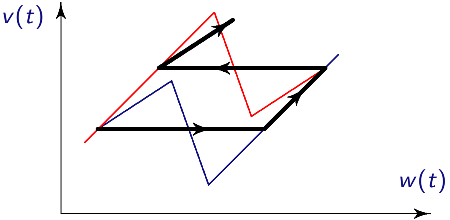

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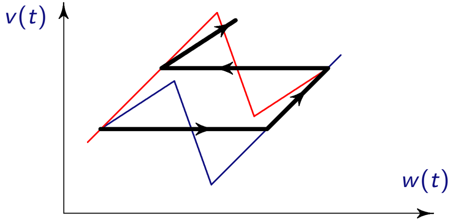

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Conclusion. Financial crush may occur when the market is controlled by a small group of dominant traders.

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- The Prandtl-Ishlinskii-Kurzweil calculus is simple and robust; error estimates can easily be derived.