Location and multiplicity results for general nonlinear fourth order BVPs and applications

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Abstract

This work concerns to the existence, non-existence, multiplicity and location results for the problem composed by the fourth order fully nonlinear equation

\[ u^{(4)}(x) + f(x, u(x), u'(x), u''(x), u'''(x)) = s \, p(x) \]  

(E)

for \( x \in [0, 1] \), \( f : [0, 1] \times \mathbb{R}^{4} \to \mathbb{R}, p : [0, 1] \to \mathbb{R}^{+} \) continuous functions and \( s \) a real parameter, with the Lidstone boundary conditions

\[ u(0) = u(1) = u''(0) = u''(1) = 0. \]  

(L)

It will be done an Ambrosetti-Prodi type discussion on \( s \). That is, there are \( s_0, s_1 \in \mathbb{R} \) such that:

- for \( s < s_0 \) or \( s > s_0 \) there is no solution of (E)-(L).
- for \( s = s_0 \) problem (E)-(L) has at least a solution.
- for \( s \in [s_0, s_1] \) (or \( s \in [s_1, s_0] \)) there are at least two solutions of (E)-(L).

The arguments used apply lower and upper solutions technique, \textit{a priori} estimations and topological degree theory.

This method will be applied to more general boundary value problems, which include the equation

\[ u^{(4)}(x) = f(x, u(x), u'(x), u''(x), u'''(x)) \]  

(1)

and the functional boundary conditions

\[
\begin{align*}
L_0 (u, u', u'', u'''(0)) &= 0 = L_1 (u, u', u'', u''(0)) \\
L_2 (u, u', u'', u'''(0), u''(0)) &= 0 = L_3 (u, u', u'', u'''(1), u''(1))
\end{align*}
\]

(2)

where \( L_i, i = 0, 1, 2, 3 \), are continuous functions satisfying some monotonicity assumptions.

Some particular cases of problem (1)-(2), such as nonlocal and multipoint problems, will be considered.

An application to a continuous model of the human spine, used in aircraft ejections, vehicle crash situations and some forms of scoliosis, will be presented.