Existence for the ODE p-Laplacian with nonlinear boundary conditions.

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Abstract

We consider

\((|y'|^{p-2}y')' = f(x, y, y'), \quad x \in [0, 1],\)
\[0 = G((y(0), y(1)); (y'(0), y'(1))),\]

where \(f : [0, 1] \times \mathbb{R}^2 \to \mathbb{R}\) and \(G : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2\) are continuous and \(p > 1\). Assuming there are lower and upper solutions, that \(f\) satisfies a suitable growth condition with respect to \(y'\), and the boundary conditions are "compatible", we prove that solutions exist. We show that if there are two suitable pairs of lower and upper solutions then there are three solutions. We apply the case \(p = 2\) result to settle a question arising in chemical reactor theory.