

Exercises for Mathematical Logic (24 Oct 2025)

16. Devise an inductive definition of the set $FV(\varphi)$ of free variables of a formula φ , of substitution $s(t_0/x_0, \dots, t_{n-1}/x_{n-1})$ and $\varphi(t_0/x_0, \dots, t_{n-1}/x_{n-1})$, and of the notion of t being free for x in φ .

17. Prove that if a term $t(x_0, \dots, x_{n-1}, y)$ is free for y in a formula $\varphi(x_0, \dots, x_{n-1}, y)$, then for all terms $s_0, \dots, s_{n-1}, r$, the formula $(\varphi(t/y))(s_0/x_0, \dots, s_{n-1}/x_{n-1}, r/y)$ is syntactically identical to the formula $\varphi(s_0/x_0, \dots, s_{n-1}/x_{n-1}, t(s_0/x_0, \dots, s_{n-1}/x_{n-1}, r/y)/y)$.

18. Let $\mathcal{A}$ be an L -structure, t a closed L -term such that $t^{\mathcal{A}} = a \in A$, and $\varphi(x)$ an L -formula. Show that $\mathcal{A} \models \phi(t)$ iff $\mathcal{A} \models \phi(\underline{a})$.

19. Consider a modification of the first-order proof system given in the lecture such that the axioms of equality are replaced with the axiom $x = x$ and the axiom schema $t = s \wedge \varphi(t/x) \rightarrow \varphi(s/x)$ for all formulas φ and terms t, s free for x in φ . Show that this is equivalent to the original proof system.