



## Letter

## Symmetric twin paradox for free-falling frames: Argument against the relativistic time dilation?

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## ABSTRACT

The twin paradox, a classical puzzle in Special Relativity (SR), is typically resolved by acknowledging the asymmetry introduced by the acceleration of one twin's frame, associated with a rocket's motion through space. A similar paradox arises without any accelerated system, involving three or more inertial systems. This is commonly resolved by employing the relativistic concepts of clock synchronization and simultaneity. Here, we reformulate the paradox for two free-falling systems, where the twins traverse identical circular orbits in opposite directions around a central mass with a spherically symmetric gravitational field. This redefined version of the paradox eliminates asymmetry inherent in the original problem. Since both frames are free-falling, they can be viewed as locally inertial according to Einstein's Equivalence Principle. Additionally, no synchronization with clocks in other frames is needed. We explore a potential resolution to the paradox and highlight that there may be a misinterpretation of the Lorentz transformation in SR.

## 1. Introduction

The twin paradox (or clock paradox) has been a subject of great interest and intense discussions for many decades [1–7]. It is one of the most famous paradoxes in Einstein's Special Theory of Relativity (SR), exploring the effects of time dilation resulting from relative motion of inertial frames. The paradox is formulated as follows: we consider two identical twins, one staying at home and the other traveling in a rocket at relativistic velocity into space and then returning home. Due to their relative motion, each twin's time should run at a different rate according to SR. When they meet again, each twin should observe the other as being younger. This is a direct consequence of the Lorentz transformation, which predicts reciprocal dilation of moving clocks.

The most widely accepted solution to this paradox is attributed to Paul Langevin [8], who noted the lack of fully symmetry in the situation, as the twin in the rocket experiences acceleration. Consequently, the traveling twin ages less than the twin at home [4,9,10].

To remove the effect of acceleration, various modifications of the twin paradox have been proposed. For instance, in a closed universe scenario, the acceleration of the traveling twin is avoided as the twin can return to Earth without changing direction [11–13]. Another modification is the 'three brothers' paradox, introducing a third clock carried by a third sibling. Instead of turning the corner and returning back, the

traveling twin's clock is synchronized with a third clock already moving at the opposite velocity towards Earth [3].

The solution to these modified paradoxes without acceleration lies in understanding the differences in the synchronization concept between classical physics and SR. Specifically, the concept of simultaneity in SR is fundamentally different from that in classical physics. Consequently, in the 'three brothers' paradox, each inertial frame - stationary, departing and returning - experiences different simultaneity in SR, leading to differential aging [14,3,4].

Despite the widely accepted solutions based on acceleration or synchronization arguments, there are dissenting voices in the relativistic literature, considering these arguments improper or unphysical. They view the twin paradox as a demonstration of the logical inconsistency of SR and challenge the concept of relativity of time and space [15–25]. Notably, physicist Herbert Dingle raised substantial criticism of SR, particularly through his examination of the twin paradox [26–29], which sparked widespread debate and controversy, being published during several years in Nature [15,30–33]. The whole story about the Dingle's criticism of SR is thoroughly described and commented in detail by Chang [34].

This paper aims to contribute to the ongoing debate about the twin paradox by presenting another simple thought experiment, the interpretation of which remains unclear or controversial within the Special

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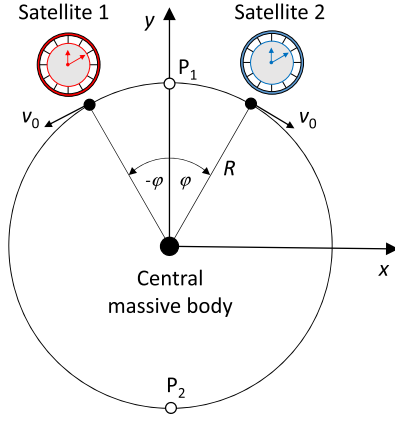


Fig. 1. Scheme of two satellites in the gravitational field, orbiting around a central massive body.  $P_1$  and  $P_2$  are points of encounter of the satellites,  $\varphi$  is the polar angle,  $v_0$  is the orbiting velocity of satellites, and  $R$  is the radius of the orbit.

and/or General Relativity (GR). Given its focus on free-falling systems in gravitational fields, it may also probe the validity of the Einstein's Equivalence Principle [35–38], being considered as an extension of the relativity principle. The paper should help enhance our understanding of relativistic time dilation resulting from the relative motion of observers in inertial and/or free-falling frames.

## 2. Formulation of the twin paradox in the gravitational field

The formulation of the twin paradox and its variations suffer from basic difficulties: (i) the inherent asymmetry present in the standard version due to the acceleration experienced by the traveling twin, or (ii) the need to introduce additional frames and clock synchronization in fully symmetric versions without acceleration. Both issues can be addressed in the following formulation of the paradox.

Consider twins (or clocks) on satellites orbiting around a massive, spherically symmetric body in a vacuum. These satellites travel along identical stationary circular orbits at the same velocities but in opposite directions (see Fig. 1). The gravitational and centrifugal forces balance out, rendering the satellites as free-falling systems where the twins perceive no acceleration. The scenario is entirely symmetrical, and the twins will encounter each other repeatedly at two defined points along the orbit (designated by polar angles  $0^\circ$  and  $180^\circ$ ).

The ‘circular’ twin paradox closely resembles the original twin paradox. From the perspective of each twin, the other twin is repeatedly receding and approaching. The mutual velocity is time-dependent: it is zero when the distance between the twins is maximum and it is maximum when the twins meet. The essential difference compared to the classical twin paradox is that neither twin perceives acceleration. Hence, each twin can consider his frame as stationary and the frame of the other twin as moving. Additionally, this formulation eliminates any need for clock synchronization with other frames because the twins meet repeatedly and their clocks can be synchronized at one of encounters. Naturally, we seek to determine the time displayed by their clocks at subsequent encounters.

Intuitively, one might expect both twins’ clocks to show the same time at every encounter due to the full symmetry of the problem. However, the situation becomes more complex when considering the principles of SR and GR. In all inertial frames, the proper speed of light remains constant at  $c$ , and the Lorentz transformation predicts time dilation between frames. Similarly, the proper speed of light remains  $c$  in all free-falling systems, suggesting that the Lorentz transformation should still apply. Consequently, time dilation due to relative motion of frames should occur, leading each twin to perceive the other as being younger. The age difference should progressively increase at successive encounters.

### 2.1. Mathematical description of the paradox

Let us consider two satellites circulating in opposite directions along an orbit of radius  $R$  with velocity  $v_0$  (see Fig. 1). The relative velocity of the satellites with respect to each other is described by the relativistic velocity-addition formula [39, their eq. 5.2]

$$v = \frac{2v_0}{1 + (v_0^2/c^2) \cos^2 \omega t} \cos \omega t, \quad (1)$$

where  $v_0 = \omega R$  represents the satellite velocity in the frame of the central massive body,  $\omega = 2\pi/T_0$  is the angular velocity,  $T_0$  is the orbital period, and  $t$  is time. The time dilation  $dt$  is expressed according to SR as

$$dt = \gamma dt' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt', \quad (2)$$

where  $\gamma$  is the Lorentz factor. Subsequently, time  $t$  of the clock at rest with respect to the moving clock showing time  $t'$  is

$$t = \int_0^{t'} \frac{dt'}{\sqrt{1 - \frac{v^2}{c^2}}} = \int_0^{t'} \sqrt{1 - \frac{4(v_0^2/c^2) \cos^2 \omega t'}{(1 + (v_0^2/c^2) \cos^2 \omega t')^2}}^{-1} dt'. \quad (3)$$

After the satellites complete one orbit with a period  $T'_0$ , the orbital time dilation  $T_0$  is

$$T_0 = \int_0^{T'_0} \sqrt{1 - \frac{4(v_0^2/c^2) \cos^2 \omega t'}{(1 + (v_0^2/c^2) \cos^2 \omega t')^2}}^{-1} dt' \\ = \frac{2T'_0}{\pi} \int_0^{\pi/2} \frac{1 + (v_0^2/c^2) \cos^2 \varphi}{1 - (v_0^2/c^2) \cos^2 \varphi} d\varphi. \quad (4)$$

Since the problem is fully symmetric, it seems evident that the twins should be of the same age and the clocks of both satellites must show the same time when the twins repeatedly meet. However, considering that  $T_0$  must equal  $T'_0$  in Eq. (4), Eqs. (2)–(4) for time dilation are either erroneous or not applicable in this case. The equality  $T_0 = T'_0$  holds true only for  $v_0 = 0$ ; for other values of  $v_0 > 0$ , we get  $T_0 > T'_0$ . Note that  $T_0$  even diverges as  $v_0$  approaches  $c$ .

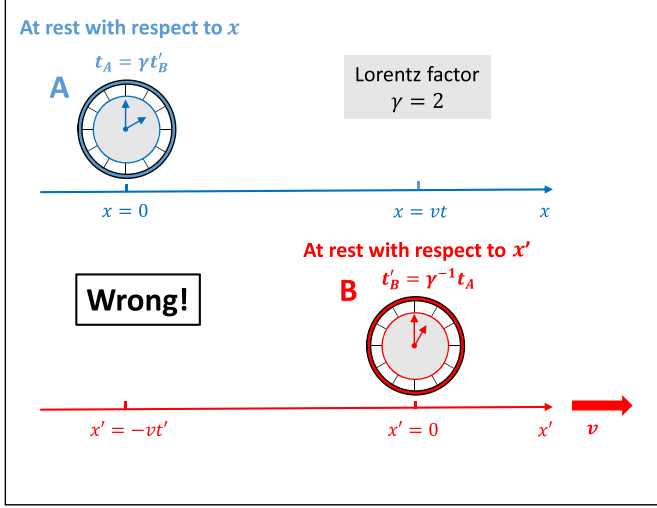
### 2.2. What is the solution of the paradox?

To resolve the aforementioned paradox, we need to assess whether the Lorentz transformation, which relates times  $t$  and  $t'$  in the mutually moving frames in SR, is applicable in this scenario. If applicable, we must then examine whether it is physically justified and properly interpreted.

Given that the satellites are free-falling frames, Einstein's Equivalence Principle [35,36] ensures that they should behave similarly to inertial frames in Minkowski spacetime. Moreover, no gravitational shift needs to be considered because the satellites maintain a fixed distance from the central body ensuring a constant gravitational potential. The only necessary correction is for relativistic time dilation, which should not pose any difficulty. Similar corrections have been applied to satellites orbiting Earth, for example, they are routinely used in GPS satellite data [40,41]. The difference here is that the GPS corrections are calculated between a satellite and an observation point on Earth's surface, whereas we aim to calculate relativistic corrections between two satellites. Thus, there is no apparent reason why the Lorentz transformation should not be applied to calculate time dilation in the presented paradox.

Next, we will examine whether the paradox could arise from an erroneous use or misinterpretation of the Lorentz transformation. The Lorentz transformation between mutually moving frames  $(x, t)$  and  $(x', t')$  is expressed by the following equations [39, their eq. 4.3]

## Time dilation predicted by the Lorentz transformation I



**Fig. 2.** Standard interpretation of the Lorentz transformation in SR. Frames  $(x, t)$  and  $(x', t')$  are moving at non-zero relative velocity  $v$ . It is incorrectly assumed that the Lorentz transformation predicts time dilation between clock A in frame  $(x, t)$  at  $x = 0$  for all  $t$  and clock B in the frame  $(x', t')$  at  $x' = 0$  for all  $t'$ . Hence, if we assume a constant Lorentz factor  $\gamma$  with value of 2, after one hour in the frame  $(x', t')$ , clock A should show twice larger time than clock B. For time-dependent  $\gamma$ , times  $t_A$  and  $t'_B$  in the plot should be substituted by the time differentials  $dt_A$  and  $dt'_B$ .

$$t' = \gamma \left( t - \frac{v}{c^2} x \right), \quad (5)$$

$$x' = \gamma (x - vt), \quad (6)$$

and its inverse reads

$$t = \gamma \left( t' + \frac{v}{c^2} x' \right), \quad (7)$$

$$x = \gamma (x' + vt'), \quad (8)$$

where  $\gamma = 1/\sqrt{1-v^2/c^2}$  is the Lorentz factor, velocity  $v$  is directed along the  $x$ -axis, and  $y' = y$ ,  $z' = z$ .

These equations satisfy the Lorentz invariance condition

$$c^2 t^2 - x^2 = c^2 t'^2 - x'^2. \quad (9)$$

For simplicity, we assume in Eqs. (5)-(9) a constant relative velocity  $v$  between the frames and consequently a constant Lorentz factor  $\gamma$ . The generalization of the Lorentz transformation to a time-dependent  $\gamma = \gamma(t)$  in Eq. (2) is straightforward as shown in Appendix A.

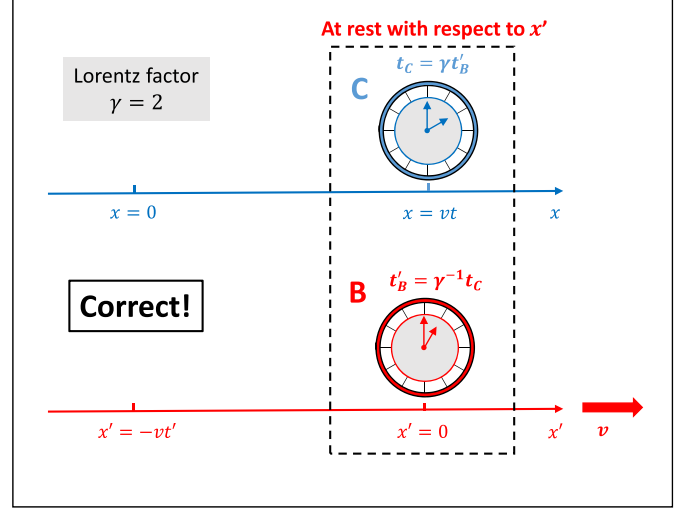
To understand the physical meaning of Eqs. (5)-(9), we first examine the relation between the original and transformed times  $t$  and  $t'$  for an object located in the origin of coordinates  $x' = 0$ . Using Eqs. (7) and (8), we obtain

$$t = \gamma t', \quad (10)$$

$$x = \gamma vt' = vt. \quad (11)$$

Eq. (10) is the standard equation for the relativistic time dilation between two inertial frames, interpreted that time  $t'$  in a moving frame runs slower than time  $t$  in the frame at rest (see Fig. 2, clocks A and B). However, this interpretation is misleading because Eq. (10) does not define the time relation between  $t$  and  $t'$  for two mutually moving objects; it does not describe a situation where one object remains fixed at  $x = 0$  and the other at  $x' = 0$  for all times  $t$  or  $t'$ . Actually, Eq. (10) defines the time relation between  $t$  and  $t'$  for two objects, which are at the same point in the moving frame ( $x' = 0$ ) for all times  $t$  or  $t'$  (see Fig. 3, clocks B and C),

## Time dilation predicted by the Lorentz transformation II



**Fig. 3.** Correct interpretation of the Lorentz transformation. Frames  $(x, t)$  and  $(x', t')$  are moving at non-zero relative velocity  $v$ , but clocks B and C are at rest to each other. The Lorentz transformation predicts time dilation between clocks B and C. For the sake of simplicity, we assume a constant Lorentz factor  $\gamma$  with value of 2. Hence after one hour in the frame  $(x', t')$ , clock C will show twice larger time than clock B. For time-dependent  $\gamma$ , times  $t'_B$  and  $t_C$  in the plot should be substituted by the time differentials  $dt'_B$  and  $dt_C$ .

$$t(x = vt) = \gamma t' (x' = 0). \quad (12)$$

This implies that both clocks (Fig. 3, clocks B and C) do not move relative to each other; they are at rest with respect to each other, both located at the same point,  $x' = 0$ . The only difference is that clock B shows time  $t'$  and clock C shows time  $t$ .

To relate times  $t$  and  $t'$  for two mutually moving objects, we need to utilize the Lorentz invariance condition defined in Eq. (9). Let us assume the first clock in frame  $(x, t)$  is situated at  $x = 0$  for all times  $t$  (Fig. 4, clock A) and another clock in frame  $(x', t')$  situated at  $x' = 0$  for all times  $t'$  (Fig. 4, clock B). Inserting  $x = 0$  and  $x' = 0$  into the Lorentz invariance condition expressed in Eq. (9), we readily obtain

$$t = t', \quad (13)$$

which means that no time dilation should be observed for mutually moving clocks (Fig. 4, clocks A and B). We arrive at the same conclusion when we take into account the time-varying Lorentz factor  $\gamma$ , as explained in Appendix A.

## 3. Discussion

We have demonstrated that the time dilation between moving frames stems from misinterpretation of the Lorentz transformation. A proper analysis of the Lorentz transformation indicates that clocks in moving frames display the same time regardless of their relative velocity. This elucidates the twin paradox in a gravitational field, where twins repeatedly meet while orbiting at high speeds in opposite directions around a massive spherically symmetric body. Clearly, the clocks must show the same time in any physically meaningful theory, and the Lorentz transformation aligns with this observation. Similar arguments can be applied to explain the classical twin paradox, where the Lorentz transformation does not predict a difference in age, when the twins meet again.

With this understanding, we need to address how to physically interpret the time dilation expressed in Eq. (10). As mentioned earlier, this equation applies to the relation between clocks B and C in Fig. 3. Both clocks are at rest relative to each other: clock B is situated at the origin of frame  $(x', t')$  and clock C is considered to be part of frame  $(x, t)$ . While clock C is at rest with respect to clock B, it is moving relative to

## Time dilation predicted by the Lorentz transformation III

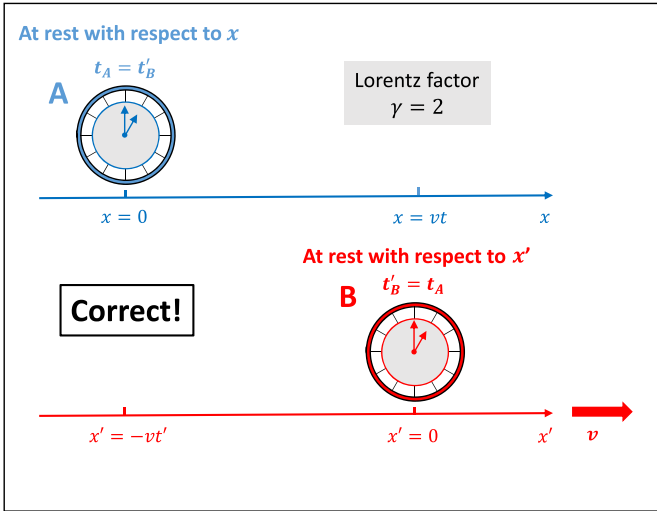


Fig. 4. Correct interpretation of the Lorentz transformation. Frames  $(x, t)$  and  $(x', t')$  are moving at non-zero relative velocity  $v$ . For the sake of simplicity, we assume a constant Lorentz factor  $\gamma$  with value of 2. The Lorentz transformation predicts no time dilation between clock A in frame  $(x, t)$  at  $x = 0$  for all  $t$  and clock B in frame  $(x', t')$  at  $x' = 0$  for all  $t'$ . For time-dependent  $\gamma$ , times  $t_A$  and  $t'_B$  in the plot should be substituted by the time differentials  $dt_A$  and  $dt'_B$ .

the origin of frame  $(x, t)$ . Eq. (10) predicts that clocks B and C would display different times, which is physically absurd. Clocks at the same point in space with a zero relative velocity cannot run differently.

However, the reason for this peculiar behavior of clocks is straightforward. The Lorentz transformation is a formal mathematical construct that aims to maintain the speed of light  $c$  as constant relative to the origins of both frames  $(x, t)$  and  $(x', t')$ . Consequently, there is no alternative way to mathematically satisfy this requirement. Thus, the transformation should be viewed as purely formal, preserving the constant speed of light by adjusting units. The units used for measuring time and length are modified to obtain the same numerical value of the speed of light. Obviously, this rescaling procedure is not physical.

This suggests that the requirement of the constant speed of light in all inertial frames may be questionable from a physical standpoint. Therefore, interferometric experiments measuring the constancy of the speed of light [42–47] should be carefully reassessed and properly reinterpreted as pointed out by several authors [48–50]. As shown by Vavryčuk [51], the Lorentz transformation is not the only transformation consistent with the null result of these experiments. For example, the Doppler transformation is physically more justified and also predicts the null result in the ether-drift experiments. In this case, the principle of the constant speed of light in all inertial frames is formulated to apply to the phase speed of light instead of the signal (energy) speed of light. Additionally, the Doppler transformation does not produce different ticking rates for clocks B and C in Fig. 3.

Note that while the Lorentz transformation lacks direct physical meaning, it still may remain a useful tool in solving problems in both SR and GR. It may serve as a formal parametrization, simplifying the mathematical treatment of various relativistic issues. Consequently, it may find applications in physics, but its physical interpretation should always be approached with caution.

## CRediT authorship contribution statement

Václav Vavryčuk: Writing – original draft, Methodology, Investigation, Conceptualization. Michal Křížek: Methodology, Investigation.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data were used for the research described in the article.

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Appendix A. Lorentz transformation for free-falling frames with time-dependent Lorentz factor  $\gamma$ 

Here, we will generalize the physical interpretation of the Lorentz transformation for free-falling frames with time-dependent Lorentz factor  $\gamma$ . In this case, we must use a relation between differentials  $(dx, dt)$  and  $(dx', dt')$  instead of the relation between  $(x, t)$  and  $(x', t')$ , commonly used for inertial frames,

$$dt' = \gamma \left( dt - \frac{v}{c^2} dx \right), \quad (\text{A.1})$$

$$dx' = \gamma (dx - v dt). \quad (\text{A.2})$$

The inverse Lorentz transformation reads

$$dt = \gamma \left( dt' + \frac{v}{c^2} dx' \right), \quad (\text{A.3})$$

$$dx = \gamma (dx' + v dt'). \quad (\text{A.4})$$

These equations satisfy the Lorentz invariance condition

$$c^2 dt^2 - dx^2 = c^2 dt'^2 - dx'^2. \quad (\text{A.5})$$

Similarly as for inertial systems, we first examine the relation between the original and transformed times  $t$  and  $t'$  for an object located at the origin of coordinates  $x' = 0$ . Since the object remains at rest in the frame  $(x', t')$  for all times  $t'$ , the distance element  $dx'$  is zero:  $dx' = 0$ . Using Eqs. (A.3) and (A.4), we obtain

$$dt = \gamma dt', \quad (\text{A.6})$$

$$dx = \gamma v dt' = v dt. \quad (\text{A.7})$$

Eq. (A.6) is the standard equation for time dilation in SR, interpreted that time  $t'$  in a moving frame runs slower than time  $t$  in the frame at rest (see Fig. 2, clocks A and B). However, this interpretation is misleading because Eq. (A.6) does not define the time relation between  $dt$  and  $dt'$  for two mutually moving objects. Actually, Eq. (A.6) defines the time relation between  $dt$  and  $dt'$  for two objects, which are at the same point in the moving frame ( $x' = 0$ ) for all times  $t$  or  $t'$  (see Fig. 3, clocks B and C),

$$dt(dx = v dt) = \gamma dt' (dx' = 0). \quad (\text{A.8})$$

This implies that both clocks (Fig. 3, clocks B and C) do not move relative to each other, both located at  $x' = 0$ . The only difference is that clock C shows time  $t$  and clock B shows time  $t'$ .

To relate times  $t$  and  $t'$  for two mutually moving objects, we need to utilize the Lorentz invariance condition defined in Eq. (A.5). Let us assume the first clock in frame  $(x, t)$  is situated at  $x = 0$  with a fixed position,  $dx = 0$  (Fig. 4, clock A). Analogously, we consider another clock in frame  $(x', t')$  situated at  $x' = 0$  with a fixed position,  $dx' = 0$  (Fig. 4, clock B). Inserting  $dx = 0$  and  $dx' = 0$  into the Lorentz invariance condition expressed in Eq. (A.5), we readily obtain

$$dt = dt', \quad (\text{A.9})$$



which means that no time dilation should be observed for mutually moving clocks (Fig. 4, clocks A and B).

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