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Twisting c_0 around Banach spaces

Joint research with **Antonio Avilés** and **Witold Marciszewski**

ABSTRACT. A twisted sum of Banach spaces Y and Z is a short exact sequences $0 \to Y \to X \to Z \to 0$, where X is another Banach spaces and morphisms are bounded linear operators. Such a twisted sum is *trivial* if it is equivalent to $0 \to Y \to Y \oplus Z \to Z \to 0$ (that is, if Y is isomorphically embedded onto a complemented subspace of X).

By the classical Sobczyk theorem, a twisted sum $0 \to c_0 \to X \to Z \to 0$ is always trivial for separable spaces Z. We discuss the class $\mathfrak{Z}$ of nonseparable spaces Z admitting a nontrivial twisted sum $0 \to c_0 \to X \to Z \to 0$. Problems concerning that class are often set-theoretic in nature. In particular, we have proved that the question of Cabello Sanchéz, Castillo, Kalton and Yost, if $C(K) \in \mathfrak{Z}$ for every nonmetrizable compactum K is undecidable within the usual set theory.