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THE PROBLEM OF CAUSALITY IN THE UNCERTAINTY-MEDIATED INFLATIONARY MODEL*

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Abstract: It was suggested in our previous works [Y.V. Dumin. In: *Cosmology on Small Scales 2018*, p. 136; Grav. Cosmol., **25** (2019), 169] that the Dark Energy (effective Λ-term) can be mediated by the time–energy uncertainty relation in the Mandelstam–Tamm form, which is applicable to the long-term evolution of quantum systems. Then, the amount of Dark Energy gradually decays with time, and the corresponding scale factor of the Universe increases by a “quasi-exponential” law (namely, proportional to the exponent of the square root of time) throughout the entire history of the Universe. While this universal behavior looks quite appealing, an important question arises: Does the quasi-exponential expansion resolve the major problems of the early Universe in the same way as the standard inflationary scenario? Here, we try to elucidate this issue by analyzing a causal structure of the respective space–time. It is found that the observed region of the Universe (the past light cone) covers a single causally-connected domain developing from the Planckian time (the future light cone). Consequently, there should be no appreciable inhomogeneity and anisotropy in the early Universe, creation of the topological defects will be suppressed, etc. From this point of view, the uncertainty-mediated Dark Energy can serve as a viable alternative to the standard (exponential) inflationary scenario.

Keywords: Dark Energy, uncertainty relation, inflationary model, causal structure

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1. Introduction

The Dark Energy (or the effective Λ-term) is one of cornerstones of the modern cosmology, however little is known about its nature and origin till now. In particular, it is unclear if it is a new fundamental physical constant or just an effective contribution from the underlying field theory. Yet another interpretation of this entity was proposed in our recent works [4, 5], where we assumed that the amount of Dark Energy could be derived from the quantum uncertainty relation between the time and energy in the Mandelstam–Tamm form [11]. As distinct from the well-known Heisenberg relation, which is used mostly in the context of measurements, the Mandelstam–Tamm relation is applicable to the long-term evolution of quantum systems and widely employed now in such branches of physics as quantum optics, quantum information processing, etc. [3]. So, its application to the quantum cosmology is well in the mainstream of the modern physics. (It should be mentioned that some qualitative conjectures about a possible role of the uncertainty relation in explanation of the Dark Energy were put forward even earlier by A. Coe in preprint [2], but the quantitative analysis undertaken there looked absolutely unreasonable.)

Briefly speaking, the main points of our treatment [4, 5] were as follows. We begin with the standard Robertson–Walker metric,

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - R^2(t)\Big[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\!\theta\,\mathrm{d}\varphi^2)\Big]\,, \tag{1}$$

whose temporal evolution is described by the Friedmann equation, e.g., [15]:

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3c^2}\,\rho \;-\; kc^2\frac{1}{R^2} \;+\; \frac{c^2}{3}\,\Lambda\,. \tag{2}$$

Here, r, θ, and φ are the dimensionless spherical coordinates, the coefficient k equals 1, 0, and -1 for the closed, flat, and open 3D space, respectively; R is the scale factor (called sometimes also the expansion function) of the Universe, H is the Hubble parameter, ρ is the energy density of matter in the Universe, G is the gravitational constant, and c is the speed of light in vacuum. (Since the resulting formulas for the uncertainty-mediated Dark Energy will look a bit unusual, we prefer to keep here all the dimensional constants.)

The Λ-term appearing in (2) can be formally associated with the vacuum energy density ρ_{v}:

$$\Lambda = \frac{8\pi G\rho_{\mathrm{v}}}{c^4}\,. \tag{3}$$

So, our main conjecture is to express ρ_{v} through the vacuum energy in the Planck volume,

$$\Delta E = \rho_{\mathrm{v}} l_{\mathrm{P}}^3 \tag{4}$$

(where $l_{\mathrm{P}}=\sqrt{G\hbar/c^3}$ is the Planck length, $\hbar = h/2\pi$), and then to estimate this energy through the uncertainty relation with the equality sign:

$$\Delta E\,\Delta t \approx \frac{C_{\mathrm{UR}}}{2}\,\hbar\,, \tag{5}$$

where $\Delta t \equiv t$ is the total time of cosmological evolution (i.e., the age of the Universe). The numerical coefficient $C_{\rm UR}$ equals 1 in the well-known Heisenberg's case (related mostly to the measurement problems) and π in the Mandelstam–Tamm's situation (referring to the long-term evolution) [3]. Anyway, this coefficient gives only the lower bound on the product of the corresponding uncertainties. So, all subsequent formulas will be valid, strictly speaking, only up to this numerical coefficient.

As a result, we get the effective time-dependent Λ-term,

$$\Lambda(t) = \frac{4\pi C_{\rm UR}}{c\, l_{\rm P}} \, \frac{1}{t} \, , \tag{6}$$

and its substitution into (2) leads to the master equation of our cosmological model:

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3c^2}\, \rho \, - \, kc^2 \frac{1}{R^2} \, + \, \frac{4\pi C_{\rm UR}}{3\tau} \, \frac{1}{t} \, , \tag{7}$$

where $\tau = l_{\rm P}/c = \sqrt{G\hbar/c^5}$ is the Planck time. From the mathematical point of view, this differential equation is non-autonomous, i.e., it involves the explicit dependence on time, which is a quite unusual situation in cosmology. However, as follows from the detailed analysis, this fact does not result in any substantial peculiarities of the solution: it turns out to be well between the solutions of usual cosmological equations obtained under the various assumptions.

The simplest case, already considered in our previous works [4, 5], corresponds to the spatially-flat Universe ($k = 0$) and the ignorable energy density of matter ($\rho \approx 0$). Then, formula (7) is reduced to

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{4\pi C_{\rm UR}}{3\tau} \, \frac{1}{t} \, , \tag{8}$$

which can be trivially integrated:

$$R(t) = R^* \exp\!\left(\sqrt{\frac{16\pi C_{\rm UR}}{3}} \, \sqrt{\frac{t}{\tau}} \right) , \tag{9}$$

where $R^* = R(0)$ is the integration constant; and we consider only the solution increasing with time, i.e., the expanding Universe.

Therefore, the entire cosmological evolution is described by the universal "quasi-exponential" function (9) instead of being composed of a few absolutely different stages (dominated by the Dark Energy, radiation, dust-like matter, and again the Dark Energy) in the standard cosmological scenario, as illustrated in Figure 1 from the paper [4]. So, the puzzle of two absolutely different Λ-terms in the very early Universe and nowadays [14] becomes naturally resolved.

Yet another important feature of the new model is a considerably increased age of the Universe T as compared to its value T^* in the standard cosmology. Really, as follows from (8),

$$T \approx (T^*/\tau)\, T^* , \tag{10}$$

where $(T^*/\tau) \approx 10^{61}$ is a huge numerical factor; for more details, see Eq. (10) in [5]. In other words, although the Universe is expanding, it becomes in some sense "quasi-perpetual". This is not surprising, because function (9) grows much slower than the pure exponent.

In principle, employing the perturbation theory, it is not difficult to get a refined solution of the equation (7), when a small contribution from the ordinary matter is taken into account; for more details, see Eqs. (10)–(14) in [4]. Unfortunately, the corresponding solution is not so interesting, because the first-order correction becomes noticeable only at the Planckian scale.

2. Application to the early Universe

As is known, the dominant modern paradigm for the description of the early Universe is the inflationary scenario; e.g., reviews [6, 12]. Its emergence in the early 1980's was caused by the fact that, in the framework of the hot Big-Bang model with a power-law expansion, the observed region of the Universe (i.e., the past light cone) contains a large number of the subregions developing independently from the Big Bang (the future light cones). Therefore, one should expect a considerable variation between the properties of such subregions, resulting in a huge inhomogeneity of the fundamental characteristics of the observed Universe, e.g., temperature of the cosmic microwave background radiation (CMB), etc. Besides, if the Universe contains the fields experiencing the symmetry-breaking phase transformations (e.g., Higgs fields, responsible for a generation of mass of the elementary particles), then various kinds of the topological defects should be formed at the boundaries between the subregions, while we do not detect them in observations.

The best remedy for this situation is to modify the expansion law of the Universe, e.g., from the power to exponential one [7]. Then, the observed region of the Universe will cover only one causally-connected subregion developing from the initial instant of time. So, its physical characteristics will be sufficiently homogenized in the course of its evolution, and all the above-mentioned problems should disappear. The stage of the exponential expansion can be naturally produced by the Λ-term in Friedmann equation (2); but it remains unclear how such Λ-term emerges in the early Universe?

The most popular idea in the early 1980's was that the effective Λ-term is associated with the potential energy of the non-linear scalar field in the overcooled state formed after the strongly non-equilibrium symmetry-breaking phase transformation; for a review of this approach, see [10]. Unfortunately, the attempts to find a suitable candidate for such scalar field in the theory of elementary particles (e.g., Higgs field in the electroweak theory) failed. So, a common tendency in the inflationary cosmology became to introduce the inflaton potentials quite arbitrarily (irrelevant to any particular model of elementary particles) and just to check the corresponding cosmological predictions.

Yet another approach to derive the approximately exponential expansion is to consider the gravitational Lagrangians with the high-order (e.g., quadratic) terms of curvature [17]. From the physical point of view, such terms could be attributed to the expectation values of the matter fields distorted by the curved space–time [16]. Unfortunately, since we still do not have a comprehensive theory of elementary particles, the exact functional form and numerical coefficients of the high-order terms remain unknown and are postulated a priori.

In view of the above-mentioned drawbacks, it would be desirable to find a more solid physical basis for the inflationary cosmology; and the effective Λ-term mediated by the quantum uncertainty relation could be one of the promising options. However, it should be kept in mind that it predicts the quasi-exponential expansion (9) rather than a pure exponent. So, it is unclear in advance if this mechanism can resolve the standard problems of the early Universe as efficiently as the exponential scenario? To answer this question, we need to analyse a causal structure of the respective space–time; and a convenient way to do so is to use the conformal diagrams.

First of all, it will be convenient to measure the time t from the instant of observation (so that the preceding cosmological evolution occurs at the negative times). Then, formula (9) should be rewritten as

$$R(t) = R_0 \exp\left(\sqrt{\frac{16\pi C_{\mathrm{UR}}}{3}} \, \frac{\sqrt{t+T} - \sqrt{T}}{\sqrt{\tau}} \right), \tag{11}$$

where R_0 is the present-day value of the scale factor, and $t = -T$ is the beginning of the Universe. (We prefer to not call it the Big Bang, because in the modern literature this term often refers to the onset of the "hot" stage, when the ordinary matter becomes dominant.

The conformal time, as usual, is defined as

$$\eta = \int \frac{\mathrm{d}t}{R(t)} \, . \tag{12}$$

Then, the domains of causality (i.e., the light cones) will be shaped just by the straight lines [13].

A crucial problem of the old, "pre-inflationary" cosmology was that the observable region of space (e.g., by the instant of symmetry-breaking phase transition, when the ordinary matter was formed, or by the instant of recombination, when the Universe became transparent), $\eta = -\eta^*$, contains a large number of the subregions developing independently from the initial instant of time, $\eta = -\eta_0$ (which is assumed to be at the Planckian scale), as illustrated in the top panel of Figure 1. Really, as follows from the evident geometrical consideration, the number of causally-disconnected domains (represented by the lower triangles) inside the observed region of space (the upper triangle) is given by

$$N_{\mathrm{CD}} \approx \left(\frac{\eta_0}{\eta^*} - 1 \right)^{-3} . \tag{13}$$

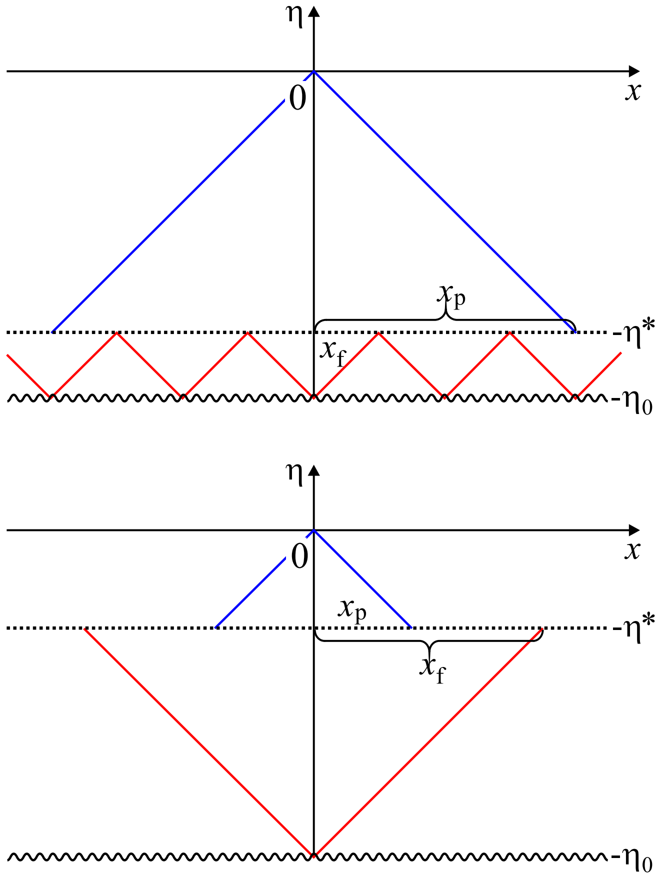

Figure 1: Conformal diagrams of the space–time corresponding to the "pre-inflationary" models, governed by the ordinary matter (top panel), and the inflationary models, governed by the Λ-term (bottom panel). Here, $x_{\rm p}$ is the size (radius) of the past light cone observable at the present time, and $x_{\rm f}$ is the size of the future light cone originating at the beginning of the Universe.

So, for the law of expansion of the ordinary matter,

$$R(t) = R_0 \left(\frac{t+T}{T} \right)^{\frac{2}{3(1+w)}} \tag{14}$$

(where $t = -T$ is the instant of the "classical" Big Bang, and w is the parameter appearing in the equation of state, $p = w\rho$), the conformal time depends on the physical time as

$$\eta(t) = \frac{cT}{R_0} \, \frac{3(1+w)}{1+3w} \left[\left(\frac{t+T}{T} \right)^{\frac{1+3w}{3(1+w)}} - 1 \right] . \tag{15}$$

Substituting this formula to (13), we get:

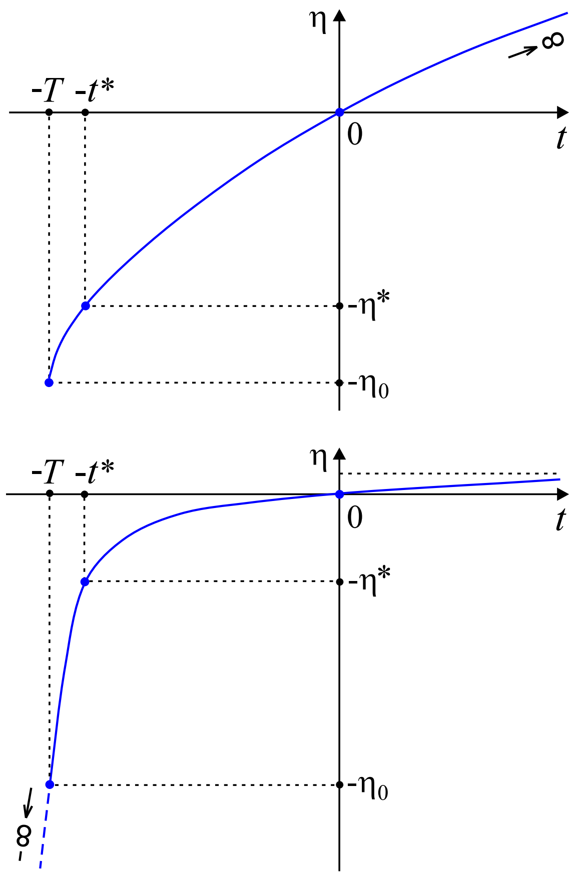

Figure 2: Characteristic behavior of the conformal time η as function of the physical time t in the "pre-inflationary" models, governed by the ordinary matter, (top panel) vs. the inflationary models (bottom panel). The dashed part of the curve, tending to $-\infty$, refers only to the standard (exponential) inflation.

$$N_{\mathrm{CD}} \approx \left\{ \left[1 - \left(\frac{T - t^*}{T} \right)^{\frac{1+3w}{3(1+w)}} \right]^{-1} - 1 \right\}^{-3} \gg 1. \qquad (16)$$

Consequently, as was already mentioned before, there should be a considerable inhomogeneity of physical characteristics within the observed region of the Universe. Moreover, if the Universe is filled with a Higgs field, giving the masses to the elementary particles, then its non-zero values will be established independently in the causally-disconnected domains [1, 18]. As a result, various kinds of the topological defects (such as monopoles, strings/vortexes, and domain walls/kinks, depending on the symmetry group involved) should be formed at the boundaries between the subregions [8, 19], while we actually do not see them in astronomical observations [9].

From the mathematical point of view, all these drawbacks stem from the fact that the conformal time η increases immediately after the Big Bang not so quickly and, as a result, the difference $\eta_0 - \eta^*$ remains quite small (top panel in Figure 2). This problem is naturally resolved in the standard (exponential) inflationary scenario, where the scale factor changes as

$$R(t) = R_0 \exp\Bigl(\sqrt{\Lambda/3}\, c\, t\Bigr) \tag{17}$$

and, consequently, the conformal time depends on the physical time as

$$\eta(t) = \frac{1 - \exp\Bigl(-\sqrt{\Lambda/3}\, c\, t\Bigr)}{R_0 \sqrt{\Lambda/3}}\,. \tag{18}$$

As a result, the interval of the conformal time before the instant at which we observe the Universe, $\eta_0 - \eta^*$, becomes much greater (bottom panel in Figure 2); it formally tends to infinity for the purely exponential inflation without singularity. Then, substituting (18) into (13), we find:

$$N_{\mathrm{CD}} \approx \exp\Bigl[-\sqrt{3\Lambda}\, c\, (T - t^*)\Bigr] \ll 1\,. \tag{19}$$

So, the entire observable region of the Universe turns out to be within the same causally-connected light cone originating in the past.

Will the same behavior take place in the quasi-exponential model, following from the principle of quantum uncertainty? In this case, relation between the physical and conformal time corresponding to the expansion law (9) will be

$$\begin{aligned} \eta(t) \;=\; & \sqrt{\frac{3}{4\pi C_{\mathrm{UR}}}}\,\frac{c\tau}{R_0}\Biggl[\Biggl(\sqrt{\frac{T}{\tau}} + \sqrt{\frac{3}{16\pi C_{\mathrm{UR}}}}\Biggr) \\ & -\Biggl(\sqrt{\frac{t+T}{\tau}} + \sqrt{\frac{3}{16\pi C_{\mathrm{UR}}}}\Biggr) \exp\Biggl(\sqrt{\frac{16\pi C_{\mathrm{UR}}}{3}}\,\frac{\sqrt{T} - \sqrt{t+T}}{\sqrt{\tau}}\Biggr)\Biggr]. \end{aligned} \tag{20}$$

Apart from the Planckian region, $t + T \sim \tau$, behavior of this function is qualitatively similar to the case of classical (exponential) inflation, except that it terminates at $t = -T$; see bottom panel in Figure 2. (In principle, our equations might be favourable for constructing the bounce-type model of the Universe, but we prefer to not speculate here about the processes at Planckian times.) In other words, the difference of the conformal times when the causally-connected subregion if formed, $\eta_0 - \eta^*$, turns out to be sufficiently large.

Next, substituting (20) to (13), we get:

$$N_{\mathrm{CD}} \approx \left(\frac{16\pi C_{\mathrm{UR}}}{3}\right)^{3/2} \left(\frac{T - t^*}{\tau}\right)^{3/2} \exp\left[-4\sqrt{3\pi C_{\mathrm{UR}}}\left(\frac{T - t^*}{\tau}\right)^{1/2}\right] \ll 1\,, \tag{21}$$

i.e., the same inequality as in the standard inflationary scenario.

3. Conclusions

1. Since the Universe expanding by law (9) possesses qualitatively the same causal structure as in the classical inflationary scenario, the problems of homogeneity and isotropy of the space–time, the absence of topological defects, etc. should be naturally resolved. Therefore, our model can serve as a viable alternative to other inflationary models. Its main advantage is a more solid physical basis, because it does not require any artificial assumptions about the form of the inflaton potential or the higher-order curvature terms in the Lagrangian.

2. Yet another advantage of the proposed model is that inflation is not anymore a separate, very specific stage in the history of the Universe; instead, all physical parameters change smoothly throughout the entire cosmological evolution. In particular, the puzzle of two very different Λ-terms in the early Universe and nowadays becomes naturally resolved.

3. A few other well-known problems of the early Universe, e.g., formation of the approximately flat three-dimensional space and the spectrum of the primordial perturbations, still need to be studied in more detail. They will be discussed in the separate papers.

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