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THE BARYONIC TULLY-FISHER LAW AND THE GAUGE THEORY OF CPT TRANSFORMATIONS

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Abstract: The gauge theory of CPT transformations is summarized. Properties of the Lagrangian and field equations are used to produce heuristic arguments explaining the Baryonic Tully-Fisher Law without need for dark matter or MOND.

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1. Introduction

General relativity (GR) is viewed as an incomplete theory because of its nonrenormalizability and, to a lesser extent, because of observable and experimental astrophysical issues difficult to explain with the Λ Cold Dark Matter (ΛCDM) paradigm such as the H_0 tension and nondetection of Dark Matter (DM). Additionally, there are the core-cusp, satellite galaxy distribution and abundance, and "too big to fail" problems when trying to model galaxies within the ΛCDM framework. Such shortcomings have led to various modified gravity theories.

The spin connection ($\omega_{\mu ab}$) formulation of GR used to handle fermions in curved spacetime allows an additional aspect on the incompleteness of GR – failure to include the entire group of proper Lorentz transformations. The spin connection formulation is derived from local Lorentz transformations acting on physical fields and the vierbein (${e_a}^{\mu}$) used to describe spacetime. These Lorentz transformations (Lorentz rotations, λ) are the familiar spatial rotations and boosts from special relativity which form a group of proper, continuous transformations. However, the discrete transformation consisting of parity and time reversal together, PT, is also a proper transformation. Therefore, it would seem interesting to include local PT transformations as a natural physical extension of GR. The use of vierbein in the spin connection formalism accommodates a local PT transformation [1–4], whereas it is unclear how to do so in the more well-known formulation of GR based on the metric tensor, $g_{\mu\nu}$, and Levi-Civita connection, $\Gamma^{\rho}_{\mu\nu}$.

Further motivation to extend GR along these lines is that PT is part of the discrete Charge Conjugation-Parity-Time Reversal (CPT) symmetry. The CPT symmetry is interesting for several reasons:

1) PT is not a universal symmetry whereas CPT is a universal symmetry.
2) The CPT symmetry has been verified experimentally and requires no extra dimensions.
3) CPT is born from the successful (i.e., renormalizable) union of special relativity with quantum theory.

The last point suggests that the CPT symmetry is a "bridge" between quantum theory and relativity and should be made a local symmetry included with local Lorentz rotations in order to construct an extension of GR which would be necessary in any approach to quantum gravity.

2. The transformations

At first glance it may not appear possible to gauge CPT, because there are no important continuously varying parameters involved with the CPT transformation. Also, locality would seem to be a problem. Except for an infinitesimal neighborhood around the origin of a Minkowski manifold, PT is not a local transformation.[1]

We examine the CPT transformation at the origin of an inertial reference frame in order to overcome the above obstacles. The effect of the global CPT transformation *at the origin* of a Minkowski spacetime coordinate system is to flip the coordinate axes and transform a Dirac wavefunction[2], ψ: $\psi \to \mathrm{i}\gamma^5\psi$. If a nontrivial *spacetime* analog of the charge conjugation operation exists, then we would have to include its effect. We assume no such operation exists, in other words, we assume there is no such thing as an "antispacetime" distinct from spacetime. An attempt to find a nontrivial "antispacetime" operation can be found in [1].

In order to extend this concept to a pseudo-Riemannian spacetime manifold, we introduce a vierbein field e^{μ}_{a}, where μ represents the manifold coordinates and a represents the local inertial frame coordinates. By viewing a vierbein at a given point x_μ as tiny coordinate axes with its origin centered at x_μ, we define a local CPT transformation at that point by $e^{\mu}_{a} \to -e^{\mu}_{a}$ (the coordinate axes flip), and $\psi(x_\mu) \to \mathrm{i}\gamma^5\psi(x_\mu)$. In the spirit of gauge theories, we define local CPT transformations as applying these "origin transformations" to the vierbein and wavefunctions at arbitrarily chosen points on the manifold.

The choice of *where* we want to perform a local CPT transformation will play the role of the continuous, arbitrary parameters appearing in gauge theories. In order to

[1]This section is revised from the version found in [3] in order to improve clarity, hopefully.
[2]We use the Bjorken-Drell conventions except for σ^{ab}, $\sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$.

make this concept precise, we introduce a real differentiable function f, defined over the entire manifold to be used as the argument of step functions Θ. In the arbitrary regions where we choose to perform the local CPT transformations, we set $f > 0$ so that $\Theta[f] = 1$. In the arbitrary regions where we choose not to perform local CPT transformations, we set $f < 0$ so that $\Theta[-f] = 1$. The boundaries between regions where local CPT is carried out and where it is not are given by $f = 0$ with the convention that $\Theta[f] = 0$ if $f \leq 0$. By setting $f < 0$ everywhere, we retrieve the original action. By setting $f > 0$ everywhere in flat spacetime, one obtains the globally CPT transformed action.

We emphasize that f is not a physical field. The Θ are parameters which define when the local CPT transformations are carried out (or not). The Θ are just like the phases, ϕ, used in the gauging of $U(1)$ except that there are only two choices regarding the CPT symmetry instead of the continuum of choices for ϕ to be used in the $U(1)$ symmetry operation $\mathrm{e}^{\mathrm{i}\phi}$. To make the $U(1)$ operation local, one makes ϕ an *arbitrary* function of spacetime subject only to the condition that ϕ is differentiable. Similarly, the function f is introduced in order to make the arbitrary choice of carrying out local CPT ($f > 0$) or not ($f < 0$) at the points of interest. The function f plays the same role as replacing a constant ϕ by $\phi(x)$ in gauging $U(1)$. The only restriction placed on f is that it be differentiable so that $\partial_\mu \Theta[\pm f] = \pm\delta[f]\partial_\mu f$ makes sense (δ being the Dirac delta functional). *The function f must disappear in the field equations and any physical predictions.*

Because we are utilizing the proper spacetime transformation PT, it would be prudent to see if the metric spin connection, $\omega_{\mu ab}$, alone could accommodate local CPT transformations. So, we also include local, proper Lorentz rotations wherever $f > 0$. The local Lorentz rotations, $\lambda(x_\mu)$, are denoted by Λ_a^b and Λ_ψ for the vierbein and Dirac wavefunction respectively. In effect, we are gauging the CPTλ transformation[3] of the Dirac field to induce the gauging of the full group of proper spacetime Lorentz transformations.

Putting all of the above together, we have the following local CPTλ transformations:

$$\begin{aligned}
e_a^{\ \mu} &\rightarrow \Theta\left[-f\right] e_a^{\ \mu} - \Theta\left[f\right] e_b^{\ \mu} \Lambda_a^{\ b}, \\
\psi &\rightarrow \Theta\left[-f\right] \psi + \Theta\left[f\right] \mathrm{i}\gamma^5 \Lambda_\psi \psi, \\
\overline{\psi} &\rightarrow \Theta\left[-f\right] \overline{\psi} + \Theta\left[f\right] \mathrm{i}\overline{\psi}\gamma^5 \Lambda_{\overline{\psi}}, \\
\omega_{\mu ab} &\rightarrow \Theta\left[-f\right] \omega_{\mu ab} + \Theta\left[f\right] \widetilde{\omega}_{\mu ab} + \delta\left[f\right] \Theta\left[-f\right] \varsigma_{\mu ab} + \delta\left[f\right] \Theta\left[f\right] \widetilde{\varsigma}_{\mu ab},
\end{aligned}$$

where the metric spin connection is

$$\omega_{\mu ab} = \frac{1}{2} e_a^\nu \left(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}\right) - \frac{1}{2} e_b^\nu \left(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}\right) - \frac{1}{2} e_a^\rho e_b^\sigma \left(\partial_\rho e_{r\sigma} - \partial_\sigma e_{r\rho}\right) e_\mu^r$$

[3] In the author's previous work this has been called $CPT\Lambda$, where Λ denotes proper Lorentz rotations and has nothing to do with a cosmological constant.

and $\widetilde{\omega}_{\mu ab}$ is the transformation of $\omega_{\mu ab}$ under CPTλ which satisfies

$$\widetilde{\omega}_{\mu ab}\sigma^{ab} = \Lambda_{\psi}\omega_{\mu ab}\Lambda_{\overline{\psi}} - 2\left(\partial_{\mu}\Lambda_{\psi}\right)\Lambda_{\overline{\psi}}.$$

The $\varsigma_{\mu ab}$, $\widetilde{\varsigma}_{\mu ab}$ are boundary terms arising from the differentiation of the vierbein transformations in the metric spin connection. Explicit expressions for $\widetilde{\omega}_{\mu ab}$, $\varsigma_{\mu ab}$, $\widetilde{\varsigma}_{\mu ab}$ are found in [3]. The volume element, $|e|\, d^4x$, transforms as

$$|e|\, d^4x \to \left(\Theta\left[-f\right] + \Theta\left[f\right]\right)|e|\, d^4x.$$

Clearly, these transformations are well defined in curved spacetime.

3. Introduction of the gauge field X_{μ}

We take the action integral as the fundamental starting point, because delta functionals need to be in a definite integral in order to have meaning. Also, the action allows us to use variational principles from which we can deduce [3]:

1) The rules on how to handle the various products containing $\Theta\left[\pm f\right]$, $\delta\left[f\right]\partial_{\mu}f$.
2) The nontrivial, nonvanishing variation of the curved spacetime Dirac action under local CPTλ and the inability of the metric spin connection to compensate for this. Hence, the requirement of introducing the new minimally coupled gauge field, X_{μ}.
3) The structure of and transformation of X_{μ}. By using the above transformations in the Dirac action in curved spacetime, one obtains the gauge covariant derivative $D_{\mu}\psi = \partial_{\mu}\psi + \frac{1}{2}\omega_{\mu ab}\sigma^{ab}\psi + \beta X_{\mu}\psi$, where β is the coupling constant associated with X_{μ}. The structure of X_{μ} is determined to be $X_{\mu} = x_{\mu\mathrm{I}}\mathrm{I} + x_{\mu 5}\gamma^5 + x_{\mu ab}\sigma^{ab}$, where the $x_{\mu(\cdots)}$ are the dynamical field components. The transformation of X_{μ} is determined to be:

$$X_{\mu} \to \Theta\left[-f\right]X_{\mu} + \Theta\left[f\right]\Lambda_{\psi}X_{\mu}\Lambda_{\overline{\psi}} + \Theta\left[-f\right]\delta\left[f\right]Y_{\mu} + \Theta\left[f\right]\delta\left[f\right]\widetilde{Y}_{\mu},$$

where

$$\begin{aligned} Y_{\mu} =& \beta^{-1}\left[\partial_{\mu}f\left(\mathrm{I} - \mathrm{i}\gamma^5\Lambda_{\psi}\right) - \frac{1}{2}\varsigma_{\mu ab}\sigma^{ab}\right], \\ \widetilde{Y}_{\mu} =& \beta^{-1}\left[\partial_{\mu}f\left(-\mathrm{I} - \mathrm{i}\gamma^5\Lambda_{\overline{\psi}}\right) - \frac{1}{2}\widetilde{\varsigma}_{\mu ab}\sigma^{ab}\right]. \end{aligned}$$

Subsequent calculations using variational principles become complicated and difficult, for example, the proof that X_{μ} is massless [3]. So, the easier and more familiar "operator" approach found in [4] makes calculations easier and avoids some subtle issues regarding the variational approach. The usual machinery of gauge theories can be used. As examples, the proof that X_{μ} is massless becomes trivial, and

$D_\mu \to U D_\mu U^{-1}$. This approach begins with rewriting the transformation of ψ as $\psi \to U\psi$, where $U = \Theta[-f]\,\mathrm{I} + \Theta[f]\,\mathrm{i}\gamma^5\Lambda_\psi$. From this, one immediately obtains:

$$
\begin{aligned}
U^{-1} =& \Theta[-f]\,\mathrm{I} - \Theta[f]\,\mathrm{i}\gamma^5\Lambda_{\overline{\psi}}, \\
\partial_\mu U =& \Theta[f]\,\mathrm{i}\gamma^5\partial_\mu\Lambda_\psi + \delta[f]\,\partial_\mu f\left(\mathrm{i}\gamma^5\Lambda_\psi - \mathrm{I}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
\left(\partial_\mu U\right)U^{-1} =& \Theta[f]\left(\partial_\mu\Lambda_\psi\right)\Lambda_{\overline{\psi}} + \Theta[-f]\,\delta[f]\,\partial_\mu f\left(\mathrm{i}\gamma^5\Lambda_\psi - \mathrm{I}\right) \\
&+ \Theta[f]\,\delta[f]\,\partial_\mu f\left(\mathrm{I} + \mathrm{i}\gamma^5\Lambda_{\overline{\psi}}\right).
\end{aligned}
$$

We denote the local CPTλ transformation by the customary U even though the transformation is not unitary. Unitarity is not necessary in making gauge theories. For example, λ is not unitary and is used to derive the metric spin connection formulation of GR as a gauge theory. For future reference, we also note that the presence or absence of a conservation law associated with a global symmetry transformation has no relevance to constructing the ensuing gauge theory. Again, returning to λ, we note that there are no conservation laws associated with global boosts.

We emphasize that X_μ *is required* in order to construct an expanded action which is well-behaved under local CPTλ. It is important to note that X_μ, its structure, and its transformation properties are *not* introduced ad-hoc. Rather, X_μ and its transformation properties are *derived* [1–4] from the requirement of constructing a Dirac action in curved spacetime which is well-behaved under local CPTλ transformations.

4. The Lagrangian

Once the transformation of X_μ is determined, then we can construct a Lagrangian density comprised of the expanded Dirac Lagrangian, free-field Lagrangian for X_μ, and a density containing the Einstein-Hilbert term of GR, κR, where $\kappa = (-16\pi G_N)^{-1}$. The total Lagrangian must satisfy the following requirements [1–4]:

1) gauge covariance under local CPTλ transformations,
2) absence of any Dirac delta functionals, $\delta[\dots]$, in the Lagrangian under local CPTλ transformations,
3) terms containing $x_{\mu ab}$ which are not solely constrained to appear within the combination $\left(\frac{1}{2}\omega_{\mu ab} + \beta x_{\mu ab}\right)\sigma^{ab}$, and
4) some components of X_μ appearing in both types of free-field Lagrangians used in GR and the standard model (SM).

The first requirement is obvious. The second prevents pathological variations of the expanded action under local CPTλ transformations. The third ensures that $x_{\mu ab}$ has physical significance and is not just a fancy way to ignore the delta functionals appearing in the transformation of $\omega_{\mu ab}$. The fourth reflects that CPT arises from both GR and SM. We also note that the minimal coupling term acting on matter,

$\left(\frac{1}{2}\omega_{\mu ab}+\beta x_{\mu ab}\right)\sigma^{ab}\psi$, implies replacing κR by $\kappa R_{X\omega}$, where $\kappa R_{X\omega}$ is the modified Einstein-Hilbert curvature term formed by replacing the metric spin connection $\omega_{\mu ab}$ in R by $\omega_{\mu ab}+2\beta x_{\mu ab}$. We obtain [4] for the Hermitian action S:

$$\begin{aligned}S=&\int\left\{\frac{\mathrm{i}}{2}e_a^{\mu}\overline{\psi}\gamma^a\left(\partial_\mu\psi+\frac{1}{2}\omega_{\mu bc}\sigma^{bc}\psi+\beta X_\mu\psi\right)-m\overline{\psi}\psi\right\}|e|\,\mathrm{d}^4x\\&-\int\left\{\frac{\mathrm{i}}{2}e_a{}^{\mu}\left(\partial_\mu\overline{\psi}-\frac{1}{2}\omega_{\mu bc}\overline{\psi}\sigma^{bc}+\beta\overline{\psi}\gamma^0X_\mu^{\dagger}\gamma^0\right)\gamma^a\psi\right\}|e|\,\mathrm{d}^4x\\&+\int\left\{\frac{1}{4}Tr\left(H_{\mu\nu}H^{\mu\nu\dagger}\right)+\frac{\kappa}{2}\left(R_{X\omega}+R_{X\omega}^{\dagger}\right)\right\}|e|\,\mathrm{d}^4x,\end{aligned}$$

where the term

$$H_{\mu\nu}=\frac{\beta}{2}\left(\omega_{\mu ab}\left[\sigma^{ab},X_\nu\right]-\omega_{\nu ab}\left[\sigma^{ab},X_\mu\right]\right)+\beta^2\left[X_\mu,X_\nu\right]+\beta\left(\partial_\mu X_\nu-\partial_\nu X_\mu\right)$$

reflects the quantum (SM) contribution to the origin of the CPT symmetry. The other symbols are the familiar Dirac terms. We note that a mass term for X_μ, $m\mathrm{Tr}\left(X_\mu X^{\mu\dagger}\right)$, is not gauge covariant and not allowed [1–4].

The Euler-Lagrange variation of S with respect to $x_{\mu ab}$ and $e_a{}^{\mu}$, respectively, gives the following two important equations: the $x_{\mu ab}$ field equation,

$$\begin{aligned}&4\beta D_\nu\left(\partial^\nu x^\mu_{cd}-\partial^\mu x^\nu_{cd}\right)\mathrm{Tr}\left[\left(\sigma^{ab}\right)^{\dagger}\sigma^{cd}\right]\\&\quad+2\beta\left\{\left(\omega_{\nu rs}+2\beta x^*_{\nu rs}\right)\left(\partial^\nu x^\mu_{cd}-\partial^\mu x^\nu_{cd}\right)Tr\left[\left[\sigma^{ab},\sigma^{rs}\right]^{\dagger}\sigma^{cd}\right]\right\}\\&\quad+2\beta D_\nu\left(2\beta x^\nu_{cd}x^\mu_{rs}+\omega^\nu_{cd}x^\mu_{rs}+\omega^\mu_{rs}x^\nu_{cd}\right)Tr\left[\left(\sigma^{ab}\right)^{\dagger}\left[\sigma^{cd},\sigma^{rs}\right]\right]\\&\quad-\left\{\beta\left(\omega_{\nu cd}+2\beta x^*_{\nu cd}\right)\left(2\beta x^\mu_{mk}x^\nu_{rs}+\omega^\mu_{mk}x^\nu_{rs}+\omega^\nu_{rs}x^\mu_{mk}\right)\mathrm{Tr}\left[\left[\sigma^{ab},\sigma^{cd}\right]^{\dagger}\left[\sigma^{mk},\sigma^{rs}\right]\right]\right\}\\&\quad+8\kappa\eta^{bc}\left(e^{a\mu}e^{n\rho}-e^{n\mu}e^{a\rho}\right)\left(\omega_{\rho cn}+2\beta x^*_{\rho cn}\right)\\&\quad+8\kappa\eta^{ac}\left(e^{b\mu}e^{n\rho}-e^{n\mu}e^{b\rho}\right)\left(\omega_{\rho nc}+2\beta x^*_{\rho nc}\right)\\&\;=4\mathrm{i}\overline{\psi}\sigma^{ab}\gamma^\mu\psi+8\kappa D_\nu\left(e^{a\nu}e^{b\mu}-e^{a\mu}e^{b\nu}\right),\end{aligned}$$

and the modified GR equation,

$$\frac{1}{2}\left\{\left(R^{\mu\nu}_{X\omega}+R^{\mu\nu\dagger}_{X\omega}\right)-\frac{1}{2}g^{\mu\nu}\left(R_{X\omega}+R^{\dagger}_{X\omega}\right)\right\}=\frac{1}{2\kappa}T^{\mu\nu},$$

where

$$\begin{aligned}T^{\mu\nu}=&\frac{g^{\mu\nu}}{4}Tr\left(H_{\rho\sigma}H^{\rho\sigma\dagger}\right)+\frac{\mathrm{i}}{2}\left[\left(D^\mu\overline{\psi}\right)\gamma^\nu\psi-\overline{\psi}\gamma^\nu D^\mu\psi\right]\\&+g^{\mu\nu}\left[\frac{\mathrm{i}}{2}\left(\overline{\psi}\gamma^\rho D_\rho\psi-\left(D_\rho\overline{\psi}\right)\gamma^\rho\psi\right)-m\overline{\psi}\psi\right].\end{aligned}$$

5. The Baryonic Tully-Fisher Law

We argue that the force arising from the $x_{\mu ab}$ components[4] follows an inverse square law for the following three reasons:

(1) $x_{\mu ab}$ affects matter (i.e., the Dirac spinor, ψ) in the exact same manner as $\omega_{\mu ab}$, since their only interaction with matter in S appears as the $\left(\frac{1}{2}\omega_{\mu ab} + \beta x_{\mu ab}\right)\sigma^{ab}\psi$ term.

(2) $x_{\mu ab}$ is massless.

(3) The free-field term for $x_{\mu ab}$ is contained in $R_{X\omega} + R^{\dagger}_{X\omega}$ and $\mathrm{Tr}\left(H_{\mu\nu}H^{\mu\nu\dagger}\right)$ both of which produce inverse square fields in GR and SM. An inverse square law allows us to use Gauss's law in the derivation of the Baryonic Tully-Fisher law (BTFL) [5].

We now argue that neutrinos are the source for the galactic scale force arising from $x_{\mu ab}$. First, by comparison with the second-order formalism definition of the metric spin connection, we notice that the terms in the $x_{\mu ab}$ field equations with coefficient κ create a new metric spin connection when replacing $\omega_{\mu ab}$ with $\omega_{\mu ab} + 2\beta x_{\mu ab}$. This interpretation is reinforced when taking the Palatini variation of the Lagrangian with respect to $\omega_{\mu ab}$ and seeing the same terms (with $2\beta x_{\mu ab}$ replaced by $\beta\left(x_{\mu ab} + x^{*}_{\mu ab}\right)$) appearing with coefficient κ. From the second-order formalism definition of the metric spin connection, these terms will cancel out. This allows us to linearize (i.e., weak field approximation) the $x_{\mu ab}$ field equations with respect to $\omega_{\mu ab}$, $x_{\mu ab}$ and focus on the remaining source term, $4\mathrm{i}\overline{\psi}\sigma^{ab}\gamma^{\mu}\psi$. Usually, this term will vanish [3], because of the random spin and momentum orientations of fermions in bulk matter. However, because of the fixed neutrino chirality, this term will not vanish for the neutrinos seen by an observer looking at a point neutrino source such as a star. So, we treat stars as point sources of this $4\mathrm{i}\overline{\psi}\sigma^{ab}\gamma^{\mu}\psi$ term which is proportional to the neutrino luminosity, I_{ν}. For a spiral galaxy the total neutrino flux will vanish at the center, so we expect no net force from the $x_{\mu ab}$ potential there. As we move towards the edge of the galaxy, the net neutrino flux starts to point outwards and increases as we move away from the center. Hence the force due to $x_{\mu ab}$ will increase along the way, and we avoid the core-cusp problem associated with the dark matter interpretation.

Because of the resemblance of the linearized $x_{\mu ab}$ field equation to electrodynamics and the expected inverse square behavior of the new force, we expect a weak field limit of the $x_{\mu ab}$ force to be of the Newtonian form $-kQ_{\nu}/r^2$, where k is the effective coupling constant, Q_{ν} is the source ("charge") term expected to be proportional to I_{ν}, and we have assumed an attractive force. So, the total field, $F(r)$ for a point source (or leading order term of an extended object such as a spiral galaxy) of mass M and I_{ν} would be [3]

$$F(r) = -\frac{G_N M}{r^2} - \frac{kQ_{\nu}}{r^2}.$$

[4] the physical implications of the $x_{\mu \mathrm{I}}$ and $x_{\mu 5}$ components are not considered in this paper

Because the neutrinos are ultrarelativistic, we would expect that the total Q_ν contained within r is given by $Q_\nu = I_\nu r/c$. So, using Gauss's law, we have

$$F(r) = -\frac{G_N M}{r^2} - \frac{kI_\nu}{cr}.$$

As $r \to \infty$, we see that

$$F(r) \to -\frac{kI_\nu}{cr}.$$

Therefore, the rotation curve, $v(r)$, flattens out at large r when

$$\frac{kI_\nu}{cr} = \frac{v^2}{r}, \quad \text{or} \quad v = \left(\frac{kI_\nu}{c}\right)^{\frac{1}{2}}.$$

Presumably, the MOND regime (a_M) is reached where the gravitational force is equal to the gauge CPT force instead of a modification to Newton's second law:

$$\frac{G_N M}{r^2} = \frac{kI_\nu}{cr} = a_M.$$

To obtain the slope of the BTFL, we have to be careful. First, we assume that the mass, M, of the galaxy is dominated by hydrogen and ignore the small amount of other elements. Then, we realize that because a small fraction of the H nuclei in the galaxy are undergoing fusion and emitting neutrinos, we have $I_\nu \propto M$. Unfortunately, this gives the asymptotic rotational speed to be $v \propto M^{\frac{1}{2}}$ from above and the wrong slope of the BTFL. To obtain the $(I_\nu)^{\frac{1}{2}}$ source dependence (instead of I_ν) required for the correct slope, we examine the behavior of the modified GR equation at the edge of a spiral galaxy. We use the weak field (linearized) version of modified GR because of the dearth of matter and the small acceleration (the deep MOND regime if one prefers) implying weak $\omega_{\mu ab}$ and $x_{\mu ab}$. Therefore, we neglect the term $(2\kappa)^{-1} T^{\mu\nu}$, because it is quadratic in the fields and because of the small value of $(2\kappa)^{-1}$. Next, we split the terms $R^{\mu\nu}_{X\omega} + R^{\mu\nu\dagger}_{X\omega}$ and $R_{X\omega} + R^{\dagger}_{X\omega}$ into $2R^{\mu\nu} + R^{\mu\nu}_X + R^{\mu\nu\dagger}_X$ and $2R + R_X + R^{\dagger}_X$, where $R^{\mu\nu}$, R are the usual curvature terms formed from the metric spin connection of the Einstein-Hilbert GR ("standard metric spin connection"), and $R^{\mu\nu}_X$, R_X are comprised of identical curvature terms with $\omega_{\mu ab}$ replaced by $2\beta x_{\mu ab}$ and additional terms containing products of $x_{\mu ab}\omega_{\nu cd}$ which are quadratic and therefore dropped. We use the standard metric spin connection as the basis for physical interpretation for a couple of reasons. First, the proof of covariance under local CPTλ [3, 4] used the standard metric spin connection of GR. Second, this allows for the standard GR interpretation of the linearized modified GR equation by moving everything containing $R^{\mu\nu}_X$, R_X, and their complex conjugates to the RHS thereby enabling us to interpret the RHS as an *effective* energy-momentum tensor source term, $\Upsilon_{\mu\nu}$: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Upsilon_{\mu\nu}$. The *direct* (i.e., enhanced without the "middleman" of $T_{\mu\nu}$) effect of $x_{\mu ab}$ acting on spacetime by replacing $\omega_{\mu ab}$ with $\omega_{\mu ab} + 2\beta x_{\mu ab}$ is reflected by the absence of a κ^{-1} factor multiplying $\Upsilon_{\mu\nu}$. Importantly, the linearized $\Upsilon_{\mu\nu} \sim (T_{X\mu\nu})^{\frac{1}{2}}$, where $T_{X\mu\nu}$ are the $x_{\mu ab}$ containing terms of $T_{\mu\nu}$. So, we take this as a cue to replace the source term I_ν of the Newtonian form of the weak field gauge CPT force by $(I_\nu)^{\frac{1}{2}}$ in order to reflect the behavior of $\Upsilon_{\mu\nu}$. This accounts for the origin of the $(I_\nu)^{\frac{1}{2}}$ dependence, hence the correct slope of the BTFL. We could also speculate that because everything is being interpreted in the standard GR framework, the new force is indeed attractive (instead of assuming it) and that light is also bent by the new force. There is no need for either dark matter or MOND.

Because the new force is sourced by neutrinos, we can comment on a few physical phenomena. First, we see that the predictions of GR near black holes remain valid, since there is no neutrino emission from the black holes. Second, there is no conflict with the Bullet Cluster, because the sources of neutrino emission are centered in the stellar medium instead of the lagging gaseous medium. Finally, it is interesting to compare the uncertainty in G_N with the variation of the square root of solar neutrino flux received at the Earth's surface between noon and midnight because the total force on the earth is due to the sun's gravity as well as the $x_{\mu ab}$ component being interpreted as a gravitational effect.

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