Conference *Cosmology on Small Scales 2020*
Michal Křížek and Yurii Dumin (Eds.)
Institute of Mathematics CAS, Prague

A POSSIBLE CONSTRAINT ON THE VALIDITY OF GENERAL RELATIVITY FOR STRONG GRAVITATIONAL FIELDS?

W. Oehm

Bonn, Germany
physik@wolfgang-oehm.com

Abstract: The axiomatic foundation of Einstein's theory of General Relativity is discussed based on an epistemological point of view, yielding a possible restriction on the range of validity regarding the strength of gravitational fields.

Keywords: General theory of relativity, Einstein's elevator, Lorentz frame, Minkowski metric

PACS: 4.20.-q

1. Introduction

As there are various forms of the equivalence principle circulating, this is the form suggested by [2, p. 22]:

> Weak Equivalence Principle (WEP): *"If an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its mass,*[1] *internal structure and composition."*

> Einstein Equivalence Principle (EEP): *(i) WEP is valid, (ii) the outcome of any local non-gravitational test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local non-gravitational test experiment is independent of where and when in the universe it is performed.*

Here the local non-gravitational test experiment is represented by the local Lorentz frame which is the freely falling reference frame in which the laws of Special Relativity are valid, and the term *local* indicates arbitrarily small spatial extensions.

[1] Here the term "*mass*" was added by the author for the sake of clarity. In [2] it is clear from the context upon discussing the equivalence of inertial and passive gravitational mass.

The world-line of a force-free test particle in a Lorentz frame with coordinates $\tilde{x}$ and proper time τ is given by

$$\frac{\mathrm{d}^2\tilde{x}^a}{\mathrm{d}\tau^2} = 0. \tag{1}$$

The transformation to the reference frame of an observer with coordinates x

$$\frac{\mathrm{d}\tilde{x}^a}{\mathrm{d}\tau} = \frac{\partial\tilde{x}^a}{\partial x^b}\frac{\mathrm{d}x^b}{\mathrm{d}\tau} \tag{2}$$

yields

$$\ddot{x}^a + \Gamma^a{}_{bc}\,\dot{x}^b\dot{x}^c = 0, \tag{3}$$

where the derivative to the proper time τ is denoted by the dot, and $\Gamma^a{}_{bc}$ is the affine (metric) connection. The metric g_{ab} of the observer's reference frame relates to the Minkowski metric η_{bc} via

$$g_{ab} = \frac{\partial\tilde{x}^c}{\partial x^a}\frac{\partial\tilde{x}^d}{\partial x^b}\,\eta_{cd}\ . \tag{4}$$

Regarding experimental and observational tests, General Relativity is *quantitatively* confirmed for *weak* gravitational fields within the Solar system only. Observations of black holes and gravitational waves are essentially of phenomenological character without thorough quantitative examination so far. Actually, this also is a matter of fact for the recently published observation of the pericentral drift of the star S2 orbiting the compact radio source Sgr A* (see [1]), which is assumed to represent the massive black hole in the Galactic centre. In this case obviously only vague estimates for the essential physical entities like the mass of the black hole are at our disposal.

2. Einstein's elevator

The assumption of a local Lorentz frame is based on Einstein's thought experiment where an observer situated in an closed elevator is not able to distinguish between gravitational forces, and inertial forces due to acceleration. Since gravitational fields are inhomogeneous, it is clear that the observer would be able to distinguish a gravitational field from an homogeneous acceleration field *if he was able to measure with sufficient accuracy*, depending on the size of the elevator and the inhomogeneity of the gravitational field. Commonly accepted is the way out to reduce the size of the elevator to infinitesimal small values, resulting in the local Lorentz frame with an approximate metric

$$\eta_{ab} + o(\tilde{x}^2)\ . \tag{5}$$

3. *A priori* vs. *a posteriori* relation between theory and accuracy of measurements

Thus the axiomatic mathematical formulation of General Relativity apparently is directly related to the issue of the measurement capabilities of observers. If the observers where able to measure with sufficient accuracy then they would not consider their elevator as inertial (Lorentz) frame, and in principal this stays true down to infinitesimal extensions.

This suggests that the variety of axiomatic formulations in physics be distinguished between those with an *a posteriori* only relation to measurement capabilities and those with an additional *a priori* relation to measurement capabilities in the following sense:

- For an *a posteriori* formulation, only the predictions derived from a mathematical axiom are compared to observations and experimental results. Clearly have the capabilities of measurement an impact on this comparison but the formulation of the mathematical axiom itself is independent of the question of measurement accuracy.
- As already explained, General Relativity represents an *a priori* formulation, because this formulation "stands and falls" with measurement capabilities. This is an additional feature to the *a posteriori* comparison of mathematical predictions with measurement results, which of course also applies here.

4. Locality, a different view

As mentioned in Chapter 1, there are no *quantitative* experiments for strong gravitational fields available. Furthermore, referring to Chapter 3, the essential issue is the fact that the *a priori* relation to the measurement capabilities of an real existing *observer on Earth* is completely separated from the objects under investigation (black holes, Big Bang). With pithy words: What is the concern of cosmological objects regarding the measurement capabilities of an *observer on Earth*? Apparently (iii) of EEP manifests a vague extrapolation to situations which are incompatible to our local situation (Solar system). In that regard the *a priori* aspect of EEP establishes a fundamental distinction from *a posteriori* axiomatic formulations of basic physical concepts applied to cosmology[2].

5. Conclusion

Based on a given size of the "elevator" and a certain measurement accuracy, it is clear that there is a limiting ceiling value for the strength of the gravitational field in that sense that beyond this ceiling value the observer would be able to distinguish between acceleration and gravitation. As a possible logical consequence, this implies

[2] Inventions like *dark matter* or *dark energy* are here not considered as basic physical concepts like the foundations of Mechanics, Electrodynamics, Quantum Mechanics etc.

an *intrinsic* restriction on the validity range of General Relativity regarding the strength of gravitational forces. However, due to the locality aspect of Chapter 4 and the *a priori* feature of EEP in combination with the arbitrariness of a given measurement accuracy, an explicit quantification appears to be hardly possible.

6. Outlook

The author perceives this document as basis for a discussion forum at the conference.

Acknowledgements

The author would like to thank P. Kroupa, M. Olschowy, and M. Křížek for encouraging discussions and communications.

References

[1] GRAVITY Collaboration et al.: Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic centre massive black hole, arXiv:2004.07187 (2020).

[2] Will, C. M.: *Theory and experiment in gravitational physics.* Cambridge University Press, Cambridge, 1993.