On Equivariant Bredon Cohomologies with Mackey Functor in Morava K-theory

Malkhaz Bakuradze

Tbilisi State University
Razmadze Mathematical Institute

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Introduction

Our consideration of the Mackey functor $\underline{M}$ for Bredon cohomology relies on the explicit Morava K-theory ring calculations for 'good' finite p-groups provided in [3].

Let G be a finite group and X a finite G-CW complex. We define the contravariant Bredon $\underline{K(n)}^*$ module on the orbit category $\mathcal{O}_G$ by setting:

$$\underline{K(n)}^*(G/H) = K(n)^*(BH)$$

for each subgroup $H \subseteq G$.

The n-th Bredon cochain group is defined as the direct sum over the n-dimensional orbit representatives:

$$C^n_G(X; \underline{K(n)}^*) = \bigoplus_{\sigma \in \mathrm{cells}_n(X/G)} K(n)^*(BG_\sigma)$$

where G_σ is the stabilizer of a chosen cell σ.

The Differential and AHSS

The coboundary operator $\delta^n : C_G^n \to C_G^{n+1}$ is defined via the restriction maps. For a cochain α, we have:

$$(\delta^n \alpha)(\psi) = \sum_{\sigma \in \mathrm{cells}_n(X/G)} \sum_{g \in G} [\psi : g\sigma] \cdot \mathrm{Res}_{G_\psi}^{gG_\sigma g^{-1}}(g \cdot \alpha_\sigma)$$

where:

- $[\psi : g\sigma]$ are the degrees of the corresponding attaching maps.
- Res is the restriction map in Morava K-theory.

The n-th Bredon cohomology group of X with coefficients in $\underline{K(n)^*}$ is then:

$$H_G^n(X; \underline{K(n)^*}) = \frac{\ker(\delta^n)}{\mathrm{im}(\delta^{n-1})}$$

Atiyah-Hirzebruch Spectral Sequence (AHSS)

While the evenness of the Bredon cohomology is a sufficient rather than a necessary criterion for the "goodness" of X, it remains the most computationally viable path.

Key Observation: If the E_2-condition is met (i.e., the cohomology vanishes in odd degrees), the equivariant Atiyah-Hirzebruch spectral sequence:

$$E_2^{p,q} = H_G^p(X; \underline{K(n)^q}) \Rightarrow K_G^{p+q}(X)$$

collapses immediately at the E_2 page.

The Example: Octahedral Radial 2-Skeleton

To study the chromatic cohomology under the Q_8 action, we define the underlying space X as a specific simplicial subcomplex of $\mathbb{R}^3$.

Let $V_{\mathrm{oct}} = \{\pm \mathrm{i}, \pm \mathrm{j}, \pm \mathrm{k}\}$ be the set of vertices of a regular octahedron centered at the origin $w_0 = (0, 0, 0)$. Let E_{oct} be the set of the 12 edges of this octahedron.

We define the complex X as the 2-skeleton of the simplicial star of the origin:

$$X = \bigcup_{\{u,v\} \in E_{\mathbf{oct}}} \mathrm{conv}(\{w_0, u, v\}) \qquad (1)$$

Combinatorial Structure

Unlike a solid ball or a filled octahedron, X is a **pure 2-dimensional complex**. Its combinatorial data consists of:

- **0-cells (7 vertices):** The central vertex w_0 and the 6 peripheral vertices V_{oct}.
- **1-cells (18 edges):** 6 radial edges connecting w_0 to V_{oct}, and 12 peripheral edges in E_{oct}.
- **2-cells (12 faces):** 12 triangular sectors (petals) spanning from the origin to each edge of the octahedron.

This structure allows us to explicitly describe the Q_8 action on the cells.

Homotopy Type

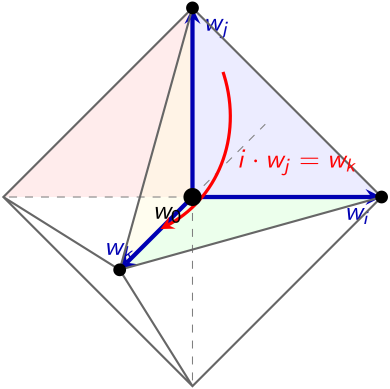
The complex X is geometrically realized as the simplicial cone over the 1-skeleton of the octahedron, $X = C(\mathrm{Oct}_1)$, where the origin w_0 serves as the cone point.

Since the cone of any topological space is null-homotopic, X is non-equivariantly contractible:

$$X \simeq * \tag{2}$$

The vanishing of ordinary cohomology in this case serves to isolate the equivariant effects. The persistence of chromatic classes in $H^1_{Q_8}(X; \underline{K(n)^*})$ is thus entirely attributable to the Q_8-action and its associated **stabilizer jumps**, rather than the underlying topology of the base space.

The underlying complex X is a contractible 2-disk with an octahedral decomposition. It consists of a central vertex w_0 and 6 peripheral vertices.



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Orbit Decomposition

The Q_8-action on X induces a decomposition into orbits of n-cells. Based on the geometric stabilizers, the equivariant structure of X is given by:

$$X \cong Q_8/Q_8 \cup \bigvee_{r\in\{i,j,k\}} (Q_8/C_4)^0_r \cup \bigvee_{r\in\{i,j,k\}} (Q_8/C_4)^1_r \cup \bigvee_{s=1}^{3} (Q_8/C_2)^1_s \cup \bigvee_{s=1}^{3} (Q_8/C_2)^2_s$$

Cellular Breakdown:

- **0-cells:** The origin (Q_8/Q_8) and 3 orbits of peripheral vertices with stabilizer C_4.
- **1-cells:** 3 orbits of radial edges (stabilizer C_4) and 3 orbits of peripheral edges (stabilizer C_2).
- **2-cells:** 3 orbits of triangular faces, each with stabilizer $C_2 = \{1, -1\}$.

This decomposition is the basis for computing the Bredon cochain groups $C^n_{Q_8}(X; \underline{K(n)^*})$.

The modified Differential and Cohomology

The differential $d^r : C_G^r \to C_G^{r+1}$ is a morphism of $K^*(BG)$-modules. Its matrix entries are determined by the interaction between the stabilizers of incident orbits through the global group G.
For a generator $\alpha = 1$ in the component corresponding to orbit $[\sigma]$, the differential is computed as:

$$d^r([\sigma]) = \sum_{[\tau] \in \mathrm{Orbits}_{r+1}} \epsilon([\tau],[\sigma]) \cdot \Phi_{[\tau],[\sigma]}([\tau])$$

where the algebraic weight $\Phi_{[\tau],[\sigma]}$ is defined using the Mackey functor structure. Specifically, let $H = \mathrm{Stab}(\sigma)$ and $K = \mathrm{Stab}(\tau)$. The entry in the matrix is given by the composition of transfer and restriction maps:

$$\Phi_{[\tau],[\sigma]} = \mathrm{Res}_K^G \circ \mathrm{Tr}_H^G : K^*(BH) \to K^*(BK)$$

Double Coset Formula and Chromatic Weights

According to the Double Coset Formula, this composition decomposes into a sum of local transfers. In our model for the Q_8-action on the octahedron, this leads to the following cases for the matrix entries:

- **Radial Differentials (d^0):** For the inclusion of the central vertex (orbit with $H = Q_8$) into radial edges (orbits with $K = C_4$), the entry is:

$$\mathrm{Res}^{Q_8}_{C_4} \circ \mathrm{Tr}^{Q_8}_{Q_8}(1) \sim v_n x^{2^n-1}$$

- **Peripheral Differentials (d^1):** For the inclusion of edges into faces (orbits with $K = C_2$), the entry involves the transfer:

$$\tau = \mathrm{Tr}^{C_4}_{C_2}(1)$$

Smith Normal Form Reduction

The resulting boundary matrices D_r are reduced to their Smith Normal Form (SNF) over the ring $K^*(BG)$. The persistence of the chromatic generator v_n in the SNF of the differential confirms the existence of non-trivial equivariant cohomology classes:

$$\mathrm{SNF}(D_r) = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_k, 0, \ldots, 0)$$

where $\delta_i \in \{1, v_n, v_n^2, \ldots\}$.

Codifferentials and Matrix Reduction

The chromatic co-chain complex is governed by two key operators. Let $k = 2^n - 1$ and τ be the transfer weight of the peripheral arcs.
This matrix connects vertices to edges. The peripheral vertices ensure that the arcs are part of the 1-skeleton:

$$d^0 = \begin{pmatrix} x^k & -1 \\ y^k & -1 \\ x^k + y^k & -1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \tag{3}$$

First Codifferential $d^1 : C_G^1 \to C_G^2$

This matrix for $d^1 : C_G^1 \to C_G^2$ connects edges to sector faces. The algebraic relation $z^k = x^k + y^k$ is embedded in the boundary of the sectors:

$$d^1 = \begin{pmatrix} x^k & -y^k & 0 & \tau & 0 & 0 \\ 0 & y^k & -(x^k + y^k) & 0 & \tau & 0 \\ -(x^k + y^k) & 0 & x^k & 0 & 0 & \tau \end{pmatrix} \tag{4}$$

Cohomological Conclusion

By applying Smith Normal Form (SNF) reduction over the Morava ring $K(n)_*[x, y]$, we find that the disk is only geometrically contractible. Equivariantly, the Q_8 action is "locked" into the sector structure. The first cohomology group is:

$$H^1_G(X; \underline{\mathcal{K}(n)}) \cong \frac{K(n)_*[x, y]}{(x^{2^n-1}, y^{2^n-1}, \tau)} \tag{5}$$

This non-vanishing group represents the persistent chromatic signature of the symmetric cluster.

Definition of Morava K-theory

Morava K-theory, denoted by $K^*(n)$, is a family of generalized cohomology theories depending on a fixed prime p and a positive integer $n \geq 1$ (the height). It serves as a fundamental building block in chromatic homotopy theory [31].
In the chromatic hierarchy, $K(n)$ occupies an intermediate position between ordinary cohomology ($n = \infty$, where $K(\infty) = H\mathbb{F}_p$) and complex K-theory ($n = 1$). Each $K(n)$ theory is associated with a formal group law of height n over the field $\mathbb{F}_p$.

The Coefficient Ring $K(n)_*$

The structure of the coefficient ring for Morava K-theory is defined as follows:

$$K(n)_* = \mathbb{F}_p[v_n, v_n^{-1}]$$

where v_n is a variable with grading $|v_n| = 2(p^n - 1)$.

This ring is a **graded field**, meaning that every non-zero homogeneous element is invertible [**?**]. The invertibility of v_n ensures the $2(p^n - 1)$-periodicity of the theory, which is a fundamental property for chromatic homotopy theory.

$K(n)^*(BG)$ as a Vector Space

For a finite group G, $K(n)^*(BG)$ possesses the structure of a $K(n)^*$-module. Its property as a finite-dimensional vector space is justified by:

- **Algebraic Structure:** Since $K(n)^*$ is a graded field, any module over this ring is a free module. Consequently, $K(n)^*(BG)$ is automatically a vector space.
- **HKR Theorem:** According to the Hopkins-Kuhn-Ravenel theorem, $K(n)^*(BG)$ is finite-dimensional over $K(n)^*$.
- **Dimension:** Its dimension is equal to the number of conjugacy classes of commuting n-tuples of elements in G whose orders are powers of p.

Computational Significance

1. **Atiyah-Hirzebruch Spectral Sequence (AHSS):** Because the coefficients form a field, the E_2 term is easily calculated:

$$E_2^{p,q} = H^p(BG; K(n)^q)$$

2. **Bredon Cohomology:** In an equivariant context, $K(n)^*(BG)$ is used to construct the relevant Mackey functors $\underline{K(n)}^*$.
3. **Good Groups:** If $K(n)^*(BG)$ is concentrated in even degrees, the AHSS collapses, simplifying the description of the cohomology ring.

Good Elements and Good Groups

Definition

An element $x \in K(s)^*(BG)$ is **good** if it is a transferred Euler class of a complex subrepresentation of G:

$$x = \mathrm{Tr}^*(e(\rho))$$

where ρ is a complex representation of a subgroup $H \subseteq G$ and $\mathrm{Tr} : BG \to BH$ is the transfer map.

Definition

A group G is called **good** if $K(s)^*(BG)$ is spanned by good elements as a $K(s)^*$-module.

The "weakly good" groups, characterized by $K(s)^{odd} = 0$, play a significant role in the classification of group cohomologies [38, 47, 58].

Families of p-groups with $K(n)^{odd}(BG) = 0$

In particular, if G belongs to any of the following families of p-groups, then $K(n)^{\mathrm{odd}}(BG) = 0$:

a. Wreath products $H \wr C_p$ with H good [31, 32];
b. Metacyclic p-groups [52];
c. Minimal non-abelian p-groups (all maximal subgroups are abelian) [54];
d. Groups of p-rank 2 [57];
e. Elementary abelian by cyclic groups (extensions $V \to G \to C$) [58, 38];
f. Central products $H \circ C_{p^m}$ with H good [47];
g. $H \lhd G$ of index p, where H is good and $\tilde{K}(s)(BH)$ is a permutation module for G/H [38].

Multiplicative Structure and Transfer

For these families, the ring structure of $K(s)^*(BG)$ is either studied in the cited works or can be derived from existing computations modulo certain indeterminacy.

Yagita and Tezuka determined the multiplicative structure modulo the transfer formula. Our main aim here is to check (specifically for groups with a maximal abelian subgroup of index 2) whether the transfer formula is sufficient to determine the ring structure via:

- Methods of characteristic classes;
- Transfer formulas (double coset formula, etc.);
- Induced representations.

The papers [9, 10, 12, 15] treat the same problem.

Examples of $K^*(BG)$: The dihedral, semi-dihedral and generalized quaternion groups

Here we consider the dihedral, semi-dihedral and generalized quaternion 2-groups. For more details we refer to [15].

Now Let

$$G = \langle a, b \,|a^{2^{m+1}} = 1,\; b^2 = a^e, bab^{-1} = a^r\rangle, \quad m \geq 1$$

and either $e = 0$, $r = -1$ (the dihedral group $D_{2^{m+2}}$ of order 2^{m+2}), $e = 2^m$, $r = -1$ (the generalized quaternion group $Q_{2^{m+2}}$) or $m \geq 2$, $e = 0$, $r = 2^m - 1$ (the semi-dihedral group $SD_{2^{m+2}}$).

Consider the following Chern classes c, x, c_1, c_2 of dimensions $|c| = |x| = |c_1| = 2$, $|c_2| = 4$:

$$c = c_1(\eta_1), \eta_1 : G/\langle a\rangle \cong C_2 \to C^*, b \mapsto -1;$$

$$x = c_1(\eta_2),\ \eta_2 : G/\langle a^2, b\rangle \cong C_2 \to C^*, a \mapsto -1;$$

and $c_i = c_i(\xi_{\pi_1})$, where

$$\xi_{\pi_1} \to B\langle a, b\rangle$$

is the plane bundle transferred from the canonical line bundle

$$\xi \to B\langle a\rangle,$$

for the double covering

$$\pi_1 : B\langle a\rangle \to B\langle a, b\rangle$$

corresponding to η_1.

Theorem

Theorem

i) $K(s)^*(BG) = K(s)^*[c, x, c_2]/R$
and the relations R are determined by

$$c^{2^s} = x^{2^s} = 0,$$

$$v_s c c_2^{2^{s-1}} = v_s \sum_{i=1}^{s-1} c^{2^s - 2^i + 1} c_2^{2^{i-1}} + \begin{cases} 0 & \textit{if } G \textit{ is dihedral,} \\ c^2 & \textit{if } G \textit{ is quaternion,} \\ cx & \textit{if } G \textit{ is semidihedral.} \end{cases}$$

$$v_s^2 c_2^{2^s} = \begin{cases} cx + x^2 & \textit{if } G = D_8, \\ c^2 + cx + x^2 & \textit{if } G = Q_8 \end{cases}$$

and for $m > 1$

Theorem

$$v_s^{2\kappa(m)} c_2^{2^{ms}} = cx + x^2$$

for G of all three types.

$$v_s x c_2^{2^{s-1}} = v_s \sum_{i=1}^{s-1} x^{2^s - 2^i + 1} c_2^{2^{i-1}} + \begin{cases} cx + x^2 & \textit{if } G = D_8, \\ x^2 & \textit{if } G = Q_8; \end{cases}$$

for $m > 1$,

$$v_s x c_2^{2^{s-1}} = v_s x \sum_{i=1}^{s-1} c^{2^s - 2^i} c_2^{2^{i-1}} + \sum_{i=1}^{ms} v_s^{1+\kappa(m)+2^{ms}-2^i} c_2^{(2^{ms}+1)2^{s-1} - (2^s-1)2^{i-1}}$$
$$+ \begin{cases} 0 & \textit{if } G \textit{ is dihedral,} \\ cx & \textit{if } G \textit{ is quaternion or semidihedral,} \end{cases}$$

where $\kappa(m) = \frac{2^{ms}-1}{2^s-1}$.

ii) $c^2 x = cx^2, \;\; c_1^{2^{ms}+1} = 0, \;\; c_2^{(2^{ms}+1)2^{s-1}} = 0.$

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Thank You!