

Exercises for Mathematical Logic (9 Jan 2024)

27. All Σ_1 -definable sets are semidecidable.

28. (Craig's trick.) Every semidecidable theory is recursively axiomatizable. [Hint: Express $\text{Thm}(T)$ as $\exists y P(x, y)$ with P decidable. Given $x = \ulcorner \varphi \urcorner$ and y , devise a sentence equivalent to φ that encodes y .]

29. Show that every decidable consistent theory T has a decidable completion. [Hint: Consider a completion procedure that enumerates sentences φ one by one, and extends the current list of axioms with φ or $\neg\varphi$, whichever maintains consistency with T .]

In the next three exercises, you will develop an alternative sequence encoding scheme due to Edward Nelson.

30. The set $\{x : \exists n \in \mathbb{N} x = 2^n\}$ of powers of 2 is definable by a Δ_0 formula, not using the 2^n function. [Hint: Consider the divisors of x .]

31. Consider an encoding of finite sets $X \subseteq \mathbb{N}$ by pairs $\langle r, w \rangle$ where the binary expansion of r acts as a “ruler” with marks at positions of 1s, and the binary expansion of w is a concatenation of binary expansions of elements of X such that each element occupies the position between two ruler marks. Show that the predicate “ x is in the set encoded by $\langle r, w \rangle$ ” is Δ_0 -definable.

32. Construct a Δ_0 encoding of finite sequences based on the previous exercise.