

Exercises for Mathematical Logic (22 Oct 2024)

16. Let \mathcal{A} be an L -structure, t a closed L -term such that $t^{\mathcal{A}} = a \in A$, and $\varphi(x)$ an L -formula. Show that $\mathcal{A} \models \phi(t)$ iff $\mathcal{A} \models \phi(\underline{a})$.

17. Consider a modification of the first-order proof system given in the lecture such that the axioms of equality are replaced with the axiom $x = x$ and the axiom schema $t = s \wedge \varphi(t/x) \rightarrow \varphi(s/x)$ for all formulas φ and terms t, s free for x in φ . Show that this is equivalent to the original proof system.

18. For any formula $\varphi(x)$ and variable y free for x in φ , show that the formula $\exists y (\exists x \varphi(x) \rightarrow \varphi(y))$ is provable.