

Exercises for Mathematical Logic (31 Oct 2025)

20. For any formula $\varphi(x)$ and variable y free for x in φ , show that the formula $\exists y (\exists x \varphi(x) \rightarrow \varphi(y))$ is provable.

21. Without using the completeness theorem, prove the following conservation property: Let T be an L -theory, φ an L -formula, and L' a language extending L . If φ has a proof from T using L' -formulas, then $T \vdash \varphi$ (i.e., φ has a proof from T using only L -formulas).

[Hint: Generalize the proof of the lemma on constants.]