Písemka č. 1 - 19. 11. 14:00

1. Nechť $a, b, c \in \mathbb{R}, a, b, c > 0$. Najděte limitu

$$\lim_{x\to 0}(\cos ax)^{\frac{1}{\operatorname{tg}(bx)(\ln(cx+1))}}$$

2. Najděte primitivní funkci k $f(x)$ na maximálních možných intervalech a najděte rovněž tyto intervaly.

$$f(x) = \frac{x+4}{x^2+4x+16}$$

Řešení

1.

$$\lim_{x\to 0}(\cos ax)^{\frac{1}{\operatorname{tg}(bx)(\ln(cx+1))}} = \lim_{x\to 0}\exp\frac{\cos(bx)\ln(\cos(ax))}{\sin(bx)(\ln(cx+1))} =$$
$$= \exp 1\cdot\lim_{x\to 0}\frac{\ln(\cos(ax))}{\sin(bx)(\ln(cx+1))} =$$
$$= \exp\lim_{x\to 0}\frac{\ln(\cos(ax))}{\cos(ax)-1}\cdot\frac{\cos(ax)-1}{\sin(bx)(\ln(cx+1))} =$$
$$= \exp 1\cdot\lim_{x\to 0}\frac{bx}{\sin(bx)}\cdot\frac{cx}{\ln(cx+1)}\cdot\frac{\cos(ax)-1}{(bx)(cx)} =$$
$$= \exp 1\cdot 1\cdot\lim_{x\to 0}\frac{\cos^2(ax)-1}{bcx^2(\cos(ax)+1)} =$$
$$= \exp\lim_{x\to 0}\frac{-\sin^2(ax)}{bcx^2(\cos(ax)+1)} =$$
$$= \exp\lim_{x\to 0}-\frac{\sin^2(ax)}{(ax)^2}\cdot\frac{a^2x^2}{bcx^2(\cos(ax)+1)} = \exp\left(-\frac{a^2}{2bc}\right)$$

2.

$$
\begin{aligned}
I &= \int \frac{x+4}{x^2+4x+16}\,\mathrm{d}x = \frac{1}{2}\int \frac{2x+4}{x^2+4x+16}\,\mathrm{d}x + 2\int \frac{1}{x^2+4x+16}\,\mathrm{d}x \\
&= \frac{1}{2}\ln|x^2+4x+16| + 2\int \frac{1}{x^2+4x+4+12}\,\mathrm{d}x \\
&= \frac{1}{2}\ln|x^2+4x+16| + 2\int \frac{1}{(x+2)^2+12}\,\mathrm{d}x \\
&= \frac{1}{2}\ln|x^2+4x+16| + \frac{1}{6}\int \frac{1}{(\frac{x+2}{\sqrt{12}})^2+1}\,\mathrm{d}x \\
&= \frac{1}{2}\ln|x^2+4x+16| + \frac{\sqrt{12}}{6}\int \frac{1}{t^2+1}\,\mathrm{d}t \\
&= \frac{1}{2}\ln(x^2+4x+16) + \frac{\sqrt{3}}{3}\mathrm{arctg}\left(\frac{x+2}{2\sqrt{3}}\right) + C.
\end{aligned}
$$

Žádný problémový bod ve jmenovateli není, takže jsme primitivní funkci našli na $\mathbb{R}$.