Michal Křížek · Lawrence Somer

Mathematical Aspects of Paradoxes in Cosmology

Can Mathematics Explain the Contemporary Cosmological Crisis?

This book provides a mathematical and numerical analysis of many problems which lead to paradoxes in contemporary cosmology, in particular, the existence of dark matter and dark energy. It is shown that these hypothetical quantities arise from excessive extrapolations of simple mathematical models to the whole physical universe. Written in a completely different style to most books on General Relativity and cosmology, the important results take the form of mathematical theorems with precise assumptions and statements. All theorems are followed by a corresponding proof, or an exact reference to the proof.

Some nonstandard topics are also covered, including violation of the causality principle in Newtonian mechanics, a critical mathematical and numerical analysis of Mercury's perihelion shift, inapplicability of Einstein's equations to the classical two-body problem due to computational complexity, non-uniqueness of the notion of universe, the topology of the universe, various descriptions of a hypersphere, regular tessellations of hyperbolic spaces, local Hubble expansion of the universe, neglected gravitational redshift in the detection of gravitational waves, and the possible distribution of mass inside a black hole.

The book also dispels some myths appearing in the theory of relativity and in contemporary cosmology. For example, although the hidden assumption that Einstein's equations provide a good description of the evolution of the whole universe is considered to be obvious, it is just a null hypothesis which has not been verified by any experiment, and has only been postulated by excessive extrapolations of many orders of magnitude.

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Never identify any mathematical model with reality.

—THE AUTHORS

Preface

Einstein's General Theory of Relativity represents a very complex theoretical discipline which intersects mathematics and physics. Its results are applied in relativistic cosmology, which is a branch of astronomy that deals with the greatest spatial and temporal distances. For some time there has been a growing crisis in cosmology. The contemporary cosmological model possesses at least 20 serious paradoxes, see e.g. Sect. 1.1. Nevertheless, it is still defended by most cosmologists. We show that the main reason for this is that the simple mathematical models used in cosmology are identified with reality and that various incorrect extrapolations are being made.

In 1999, the first author of this monograph published the paper [157] about delay differential equations: *Numerical experience with the finite speed of gravitational interaction* which indicated that the laws of conservation of energy and angular momentum can be slightly violated in our universe. This was the beginning of a detailed analysis of mathematical and numerical methods used in current astrophysics and relativistic cosmology.

This treatise is based on our more than 50 works in Special and General Relativity and related topics that we published mostly in prestigious international journals such as *Journal of Computational and Applied Mathematics, Gravitation and Cosmology, Communications in Computational Physics, Astrophysics and Space Science, Mathematics and Computers in Simulation, International Journal of Astronomy and Astrophysics, New Astronomy, Neural Network World, Astronomische Nachrichten, and the Czech journal Pokroky (=Advances) matematiky, fyziky a astronomie* (see e.g. `dml.cz`). Most of our results were reported at many international conferences (see e.g. [162, 175, 177–180, 195]) and also at our regular seminars *Current Problems in Numerical Analysis* and *Cosmological Seminar* which take place at the Institute of Mathematics of the Czech Academy of Sciences in Prague [387, 398].

Prague stood at the birth of two revolutions in building the theory of gravity:

1. At the beginning of the seventeenth century in Prague, Johannes Kepler formulated his first and second laws of planetary motion with the help of Tycho

Brahe's observational data. This discovery played a key role in Newton's theory of gravitation.

2. In the period 1911–1912, Albert Einstein received his first full professorship at Charles-Ferdinand University in Prague. His colleague Georg Pick worked on non-Euclidean geometries and taught Einstein the foundations of tensor calculus. In Prague, Einstein found the calm atmosphere necessary to formulate the basic ideas of his General Theory of Relativity. Here he wrote several important papers on this topic.

There are many books on General Relativity and cosmology. Let us mention, e.g., [107, 184, 249, 270, 272, 332, 337, 369, 371]. However, our book is written in a different style. Most of the important results are formulated in the form of mathematical theorems with precise assumptions and statements. Our book also contains some nonstandard topics such as: the violation of the principle of causality in Newtonian mechanics, critical mathematical and numerical analysis of Mercury's perihelion shift, and the large computational complexity of Einstein's equations which renders them factually inapplicable for the classical two-body problem. Further topics include the nonuniqueness of the notion universe, topology of the universe, various descriptions of a hypersphere, regular tessellations of hyperbolic spaces, flight around the universe, local Hubble expansion of the universe, neglected gravitational redshift in detection of gravitational waves, possible distribution of mass inside a black hole. We would also like to dispel some myths appearing in the theory of relativity and contemporary cosmology.

Our book is intended for a general audience—especially for those who can appreciate the beauty of both mathematics and cosmology, who are open to new ideas, and who want to break out of the rut. We only assume that the reader is familiar with the basic rules of arithmetic and has no problem with adjustments of algebraic formulas. Only very rarely it is necessary to understand some deep relationships from linear algebra or calculus. Most chapters can be read independently from one another. Some parts are quite simple, others more complicated. If some part is too difficult, there is no problem in skipping it.

For inspiration, there are also several links to websites, although we are well aware that they are not subjected to any review and change quite frequently. Newly defined terms are highlighted in italics in the text for the convenience of the reader. They can also be found in the Index.

In order to read the individual chapters, it is not necessary that the reader understands all assertions. Mathematicians formulate their ideas in the form of theorems that contain only what is relevant in the problem in question. To avoid any discussion whether a particular statement is true or not, we provide proofs of most statements so that everyone can verify their validity. For more complicated proofs, we only give suitable references to the corresponding literature.

In a number of discussions, many researchers helped us to improve the content of this book, in particular, Jan Brandts, Jan Chleboun, Caroline Griffis, Jiří Grygar, Vesselin Gueorguiev, Ivan Hlaváček, Kurt Koltko, Pavel Křížek, Monika Lanzendörferová, František Lomoz, André Maeder, Jan Maršák, Jaroslav Mlýnek,

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Finally, we are profoundly indebted to Elena Griniari, Francesca Ferrari, and Harini Rajesh from Springer-Verlag for their helpful cooperation in the preparation of this book. Our great thanks go to the Springer Publishing House for its care in the graphical design of the book and to the referees for invaluable suggestions. We are also grateful to our families for patience and understanding.

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Prague 1, Czech Republic — Michal Křížek
Washington, DC, USA — Lawrence Somer

Contents

Glossary of Symbols

$\mathbb{N} = \{1, 2, 3, \dots\}$	Set of natural numbers
$\mathbb{E}^n$	n-dimensional Euclidean space
$\mathbb{S}^3$	Three-dimensional sphere (hypersphere)
$\mathbb{H}^3$	Three-dimensional hyperbolic pseudosphere
k	Curvature index from $\{-1, 0, 1\}$
$\mathbb{C}$	Set of complex numbers
(a, b)	Open interval
$[a, b]$	Closed interval
$\lvert \cdot \rvert$	Absolute value
$\log_b$	Logarithm to the base b
ln	Natural logarithm
e	Euler number 2.718 281 828...
$\forall$	For all
$\exists$	There exist(s)
diag	Diagonal matrix (tensor)
det	Determinant
vol	Volume
$\top$	Transposition
$\mathcal{O}(\cdot)$	Landau symbol: $f(\alpha) = \mathcal{O}(g(\alpha))$ if there exists $C > 0$ such that $\lvert f(\alpha)\rvert \le C\lvert g(\alpha)\rvert$ for $\alpha \to 0$ or $\alpha \to \infty$
$\emptyset$	Empty set
π	Ludolph number 3.141 592 653...
$\prod$	Product
$\sum$	Sum
$\cap$	Intersection
$\cup$	Union
$\setminus$	Set subtraction
$\subset$	Subset
$\in$	Is element of
$\notin$	Is not element of
$\{x \in A;\ \mathcal{P}(x)\}$	Set of all elements x from A which possess property $\mathcal{P}(x)$

$f: A \to B$	Mapping (function) f from the set A to the set B
$x \mapsto f(x)$	Function which assigns value $f(x)$ to x
$\Longrightarrow$	Implication
$:=$	Assignment
$a = a(t)$	Expansion function
$\dot{a}$	Time derivative of $a = a(t)$
$\square$	Halmos symbol
$\odot$	Sun
$R_\odot$	Radius of the Sun 695 700 km
$M_\odot$	Mass of the Sun $1.988547 \cdot 10^{30}$ kg
m_p	Mass of the proton $1.67 \cdot 10^{-27}$ kg
z	Redshift
c	Speed of light in vacuum 299,792,458 m/s
G	Gravitational constant $6.674 \cdot 10^{-11}$ $\mathrm{m^3kg^{-1}s^{-2}}$
Λ	Cosmological constant $\approx 10^{-52}$ $\mathrm{m^{-2}}$
σ	Stefan-Boltzmann constant $5.669 \cdot 10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$
h	Planck constant $6.626\,070 \cdot 10^{-34}$ Js
au	Astronomical unit 149,597,870,700 m
pc	Parsec 3.262 ly = 206 265 au = $3.086 \cdot 10^{16}$ m
H_0	Hubble-Lemaître constant $\approx$ 70 km/(s Mpc)
$H(t)$	Hubble parameter
yr	Sidereal year 365.256 36 days = 31 558 149.54 s $\approx \pi \cdot 10^7$ s
ly	Light-year 63,242 au = $9.46 \cdot 10^{15}$ m
cy	Century
eV	Electronvolt = $1.602 \cdot 10^{-19}$ J

Chapter 1
Mathematical Modeling

1.1 Introduction

Cosmology is a branch of astronomy that deals with the greatest spatial and temporal distances and the questions of the origin of the universe as a whole. However, the current standard cosmological model leads to a wide range of serious paradoxes (or even catastrophes—see Index) and does not give acceptable results (see e.g. [24, 25, 79, 184, 205–207, 217, 226, 360]). Therefore, in this book we will analyze mathematical models used in cosmology and investigate their theoretical background.

The main problems of the standard ΛCDM (i.e. Lambda-Cold Dark Matter) cosmological model, which should not be ignored, can be summarized as follows:

1. The problem of the existence of dark matter (see Sect. 8.1).
2. The problem of the existence of dark energy (see Sect. 9.1).
3. The problem of isotropy of the universe (see Remark 6.1).
4. The problem of homogeneity of the universe (see Remark 6.2).
5. The flatness problem (see Remark 6.19).
6. The problem of the existence of the cosmological constant (see Remark 3.15).
7. The problem of the accelerated expansion of the universe and the energy non-conservation problem (see Remark 3.16 and Sect. 10.6).
8. The problem with observed values of the cosmological parameter σ_8 (see Remark 6.22).
9. The problem that the standard cosmological model admits a division by zero and that cosmological parameters have a strange behavior in time (see Sect. 6.7).
10. The problem with inconsistency of different values of the Hubble-Lemaître constant obtained by different methods (see Remark 6.23).
11. The problem of hierarchical structures (see Remark 1.12).

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12. The problem of the existence of stars that are older than the universe according to the standard cosmological model (see Remark 7.28).
13. The problem of setting up accurate initial conditions (see Remark 10.2).
14. The problem of the existence of an infinite universe (see Remarks 6.27 and 6.28).
15. The inflation problem (see Remark 7.7).
16. The problem of the existence of giant black holes in the early universe (see Sect. 3.6).
17. The horizon problem (see [122]).
18. The problem of the large angular momnetum and velocities of all spiral galaxies (see Remark 9.7).
19. The problem of a 120-order-of-magnitude discrepancy between the density of fluctuations in a vacuum, which should have gravitational effects, and theoretically derived density of dark energy (see [11] for the vacuum catastrophe).
20. The Big Bang problem itself (see Remarks 6.8 and 6.21).

Some of these problems are overlapping, but their number is growing. Most mainstream cosmologists do not mind that these problems exist and still defend the standard cosmological model. Their frequently used argument: *"We have nothing better"* does not hold up.

Why is this so? The main reason is that physical reality is identified with very simple mathematical models (see e.g. (7.21) below) ignoring the modeling error.

The second reason is that people like sensationalism: Our universe consists of 25 % of some invisible mysterious dark matter and 70 % of even more mysterious dark energy, and nobody knows what it is, etc. However, these mysterious entities could be mostly the above-mentioned modeling errors.

The third reason is that cosmology is developing too rapidly and spontaneously. It usually does not rigorously use exact mathematical procedures and methods.

From 1–20 it is clear that the standard cosmological model, which is a direct mathematical consequence of Einstein's equations, does not approximate the physical universe well [200]. In our book we will investigate mathematically some of the above-mentioned problems and puzzles.

Mathematical models of physical phenomena, in general, are typically described by algebraic equations, ordinary or partial differential equations, integro-differential or integral equations, systems of these equations, variational inequalities, systems of differential-algebraic equations, etc. Their exact analytical solution is usually not known and there is no universal method capable of solving the above-mentioned models. Therefore, each class of problems has to be investigated individually. This consists of several steps including, as a rule, the proof of the existence (and, if applicable, also uniqueness) of the true and approximate solution, numerical solution of approximate problems and its stability, the treatment of convergence questions, continuous dependence of solutions on data, etc.

We should have in mind several important features in the mathematical modeling of physical problems. First, one should realize that no physical relation (except for definitions) holds entirely exactly, since the universe is composed from elementary