The sculpture of Tycho Brahe and Johannes Kepler in Pohořelec near Prague Castle commemorates the meeting of these famous scientists in 1600.

COSMOLOGY ON SMALL SCALES 2026

Proceedings of the International Conference

Cosmology on Small Scales 2026

Dark Matter Problem and Other Cosmological Puzzles

Prague, September 17–19, 2026

Edited by
Michal Křížek and Yurii V. Dumin

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LaTeX typesetting prepared by Hana Bílková

All truth passes through three stages:
First, it is ridiculed.
Second, it is violently opposed.
Third, it is accepted as self-evident.

Arthur Schopenhauer (1788–1860)

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Alternative cosmological theories

Preface

The term dark matter first appeared in an article by Jan Oort in 1932. A year later, to explain the gravitational binding of galaxy cluster Abell 1656, Fritz Zwicky postulated that it contains 400 times more non-luminous matter than luminous matter. Later, this enormous ratio was dramatically reduced. According to the standard cosmological model, there is six times more dark matter than ordinary baryonic matter. The aim of this conference is to bring together experts from various scientific fields to discuss whether dark matter actually exists or whether it is merely a consequence of excessive extrapolations of common physical laws to the large scales. Yet another issue to be discussed is the global geometry and shape of the physical universe, which may be revealed by observing objects with large cosmological redshifts using the James Webb Space Telescope. At last, we will also focus on our traditional topic – the local Hubble expansion of the universe.

The 6th International Conference *Cosmology on Small Scales 2026: Local Hubble Expansion and Other Cosmological Puzzles* was held at the Institute of Mathematics of the Czech Academy of Sciences at Žitná 25, Prague 1, from 17 to 19 September 2026 (see `css2026.math.cas.cz`). It was a continuation of our five previous conferences:

1. *Cosmology on Small Scales 2016: Local Hubble Expansion and Selected Controversies in Cosmology*,
2. *Cosmology on Small Scales 2018: Dark Matter Problem and Selected Controversies in Cosmology*,
3. *Cosmology on Small Scales 2020: Excessive Extrapolations and Selected Controversies in Cosmology*,
4. *Cosmology on Small Scales 2022: Dark Energy and the Local Hubble Expansion Problem*,
5. *Cosmology on Small Scales 2024: Local Hubble Expansion and Other Cosmological Puzzles*.

The main topics of the conference "Cosmology on Small Scales 2026" were:

- ▷ Arguments for and against dark matter, and revisiting the foundations of physics
- ▷ Alternative models for dark matter and dark energy
- ▷ Geometry of the physical universe according to JWST data
- ▷ Local Hubble expansion – searching for observational and laboratory evidence
- ▷ Cosmological effects in the localized astronomical systems

- ▷ Mathematical aspects of the extrapolations used in cosmology
- ▷ Explanations of various cosmological paradoxes

Only a fraction of the presented reports in CSS2026: *Dark Matter Problem and Other Cosmological Puzzles* is included into these proceedings; another part has been already published elsewhere. We incorporated also several abstracts and papers on "alternative cosmological theories". Although they may be questionable and the Organizing Committee is not responsible for their content, we believe that it is reasonable to present them to the wide audience.

The Organizing Committee consisted of

Prof. Michal Křížek — Chair (Czech Academy of Sciences, Czech Republic)
Assoc. Prof. Yurii Dumin — Vice-Chair (Moscow State University & Space Research Institute of RAS, Russia)
Dr. Avril Styrman (University of Helsinki, Finland)

We are deeply grateful to all authors for their contributions and the support of RVO 67985840 (Institute of Mathematics of the Czech Academy of Sciences). Out sincere thanks go also to all active members of the Cosmological Section of the Czech Astronomical Society for their continual help. Finally, we are indebted to Hana Bílková, Eva Ritterová, and Jarmila Štruncová for their technical assistance in the final typesetting.

These Proceedings can be downloaded from the website:

`http://users.math.cas.cz/~krizek/list.html`

Michal Křížek and Yurii V. Dumin

Conference *Cosmology on Small Scales 2026*
Michal Křížek and Yurii Dumin (Eds.)
Institute of Mathematics CAS, Prague

TEN OBJECTIONS TO THE PROCLAIMED AMOUNT OF DARK MATTER

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Abstract: According to the standard ΛCDM cosmological model, the physical universe consists of a significant amount of dark matter, about six times that of the usual baryonic matter, besides even larger amount of dark energy. But to date, both dark matter and dark energy have remained conceptually elusive, without concrete evidence based on direct physical measurements. We present ten mostly mathematical objections showing such a claimed amount of dark matter can be a result of vast overestimation and does not conform to reality. Some of them can be easily recalculated by hand. We also draw attention to argumentation, where staunch supporters of nonbaryonic dark matter fail. This paper is an upgrade of a previous paper in 2018 by the first author and updates some parameters from this paper.

Keywords: excessive cosmological extrapolation, red dwarf, spiral galaxy, rotational curve, gravitational redshift, galaxy cluster, orbital speed.

PACS: 95.35+d; 98.35.Hj; 98.65.Cw; 98.80-k

Cordially dedicated to Professor Ivo Babuška on his missed 100th birthday

Introduction

The term *dark matter* (*dunkle Materie* in German) can be found in Zwicky's 1933 paper [60, p. 125], and stands for invisible matter that could not be accounted for from estimated luminous mass. It was also briefly mentioned in Oort's earlier 1932 paper [42, p. 285], again for mass that was not visible. None of those earlier authors claimed that the dark matter must be nonbaryonic other than just optically invisible. But in modern terms, dark matter stands for nonbaryonic dark matter. It does not include the baryonic matter that is not luminous.

According to the standard ΛCDM cosmological model (i.e. Λ-Cold Dark Matter), we have

$$\text{the ratio of masses of dark matter to baryonic matter} \approx \mathbf{6} : \mathbf{1}. \tag{1}$$

The aim of this paper is to collect mathematical arguments showing that this ratio is highly exaggerated. We do not claim that dark matter does not exist. However, we do claim that the ratio on the left-hand side of (1) is much smaller and it can even be zero. We also update some parameters from our previous paper [21].

1. Too many simplifying assumptions in Zwicky's method

In 1933, Fritz Zwicky predicted the existence of some hypothetical dark matter which holds together the Coma galaxy cluster A1656 (see Figure 1). Using the Virial Theorem

$$V + 2T = 0, \tag{2}$$

he estimated its mass M by the following very simple algebraic formula

$$M = \frac{5R\overline{v}^2}{3G}, \tag{3}$$

where G is the gravitational constant, $V = -\frac{3}{5}GM^2/R$ is the total potential energy of the cluster, $T = \frac{1}{2}M\overline{v}^2$ is the kinetic energy, R is the radius of the cluster, and $\overline{v}$ is the mean quadratic speed with respect to the center of gravity of the cluster.

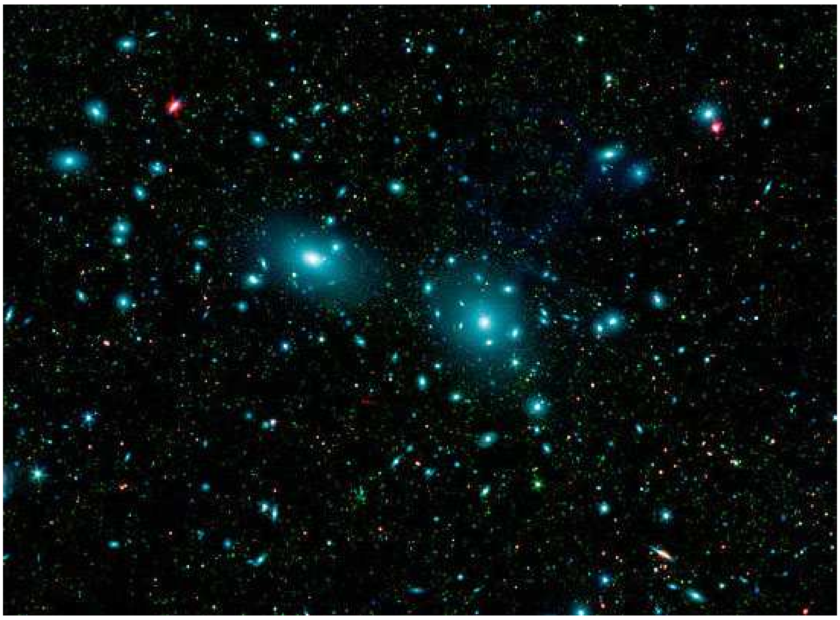

Figure 1. Giant galaxy cluster Abell 1656 in the constellation Coma Berenices. In the middle there are two supergiant elliptic galaxies NGC 4889 and NGC 4874 that grew larger due to galactic cannibalism (photo NASA).

In [60], Zwicky stated that in order to explain fast movements of galaxies for which $\overline{v} > 1000$ km/s in the Coma cluster, he had to assume the existence of a 400 times larger amount of nonluminous than luminous matter to keep the cluster gravitationally bound together. In his 1937 paper [61], he reduced this ratio to 150. Of course, he could use only very imprecise data and made implicitly many hidden simplifying assumptions (see [25]), otherwise he would be not able to estimate the mass M. For instance,

1) Zwicky was mistaken by one order of magnitude when estimating the distance of the cluster 13.8 Mly instead of currently accepted value ~100 Mly.

2) He was mistaken by two orders of magnitude when calculating masses of galaxies from their luminosity. He assumed that each galaxy has on average only 10^9 of solar masses.

3) He replaced galaxies of diameters $\sim 10^{10}$ au by point masses, which does not allow the consideration of tidal forces or spin angular momenta of rotating galaxies.

4) His total number of galaxies $N \approx 800$ in the Coma cluster is nowadays estimated as follows $N > 2000$.

5) The size $2.7° \times 2.5°$ of the cluster is much larger than his measured angular diameter $\beta = 1.7°$.

6) Zwicky assumed a uniform distribution of galaxies in the cluster, even though now we know that the central region is much denser than the region around the boundary.

7) He used classical Newtonian mechanics with an infinite speed of gravitational interaction in flat Euclidean space.

8) He assumed that the Coma cluster is in equilibrium and that the Virial Theorem (2) holds absolutely exactly.

9) He measured recession velocities with accuracy ≈ 100 km/s. In [60, p. 119], the radial velocity $\overline{v}$ appearing in (3) was obtained only from the 8 largest galaxies.

10) Intracluster matter (gas, dust, ejected solitary stars) was not taken into account. The Coma cluster may contain at least five times more nonluminous baryonic matter than luminous matter contained not only in galaxies.

Consequently, the ratio of masses of dark matter to baryonic matter was later reduced to (1) by the ΛCDM model. If the *virial parameter* $Q := T/|V| = 0.5$, then the virial mass M is given by (3). However, it seems that $Q > 0.5$ (i.e. the cluster dissolves) and then from (2) we get

$$M < \frac{5R\overline{v}^2}{3G}.$$

Considering a nonuniform mass distribution, the coefficient $\frac{5}{3}$ in (3) can be made smaller (see [25, pp. 112–115] for details). Also Sinclair Smith [53] assumes that this coefficient is only $\frac{1}{2}$ or 1 for the Virgo cluster. Taking into account relativistic effects of high velocities, gravitational redshift, gravitational lensing in a curved space, decreasing Hubble parameter in time, intergalactic baryonic matter, gravitational aberration, etc., the proclaimed virial mass M and also the ratio (1) can be essentially reduced by means of actual data (see Figure 2). These new arguments are not accounted for in Zwicky's method, see [23] for details.

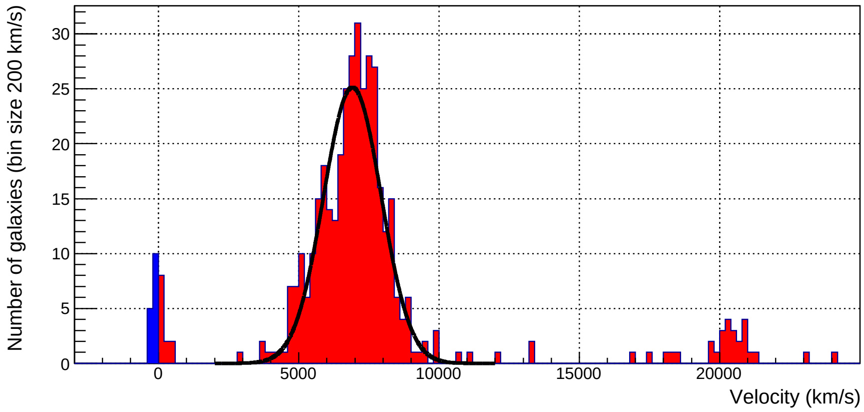

Figure 2. Histogram of radial velocities less than 25 000 km/s of galaxies which are projected to the Coma cluster region. Here we consider only those galaxies whose magnitude does not exceed 20. These data are available in [1], [6], and [10]. Galaxies possessing blueshift are on the left. The dark line represents the Gaussian curve fitted to those data that correspond to galaxies belonging to the Coma cluster. The other galaxies are not contained in this cluster, since their radial velocities with respect to the center of gravity of the cluster are too large. They should not be used to calculate the virial mass (3), because they do not form a bound system.

Finally note that the Coma cluster is located near the north Galactic pole. Therefore, the recession speed of the Coma cluster from the Sun is practically equal to the recession speed of the Coma cluster galaxy from the Milky way, even though the orbital speed $v_\odot = 230$ km/s of the Sun about the Galactic center is relatively high. The mean recession speed can be established by Pogson's equation, for details see [25, p. 111].

2. Excessive cosmological extrapolations

Each equation of mathematical physics has certain restrictions on the size of investigated objects, where reality is modeled well and, on the other hand, where its description fails, i.e., the modeling error essentially depends on the size of these objects. For instance, Archimedes' law is not valid on atomic level $\sim 10^{-10}$ m and also cannot be applied to objects of size $\sim 10^{10}$ m. The same is true for Ohm's law, Ampère's law, Biot-Savart law, etc. See [28] for other real-life examples.

The present ΛCDM model is based on the Friedmann equation, which describes the time behavior of the radius $a = a(t)$ of the expanding universe described by the three-dimensional sphere

$$\mathbb{S}_a^3 = \{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 + w^2 = a^2\},$$

where $\mathbb{E}^4$ stands for the four-dimensional Euclidean space. In 1922, this enabled Alexander Friedmann [14] to avoid boundary conditions. He applied the scale non-invariant Einstein's equations to the whole universe, even though they are formulated only locally. In this way he committed a questionable extrapolation, since these equations "are verified" (cf. [20]) on scales of the Solar system while the universe is at least a 15 orders of magnitude larger object.

The current cosmological model is based on the *normalized Friedmann equation*

$$\Omega_{\mathrm{M}} + \Omega_\Lambda + \Omega_k = 1$$

which is assumed to be valid for any time. Here the so-called *mass density parameter* (see [44]) is defined by

$$\Omega_{\mathrm{M}}(t) := \frac{8\pi G \rho(t)}{3H^2(t)}, \tag{4}$$

where $\rho(t)$ is the mass density at time t, $H(t) := \dot{a}(t)/a(t)$ is the *Hubble parameter*, $a = a(t)$ is the expansion function of the universe, and the dot denotes the time derivative. The *density of dark energy* is defined by

$$\Omega_\Lambda(t) := \frac{\Lambda c^2}{3H^2(t)}, \tag{5}$$

where Λ is the cosmological constant, c is the speed of light in a vacuum, and

$$\Omega_k(t) := -\frac{kc^2}{\dot{a}^2(t)}$$

is called the *curvature parameter* (or the curvature density), see [45], where $k \in \{-1, 0, 1\}$ is the *curvature index* ($k = 1$ in the case of sphere). Note that the Friedmann normalized equation is a differential equation, because the derivative $\dot{a}$ is hidden in the Hubble parameter.

Approximate values of Ω-cosmological parameters were determined by the three different methods of Baryonic Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB), and Supernovae type Ia explosions (SNe). They demonstrate that the universe consists of approximately 30 % of dark and baryonic matter (see (4)), 70 % of dark energy (see (5)), and the curvature parameter is almost zero (see [11]). However, these methods are not independent, since they are all based on the same normalized Friedmann equation which was derived by questionable extrapolations, see [28]. In truth, it is more likely that the measured data just indicate that the extrapolation is wrong, since it requires one to introduce some exotic dark matter and dark energy (see e.g. [29, 55, 56, 58]). Consequently, it seems that the real dynamics of the universe cannot be described by such a simple equation.

The solution of many dynamical systems essentially depends on history, i.e., on the way in which the system got into its immediate state. However, the Friedmann equation does not have this property, since it is an ordinary differential equation (see [28, p. 274]). From this equation far-reaching conclusions about the deep past and the far future are made in [3], [46], ...

Finally note that we do not know any significant digit of the cosmological constant Λ, yet. We even do not know whether it is positive or negative, if it exists at all, although there are hundreds of papers about this idea of Einstein. Notice also that the term (5) with Λ is dominant, because $\rho(t)$ in (4) is at present decreasing.

3. Red dwarfs

In 1960 Jan H. Oort [43] (see also [50, p. 27]) showed that the observed oscillations of stars perpendicularly to the galactic plane require double the mass density of the galactic disk. At the end of the 20th century it was thought that red dwarfs of the spectral class M form only 3 % of the total number of stars, see [5, p. 93]. Nevertheless, at present it is estimated that red dwarfs are the most common stars making up about 73–76 % of all stars in the Milky Way. To support this statement we point out that among the 20 nearest stars to our Sun, 13 red dwarfs are currently known. Some suggest even higher percentages of red dwarfs in elliptical galaxies. The mass of a typical red dwarf ranges from

$$0.08 M_\odot \quad \text{to} \quad 0.45 M_\odot\,.$$

The above-mentioned observed motion of stars perpendicularly to the galactic plane can thus be explained by a large amount of red dwarfs without dark matter.

Distribution of stars in our Galaxy according to their spectral classes is based on the Gaia satellite data (see [15]). The Harvard Spectral Classification (see [63]) shows a similar relative representation of stars. Furthermore, Gaia is able to look at the center of our Galaxy and in the opposite direction at the galactic edge which was not possible for the Hipparcos satellite. In the last century, astronomers, of course, could not know about the existence of so many red dwarfs of the spectral class M. This growth is due to the continual improvements of the sensitivity of space telescopes. In this way, the estimated mass of the baryonic matter in our Galaxy has considerably increased.

The radius of the visible part of the disk of our Galaxy is estimated by

$$r_\mathrm{G} = 16 \text{ kpc} = 4.938 \cdot 10^{20} \text{ m}. \tag{6}$$

Unfortunately, we cannot so far reliably determine the contribution to the total mass of our Galaxy from black holes, neutron stars, and infrared dwarfs whose luminosities are very small. Note that there are three supplementary spectral classes for small cold dim stars: L (red-brown dwarfs), T (brown dwarfs), and Y (black dwarfs). For instance, in 2013 Kevin Luhman discovered a pair of brown dwarfs only 6.5 ly from the Sun. Another brown dwarf WISE J085510.83-071442.5 is located 7.2 ly from us. It seems that there exist over 10^{11} brown dwarfs in the Milky Way. According to [38, p. 393], the mass of the baryonic matter of all the stars in the Galaxy is about

$$175 \cdot 10^9 M_\odot = 3.5 \cdot 10^{41} \text{ kg} \tag{7}$$

including further stars of the luminosity classes I–IV (i.e. supergiants, giants, and subgiants). The disk and bulge contain also a large amount of non-luminous baryonic matter in the form of dust, gas, and plasma. In [38, p. 353], the amount of interstellar matter (without hypothetical dark matter) is estimated at about 10% of the total mass of the Milky Way's stars, i.e., by (7) we obtain

$$\mathcal{M}(r_{\mathrm{G}}) \geq 3.85 \cdot 10^{41} \text{ kg}, \tag{8}$$

where $\mathcal{M}(r)$ is the *mass of baryonic matter* within the ball of radius r and the center in the middle of our Galaxy, see [21, p. 353]. The mass in (8) is much larger than that predicted in the 20th century. Thus the ratio (1) should be smaller.

4. Winding problem of spiral galaxies

Vera Rubin's greatest discovery was the fact that spiral galaxies have "flat" rotational curves (see Figure 3 and [47]). On that basis, in the 70's of the last century she developed her own theory of rotational curves of galaxies. From the high orbital speed of stars she concluded that galaxies should contain much more nonluminous than luminous matter to be kept together by gravity — see e.g. her review articles [48], [50], and [51] on dark matter.

Now let us look more closely at her hypothesis. Consider a test particle of mass m (typically this will be a star) and let $M \gg m$ be the mass of some body generating the central force field. Assume that the test particle revolves about the center along a circular orbit with radius r and speed v. Then from Newton's law of gravitation and the relation for centripetal force Rubin easily obtained that (see [48] and [49])

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}, \quad \text{i.e.,} \quad v = \sqrt{\frac{GM}{r}}. \tag{9}$$

The velocity v of a particle on a circular orbit is thus proportional to $r^{-1/2}$. Such orbits are called *Keplerian* (see Figure 3).

In 1962, Rubin et al. in [47, p. 491] stated:

...the stellar curve does not decrease as is expected for Keplerian orbits.

To explain this discrepancy, we note that spiral galaxies do not have a typical central force field. For instance, the mass of the central black hole is only 0.01 % of the total mass of our Galaxy; while in the Solar system 99.85 % of the mass is concentrated at the Sun. The planets barely interact gravitationally among themselves and their movements are determined mainly by the central force of the Sun. On the other hand, trajectories of stars in a galactic disk are substantially influenced by neighboring stars, since the central bulge contains only about 10 % of the total mass of a galaxy. Therefore, the speed v of stars on circular orbits in a spiral galaxy should be higher than for Keplerian orbits (cf. Figure 3). For details see [24]. The gravitational potential of the Solar system has a completely different shape than the gravitational potential of a spiral galaxy.

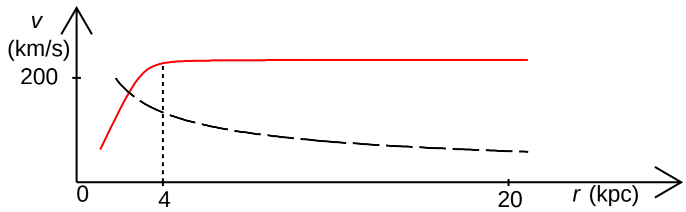

Figure 3. The solid line shows an idealized rotational curve whose shape was derived by Rubin by means of a variety of measurements. It is flat for $r > r_0 \approx 4$ kpc. The dashed line shows the rate of decrease of velocities for Keplerian orbits that depend on the distance r from the center of a spiral galaxy.

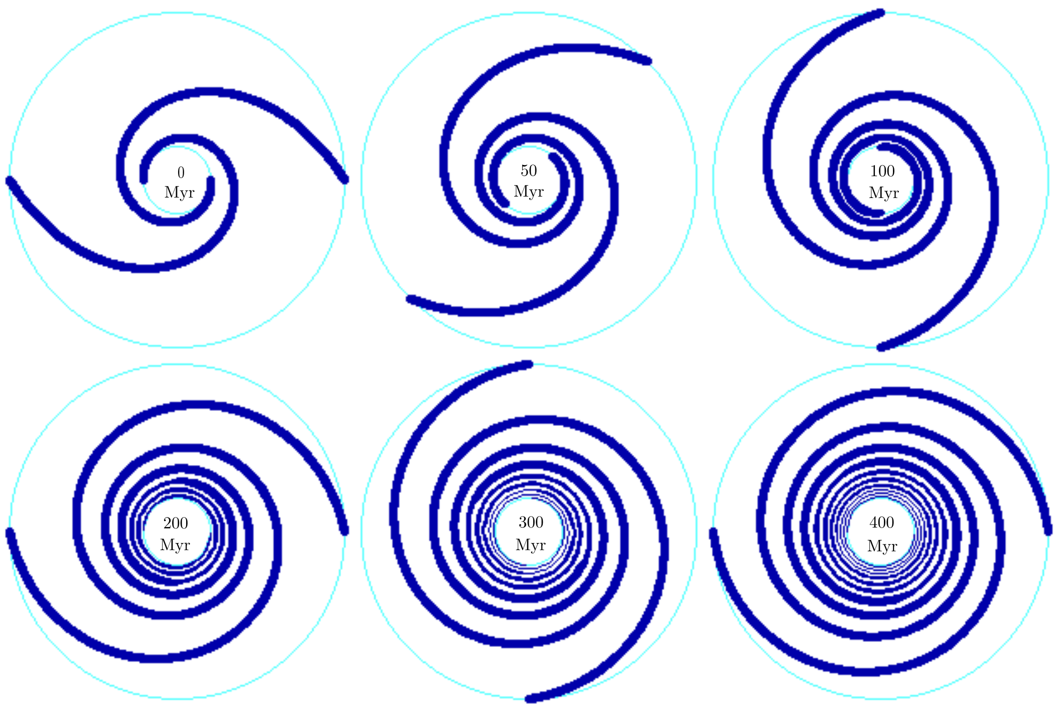

Figure 4. Winding problem of spiral galaxies: The flat rotational curve from Figure 3 causes a very quick tightening of spiral arms after one revolution of the most external stars. This takes approximately 400 Myr for a typical spiral galaxy (note

that the age of the Sun is more than 10 times larger). Here stars on the galactic edge revolved about the angles: 0 (an idealized initial state), $\pi/4$, $\pi/2$, π, $3\pi/2$, and 2π. After a relatively short time period of 100 Myr we would get a shape similar to Figure 9. After a few hundreds million years the spiral arms would be highly twisted. After 1 Gyr they would be completely wrapped up. However, observed spiral galaxies exist for many Gyr.

It is noteworthy that the stars of spiral galaxies are measured to move at almost constant speed (see [51, p. 7]), but these galaxies are not winding up and surprisingly do not show an expected tightening of arms as shown in Figure 4. Therefore, their shape cannot be stable if they have gone through many revolutions.

To see this, consider a spiral galaxy as marked in the upper left part of Figure 4. Assume that the outer radius is 20 kpc and the inner radius $r_0 = 4$ kpc (see Figure 3). Then after one revolution of a star on the outer orbit a star on the inner orbit makes $20 : 4 = 5$ revolutions if it has the same speed. This contradicts observations (see e.g. Figure 9).

With difficulty it can also be assumed that the arms of e.g. type Sbc galaxies are formed by some kind of density waves as suggested in [5, p. 544]. Such barred spiral galaxies resemble an open letter S. Therefore, many astronomers did not become convinced of the need for dark matter halos in spiral galaxies.

5. Orbital speed of stars

Our Sun orbits the center of the Milky Way with the speed

$$v_\odot = 230 \text{ km/s} \tag{10}$$

on a path of radius $r_\odot = 8$ kpc, i.e., it stays about halfway (cf. (6)) out from the center O of the Galaxy. According to (10), its orbital period is

$$T_\odot = \frac{2\pi r_\odot}{v_\odot} = \frac{2\pi \cdot 2.5 \cdot 10^{20}}{2.3 \cdot 10^5} \text{ s} = 2 \cdot \pi 10^7 \cdot 10^8 \text{ s} = 200 \text{ Myr},$$

compare with Figure 4. Stars orbiting the center of our Galaxy at any distance $r > r_0 \approx 4$ kpc should have a speed similar to $v_\odot$ due to the expected flat rotational curve (see Figure 3).

Denote the mass of all baryonic matter contained inside the ball of radius (6) centered at O by $\mathcal{M}(r_\mathrm{G})$. Further, we shall proceed in two steps:

1. First, let us concentrate all baryonic matter contained inside this ball to the central point O. Then from relations (6), (8), and (9) the velocity of a test particle on the orbit of radius r_G is

$$v = \sqrt{\frac{G\mathcal{M}(r_\mathrm{G})}{r_\mathrm{G}}} \geq \sqrt{\frac{6.674 \cdot 10^{-11} \cdot 3.85 \cdot 10^{41}}{4.938 \cdot 10^{20}}} = 228 \cdot 10^3 \text{ (m/s)}. \tag{11}$$

By this very simple "back of the envelope" calculation, we see that v is indeed comparable to the measured speed (10). Although these relations are only approximate, to postulate the existence of six times more dark matter than baryonic matter (see (1)) to hold the Galaxy together by gravity seems to be highly overestimated.

2. Second, we claim that the orbital velocity around a flat disk of the same mass is even higher than that in (11). This results from the following theorem proved in [25, pp. 131–134].

Theorem. *A particle orbiting a central mass point along a circular trajectory of radius R has a smaller speed than if it were to orbit a flat disk of radius R and the same mass with an arbitrary rotationally symmetric density distribution.*

To explain the main idea of the proof, consider the situation in Figure 5. Let two point masses $m_1 = m_2$ be located inside a ball placed symmetrically with respect to the horizontal plane. Let a test particle with mass m be in this plane. Then the total force F of both point masses acting on the test particle of mass m will be less than the force $\overline{F}$ of both point masses projected perpendicularly to the disk and acting on m. Let d be the distance between m_1 and m. Denoting by b its orthogonal projection on the horizontal plane, we find that

$$F = G\frac{2m_1 m}{d^2} \cdot \frac{b}{d} \quad \text{and} \quad \overline{F} = G\frac{2m_1 m}{b^2}.$$

Thus we see that the ratio of forces $\overline{F}$ and F is equal to the third power of the fraction d/b

$$\overline{F} = \left(\frac{d}{b}\right)^3 F \geq F.$$

This cubic nonlinearity causes a greater attractive gravitational force by the disk than by the ball (cf. (9)), and thus also a higher orbital speed around the disk. For instance $\left(\frac{5}{4}\right)^3 \approx 2$. The above theorem thus explains the large orbital velocities of stars in our Galaxy, even though spiral galaxies do not have rotationally symmetric distribution of mass. It also indicates why (1) is overestimated and why Newton's law of gravity on galactic scales could be still a fairly good approximation of reality.

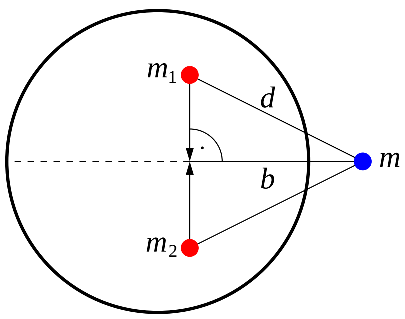

Figure 5. A ball with symmetrically distributed mass with respect to the horizontal plane acts on a test particle by a smaller force than the mass projected perpendicularly to the horizontal plane of the disk — dashed.

Finally, recall the Shell Theorem which states that a spherical layer with spherically symmetric mass density distribution exerts no force on a point mass located inside (see [25, p. 45]). Hence, for a spherically symmetric mass distribution of the halo we may neglect the influence of baryonic (and possible dark) matter outside a ball that contains the galactic disk.

6. Tidal tails

Figure 6 presents a collision of two galaxies called *Antennae*. Behind each galaxy there is a clear "tidal tail" showing their original trajectories. If there were to be six times more dark matter than baryonic matter (see (1)) around this pair, then we would not observe such nice almost elliptic Keplerian orbits as in the classical two-body problem.

Figure 6. The Antennae Galaxies, also known as NGC 4038/NGC 4039, are a pair of interacting galaxies in the constellation Corvus.

Moreover, Pavel Kroupa (see e.g. [32, 33]) presents a number of observations which rule-out the existence of dark matter particles. He shows that there are no dark matter halos, since then our neighboring dwarf galaxies would slow down due to dynamical friction with dark matter and they would spiral into the center of the Milky Way. This is not observed.

7. Collision of galaxy clusters

Douglas Clowe et al. in [9] propose a collision of two galaxy clusters, where the intergalactic gas is stopped, while the galaxies continue in an unchanged direction together with presumed dark matter. The title of this paper *A direct empirical proof of the existence of dark matter* should impress that dark matter was finally found. Nevertheless, we are not able to measure tangential components of the velocities of these clusters to prove that the collision really happened. The authors neglect dynamical friction of particular galaxies and suppose an unrealistically high mutual infall velocity (see [9, p. L112])

$$v \approx 4700 \text{ km/s} \ > 0.01c$$

of these clusters to guarantee that the collision does not last more than several billion years. The expected tidal tails are not visible as in Figure 7. Moreover, the proposed infall velocity v has the opposite sign to the overall Hubble expansion speed of the universe. How could these two galaxy clusters get such unlikely high velocities several Gyr ago? This would produce an extremely large kinetic energy proportional to v^2 in an almost isotropic and homogeneous universe, where the local peculiar speed of galaxies is usually only several hundred km/s.

The regions with dark matter are artificially colored in blue [9] by some numerical simulations based on gravitational lensing. In case of light bending near our Sun during total eclipses, we know exactly the bending angle. However, in case of hypothetical dark matter regions we have to apply only some inexact heuristic algorithms and interpolation techniques between galaxies, since galaxies are represented only by several pixels on photos.

Figure 7. This figure should illustrate the collision of two galaxy clusters MACS J0025.4-1222. Hypothetical dark matter is artificially colored on the left and right. The central region produces X-rays, which should demonstrate the collision of gas from both the clusters [9].

8. Revolution of two giant galaxies in the Coma cluster

Now we shall present another "back of the envelope" calculation illustrating whether it is necessary to assume some extragalactic dark matter in the center of the Coma cluster satisfying (1). Each of the two supergiant elliptic galaxies NGC 4889 and NGC 4874 in the middle of Figure 1 has a 10 times larger mass than the Milky Way (see e.g. [62]). Hence,

$$m = 10M_{\rm G} = 10^{13}M_\odot = 2 \cdot 10^{43} \text{ kg}, \tag{12}$$

where the total mass of our Galaxy

$$M_{\rm G} = 10^{12} M_{\odot} = 2 \cdot 10^{42} \text{ kg}$$

is given in [34, p. 127]. If one of these two galaxies were to be smaller, it would orbit the larger one by a higher velocity and along a longer path. Then it would absorb more additional galaxies than the larger one. By this mechanism the masses of both galaxies are well balanced, see (12). The mass $M_{\rm G}$ is, of course, larger than the lower bound for $\mathcal{M}(r_{\rm G})$ in (8), where $r_{\rm G}$ is the radius of the visible part of the galactic disk only. Assume that both the giant galaxies orbit along a circular trajectory with center O, radius r, and velocity v.

By the Shell Theorem (see [25, p. 45]) the gravitational potential inside a homogeneous spherical layer is constant. External galaxies and possible dark matter outside the sphere with center O and radius r have almost no influence on this motion. From Newton's laws and the relation for centripetal force we get

$$\frac{Gm^2}{4r^2} = \frac{mv^2}{r}. \tag{13}$$

The distance of both galaxies on the celestial sphere is $8.15'$ which in projection on the distance 100 Mpc gives $7.32 \cdot 10^{21}$ m. Thus for the radius r we have

$$r \geq 3.66 \cdot 10^{21} \text{ m}. \tag{14}$$

According to [1, p. 19], the measured radial velocities of both supergiant galaxies are 6472 km/s and 7189 km/s. Their average velocity $\tilde{v} = 6830.5$ km/s nicely corresponds to the mean recession speed 7000 km/s (see Figure 2) of the whole cluster. For the radial velocity $v_{\rm radial}$ with respect to O we get by (12) and (13), and (14)

$$3.585 \cdot 10^5 = \frac{7\,189\,000 - 6\,472\,000}{2} = v_{\rm radial} \leq v = \sqrt{\frac{Gm}{4r}}$$
$$\leq \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{43}}{4 \cdot 3.66 \cdot 10^{21}}} = 3.02 \times 10^5 \text{ (m/s)}. \tag{15}$$

Comparing the left-hand and right-hand sides, we find a small discrepancy. However, by the Shell Theorem, the velocity of the two giant galaxies is mainly influenced by matter situated inside the sphere of radius r and the center O. Thus considering the gravitational influence of other matter (small galaxies, large amount of solitary stars, and hot gas) that are inside this sphere, the right-hand side of (15) should be much larger. For instance, according to [57], the intracluster medium contains 30–50 % of stars from all the stars of the cluster. Inside galaxy clusters there is at least five times more baryonic matter in the form of hot gas emitting X-rays than baryonic matter contained in galaxies (see [2], [7], [18], [59]).

Assume, for simplicity, that this additional baryonic matter has a spherically symmetric distribution and denote its mass by M. Concentrating the mass M to the center O, the speed v in (15) can be, using the First Newton Theorem (see [25, p. 43]), replaced by

$$\overline{v} = \sqrt{\frac{G(m + 4M)}{4r}} \gg v.$$

To see this, it is enough to consider the relation

$$\frac{Gm^2}{4r^2} + \frac{4}{4} \cdot \frac{GmM}{r^2} = \frac{m\overline{v}^2}{r}$$

instead of (13). Zwicky's paradox of large observed velocities thus vanishes, since it has a quite natural explanation without dark matter (cf. (1)).

9. Millennium simulation

The Millennium simulation [8] seeks to prove that without dark matter galaxies could not form after the Big Bang. However, this simulation is based on a Newtonian model with unclear definition of initial and boundary conditions. This model possesses several classical drawbacks, for instance, the mirror image of Figure 8 should be also a solution of this problem. The modeling error of the N-body simulation with 10^{10} dark matter particles as presented in [54] is ignored. It is also not evident whether Newtonian mechanics can be applied to the early superdense universe and such large scales. Moreover, the millennium simulation assumes the infinite speed of gravitational interaction which obviously contradicts causality.

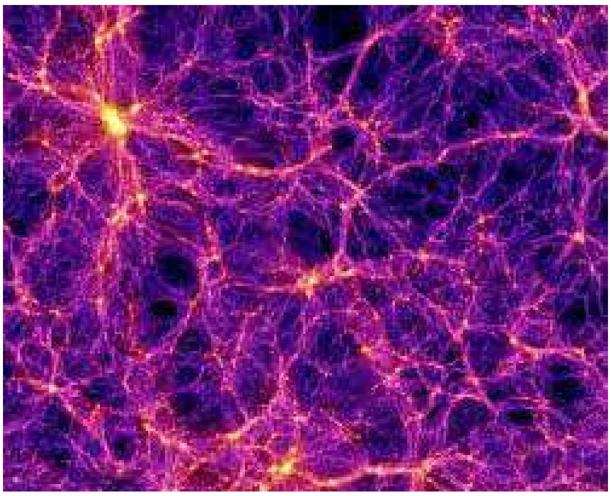

Figure 8. Millenium simulation

Note that Newtonian mechanics is not a causal theory, since it assumes an infinite speed of gravitational interaction. This means that every gravitational event in the Newtonian mathematical model immediately spreads beyond the corresponding light cone. On the other hand, gravitational events in the actual physical world propagate inside the corresponding light cone and satisfy the Principle of Causality which states that

All real events necessarily have a cause.

Therefore, any conclusion about the existence of a large amount of dark matter from Newtonian theory is questionable.

10. High symmetry of spiral galaxies

Gravity is the only interaction that rules our universe on galactic scales. Dark matter should only interact gravitationally (and perhaps locally also weakly). Most of the observed galaxies have spiral structure. If these galaxies were to contain six times more uniformly distributed dark matter than baryonic matter, then they could not exhibit such a high symmetry of structured baryonic matter as seen in Figure 9. Although non-discovery of dark matter does not mean that it does not exist, the ratio (1) seems to be again overestimated.

Figure 9. Spiral galaxies could not have such a large symmetry if there were to be six times more almost uniformly distributed dark matter than structured baryonic matter.

11. Final remarks

In previous sections we introduced 10 arguments showing that the amount of dark matter as given in (1) seems to be considerably overestimated. It then follows that the amount of dark energy is also mistakenly determined, or that dark energy does not exist at all, see [64].

It is very probable that Newton's law of gravity and the general theory of relativity on large cosmological scales approximate physical reality only very roughly [22, 29, 40] and thus the proclaimed dark matter is nothing else than a modeling error. Pavel Kroupa in [30] and [31] states other arguments that point to the absence of dark matter around our Galaxy and M31. A number of other papers (see [4], [12], [13], [16], [19], [32], [36], [39], [41], and [52]) also confirms that on scales of galactic disks it is not necessary to assume the existence of dark matter. The main arguments are the following:

• The influence of dark matter in the Solar system has not been observed, even though our Sun is a large gravitational attractor [39].

• The ratio between virial mass and luminous mass in globular clusters is less than two (see [37]).

• The cosmic microwave background corresponds to the redshift $z \approx 1089$, see [46]. The most typical diameter in fluctuations of the temperature angular power spectrum is about 1°. Since the radius of the universe is about 10^{27} m, the size of these fluctuations is about 10^{21} m, which is comparable with the present diameter 100 000 ly of our Galaxy. However, there is no physical process that could produce e.g. polarization of the CMB or acoustic baryonic oscillations on such a large scale during a period of only 10 000 years when the CMB was generated. At that time the mean mass density was $(z+1)^3 \approx 10^9$ times larger than now. Hence, it is difficult to state any reliable conclusions about the present value of the mass density parameter (4) from the CMB map.

• Also weak gravitational lensing essentially deforms CMB for $z \gg 1$ (see [28, p. 277]).

• Several modifications of Newtonian theory, e.g. MOND (Modified Newtonian Dynamics) and its relativistic generalization TeVeS (Tensor-Vector-Scalar), are at present being developed and studied. Effects that are attributed to dark matter are explained by a different form of the gravitational law on large scales. However, MOND contradicts causality, since it assumes an infinite speed of gravitational interaction.

Remark. In the standard model of particles and their interactions there is no place for dark matter particles such as axions, neutralina, wimps, etc. Also the LHC in CERN has not found any signs of new physics that could explain dark matter.

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IMPLICATIONS OF A LOCAL HUBBLE-LEMAÎTRE PARAMETER $H_0 = (49 \pm 11)$ km/(s Mpc)

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Abstract: Despite a widespread belief that cosmological expansion does not occur in gravitationally bound systems, multiple lines of evidence show that the Hubble-Lemaître parameter H_0 can not only be measured in the Solar System, but that its value is lower than estimates of H_0 derived from the wider universe. Apart from effects on the dynamics of planets and their satellites, this result has implications for cosmological theories. In particular, it aligns with the large scale Hubble tension in suggesting that dark energy is unrelated to Einstein's cosmological constant. It shows that quantitative studies of tidal drag may have been overestimated, and it raises questions about the physical determinants of H_0.

Keywords: Hubble-Lemaître parameter, Solar System, expansion, planets, satellites, tidal theory, gravitation, dark energy, cosmological constant

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1. Introduction

There is considerable evidence that cosmological expansion occurs in the Solar System [6–11, 14–22, 23–27, 31, 32]. The Hubble-Lemaître parameter H_0 can be measured locally: a finding that conflicts with the standard assumption that the orbits of gravitationally bound bodies are unaffected by the Hubble flow.

Much of the observational support for orbital expansion relates to the Earth-Moon system and to the natural satellites of the other planets. As noted over fifty years ago [14], the measured rate of recession of the Moon is close to that predicted by the Hubble-Lemaître law. Similar 'coincidences' occur throughout the Solar System. Thus, the relative mean motion acceleration of some moons has been determined. This metric has units of reciprocal time and is found to be numerically close to H_0. For the Galilean moons of Jupiter (Io, Europa, Ganymede and Callisto), the large inner moons of Saturn (Rhea, Dione, Tethys, Enceladus and Mimas) and two of the larger moons of Uranus (Miranda and Ariel), a Hubble-Lemaître parameter equal to $H_0 = 70$ km/(s Mpc) provides an almost identical rate of recession to that calculated from tidal theory. The observed rate of recession of Titan, Saturn's large outer moon, is close to the rate predicted by H_0 [26]. Other evidence for local expansion is provided by the Faint Young Sun paradox [10, 11, 25] and the anomalously large orbit of Neptune as well as the origin and recession of the Martian satellite Deimos [16] and the Neptunian satellites Proteus and Hippocamp [20].

In many studies of local Hubble-Lemaître flow, it has been convenient to use $H_0 = 70$ km/(s Mpc), a value midway between the two large-scale estimates based on the cosmic background radiation and the red shift of supernovae [5, 30]. However, a number of measurements [22] in the Solar System lead to the conclusion that the best estimate for the local Hubble-Lemaître parameter is $H_0 = (49 \pm 11)$ km/(s Mpc). Despite the uncertainty, that is much lower than has been obtained from measurements in the wider universe. This has implications not only for the dynamics of bodies in the Solar System but for cosmological theories and the physical origin of dark energy.

2. The Solar System

2.1. The Faint Young Sun Paradox

This paradox arises because there is a contradiction between observations of liquid water early in Earth's history and the astrophysical expectation that the Sun's output was lower in the distant past. Although estimates vary, that luminosity may be taken here as 80 % as intense around 3×10^9 yr (i.e., 3 Gyr) ago as it is today [13]. If the orbit of the Earth had been unchanged, it would have meant that the Earth had been completely frozen at that earlier epoch. However, it is clear that conditions on Earth were suitable for life to evolve at that time. Many explanations have been suggested for this paradox including denser cloud cover or greenhouse gas effects, yet none is satisfactory nor widely accepted. The relationship between the thermal history of the Earth and the Hubble-Lemaître parameter has recently been discussed [10, 11, 25].

The Solar radiation experienced by a body at distance r is proportional to $1/r^2$. If the rate of recession of the Earth from the Sun is $H_0 = 49$ km/(s Mpc), then the Earth-Sun distance at 3 Gyr ago was 0.85 au. If the current radiation experienced by Earth is taken as unity, then the radiation experienced by Earth at that time is given by $0.8/0.85^2 = 1.11$ of the current value. However, this result should be treated with caution, since the thermal history of Earth would have also depended on the types of gases in the atmosphere, the extent of cloud cover, the albedo, major collisional events in the early epoch and other factors.

2.2. Earth-Moon Recession

When $H_0 = 49$ km/(s Mpc), the expected cosmological expansion of the Earth-Moon system is 1.96 cm yr^{-1}. Since the observed rate is 3.82 cm yr^{-1} (see [4]), then Hubble-Lemaître flow accounts for 51 % of the retreat. In a separate study of Lunar data, Křížek and Somer [27] concluded that the best estimate is $H_0 = 40$ km/(s Mpc).

2.3. The Origin and Orbit of Deimos

The semi-major axis of the Martian moon Deimos is 23 500 km. Bagheri et al. [2] have described a mechanism whereby Deimos and the second Martian moon Phobos originated from the fragmentation of a common progenitor. That event is believed to have taken place between 1 and 2.7 Gyr ago when that original body was close to a synchronous equatorial orbit, which has a current radius of 20 428 km. The orbits of the two fragments, initially eccentric, became circularised through tidal drag causing Phobos to adopt an orbit closer to Mars while Deimos receded. The current rate of recession of Deimos based on $H_0 = 49$ km/(s Mpc) is therefore 0.12 cm yr^{-1}. Since Deimos has receded by 3 072 km, then it left the stationary orbit 2.6 Gyr years ago. That is now close to the earlier time in the range provided by Bagheri et al. [2] and close to the epoch of the 'Late Heavy Bombardment' [3] when impacts between bodies in the Solar System were more frequent.

2.4. The Moons of Jupiter, Saturn and Uranus

In the case of Titan, the large outer moon of Saturn, the measured rate of orbital expansion is 11.3 cm yr^{-1}, see [28]. With $H_0 = 49$ km/(s Mpc), cosmological expansion equates to 6.0 cm yr^{-1}, which represents 53 % of this recession. Tidal effects then account for the remaining 47 %. For the other moons of these three major planets, where an analysis has been published [17–19] then, as discussed below, a lower value of H_0 may still leave the total rate of recession (cosmological + tidal) too high.

2.5. The Neptunian System

It is well recognised that the mass and orbit of Neptune (30 au) appear too large to be consistent with its origin in a Solar nebula. With $H_0 = 49$ km/(s Mpc), then Neptune would have originated at 23 au from the Sun 4.6 Gyr ago.

When $H_0 = 70$ km/(s Mpc), it has been shown [20] that the Neptunian satellites Proteus and Hippocamp probably originated from the fragmentation of an original body around 3.3 Gyr ago. Using $H_0 = 49$ km/(s Mpc), and assuming that Proteus, the larger moon, experiences similar rates of tidal and cosmological recession, while the tiny Hippocamp is unaffected by tides, then the orbits of Hippocamp and Proteus would have coincided around 2.5 Gyr ago just above the current location of the synchronous orbit. As with the origin of Deimos and Phobos in the Martian system, that is close to the period of 'Late Heavy Bombardment' [3].

2.6. Tidal Theory

There is no doubt that tidal drag can play an important role in the recession of orbiting bodies, but it appears to have often been overestimated. Thus, the retreat of the Moon from Earth has historically been regarded entirely as a tidal effect, but it is now seen to contribute only around half of that recession. For the satellites of Jupiter, Saturn and Uranus where detailed calculations have been made of tidal effects, it appears that they are again too high. Thus, the orbits of some of their natural satellites would have originated just outside the current respective synchronous (stationary) orbits even in the absence of cosmological expansion. It is considered unlikely that those moons could have originated below the local synchronous orbit since tidal forces will have caused their orbits to contract. Thus, in the case of the Martian moon Phobos, whose orbit is located below the synchronous orbit, its rate of descent towards the surface of Mars is nearly 200 times the expected cosmological recession velocity [16]. Since tidal and cosmological expansion of satellite orbits cannot therefore both have occurred to the same extent, it suggests that either the cosmological recession velocity or the tidal retreat velocity or both have been overestimated.

3. Cosmological Implications

3.1. Gravitationally bound Bodies

It is widely stated that cosmological expansion cannot overcome gravitation. That assumption is applied not just to planetary systems, but to entire galaxies and even galaxy clusters. As noted by Křížek and Somer [27], it is difficult to see where gravitational dominance ends and expansion begins. The idea of 'no local expansion' could have originated with the Einstein-Straus model [12] published eighty years ago. That envisaged a central mass surrounded by a spherical vacuum, with a radius at that point where gravitation balanced recession. Cosmological expansion could then only occur outside that vacuole. That theory may have been acceptable in the long period when the universe was believed to be matter-dominated, but the concept of a vacuole is meaningless in a universe where dark energy is the dominant form of mass-energy and is distributed everywhere. Although expansion is often described as a force, it is clearly not a force in the Newtonian sense since it appears to operate equally on all bodies regardless of their mass.

3.2. Dark Energy and the ΛCDM Model

The traditional view is that around 13.8 Gyr ago a 'Big Bang' was responsible for the expansion of the universe. According to Peebles [29] that is an unfortunate term, but a 'Big Bang' cannot explain why the Solar System should expand since it only formed around 4.6 Gyr ago. In the past thirty years, it has been hypothesised that an accelerating expansion is being driven by dark energy, and that this constituent comprises close to 70 % of the total mass-energy of the universe. In the standard Lambda cold dark matter (ΛCDM) paradigm, the term Λ encapsulates this dark energy as being none other than the cosmological constant introduced by Einstein to ensure stability of his original static cosmological model. As such, Λ is now equated with the negative pressure of the quantum mechanical vacuum. Since Λ is a constant, dark energy must be spatially and temporally constant, yet this appears to be contradicted by the continuing failure to resolve the Hubble tension [5, 30] as well as the low value of H_0, as described here, found in the Solar System. The cosmological validity of Λ is also questionable in the light of other concerns such as the 'cosmological constant paradox' [1], where theoretical calculations suggest Λ could be at least 120 orders of magnitude greater than is observed. It seems to many that this theoretical expectation must be completely cancelled by other (unknown) terms rather than leave a minute residual component. A further curiosity is that, in Einstein's equations, Λ has dimensions of m^{-2}, whereas it manifests as an energy density, i.e.,$\mathrm{kg\,m^{-1}s^{-2}}$. These and other problems with Λ have been discussed by Křížek and Somer [27].

3.3. What determines H_0?

If we are to retain the concept of dark energy then it must have a physical basis which allows it to vary in space and time. This in turn leads us to ask what determines the magnitude of the Hubble-Lemaître parameter in the Solar System. The Friedmann equation [27] relates H_0 to the overall mass density of the universe, but that is an idealised derivation since it is based on an isotropic and homogeneous universe, a situation that does not apply to the Solar System. We therefore seem to be left with the conclusion that local expansion must be driven largely, if not entirely, by dark energy. This might be pictured as the vacuole in the Einstein-Straus model, where no expansion occurs, now being filled with dark energy that provides the means for expansion. It might also be asked if local masses can modulate local expansion, in other words how H_0, or the dark energy field that drives it, is coupled to the gravitational field. For example, might satellites with similar masses and with similar orbital radii retreat from their respective planets at different rates if those planets have different masses? At present, the local value of H_0 is not known with sufficient precision to answer this question.

4. Conclusions

The finding that the Hubble-Lemaître parameter has a finite value in the Solar System shows that gravitationally bound bodies do experience cosmological recession. The best local estimate for that parameter is $H_0 = (49 \pm 11)$ km/(s Mpc), a result that is much lower than has been determined in the wider universe. However, this adds a further dimension to the well-documented Hubble tension, and supports the suggestion that H_0 may be scale dependent. That then casts doubt on the validity of Einstein's cosmological constant as a source of dark energy. At a local level, cosmological recession of some satellite orbits is similar to their rate of tidal recession, but it is concluded that published estimates of tidal recession often appear too high.

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ON THE VALIDITY OF THE LAW OF CONSERVATION OF ENERGY

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Abstract: According to Emmy Noether's Theorem, the energy of each isolated system is conserved if it possesses symmetry with respect to time translations. In Newtonian models with infinite speed of gravitational interaction, this theorem is perfectly applicable. However, the situation is different in the actual physical universe, where the speed of gravitational interaction is finite which causes that the trajectories of planets and their moons form slightly expanding spirals. This process is not reversible. We present ten independent real-life examples showing that the Solar system very slowly expands and thus is not symmetric with respect to time translations as required by Noether's theorem. Some of these examples are based on very precise measurements. Consequently, we hypothesize that the law of conservation of energy (and also angular momentum) is slightly violated in the actual physical universe. Then we demonstrate why mathematical models should not be identified with reality which often happens. Finally, we present ten main problems of the contemporary cosmological model which is based on the energy and angular momentum conservation.

Keywords: dark energy, Solar system, spiral galaxy, orbital speed, excessive cosmological extrapolation

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1. Introduction

More than 100 years ago Emmy Noether proved the following theorem, see [38].

Theorem (Noether). *The energy of each isolated system is conserved if it possesses symmetry with respect to time translations.*

This ground-breaking result has completely changed the face of modern physics. We know that Newtonian celestial mechanics is constructed so that the law of conservation of energy is satisfied absolutely exactly. However, Newtonian mechanics is not a causal theory, since it assumes an infinite speed of gravitational interaction.

This means that every gravitational event in the Newtonian mathematical model immediately spreads beyond the corresponding light cone. On the other hand, gravitational events in the actual physical world propagate inside the corresponding light cone and satisfy the Principle of Causality which states that

All real events necessarily have a cause.

For simplicity, let us consider now only two mutually orbiting material bodies. According to Newtonian mechanics, their trajectories are fixed Keplerian ellipses. During each orbital period, their kinetic energy K is partially converted into potential energy P and vice versa, but their total energy $K + P$ is conserved.

However, the further away a body orbits from the Sun, the greater its total energy. This property is usually modeled mathematically using a system ordinary differential equations for the N-body problem, in general. Since 2017 we know that the speed of gravity is with a very high accuracy equal to the speed of light in vacuum due to observations of a binary neutron star merger. The associated gravitational event GW 170817 was detected practically at the same time as the corresponding electromagnetic waves, see [1]. The finite speed of gravitational interactions of N bodies is mathematically modeled by delay differential equations [29, p. 253]. Each dynamical system of free bodies, which act only gravitationally, expands on "average" due to gravitational aberration, because the bodies interact mutually with delays. The term on "average" should be understood as a gradual long-term (i.e. secular) change in distances. For instance, for a finite speed of gravity the two bodies would be alternately approaching and receding, but their trajectories would take the shape of slightly evolving elliptical spirals. In the next section, we introduce 10 independent real-life examples showing that the trajectories of two and more free bodies slightly expand which supports our hypothesis that the Law of Conservation of Energy may not hold in the physical universe. To see this let us again consider only two point masses and, for simplicity, let they have almost circular orbits. For their observed slowly expanding spiral trajectories, the total kinetic energy K slowly decreases, but the total potential energy P increases twice as fast. This means that the total energy $K + P$ slowly increases. For a mathematical derivation of this statement see e.g. [29, p. 210]. In Section 3, we explain why attractive physical and mathematical models should not be identified with reality, otherwise this may lead to various problems in physics and especially in cosmology. They are surveyed in Section 4. Section 5 is devoted to conclusions.

2. The breakdown of the law of conservation of energy in the Solar system

In the following 10 examples all trajectories of the actual Solar system bodies are slightly evolving spirals that contradict the law of conservation of energy (see Noether's theorem). This process is not reversible, since the finite speed of gravitational interaction causes gravitational aberration effects, see [29, p. 250]. All planets and theirs moon in the Solar system are sufficiently isolated from the gravitational influence of other stars. There are many paradoxes, mysteries and puzzles in the actual Solar system that can be easily explained by orbits spiraling outward which are of the same order as the Hubble-Lemaître constant

$$H_0 \approx 70 \text{ km}\,\text{s}^{-1}\text{Mpc}^{-1} = 70/(3.086 \cdot 10^{19})\,\text{s}^{-1} = 2.27 \cdot 10^{-18}\,\text{s}^{-1} \tag{1}$$

Some of the following arguments are based on very precise measurements.

Example 1. The Earth is located inside the so-called habitable zone (ecosphere) in which liquid water is present permanently on its surface. However, 4 billion years ago, there were hot oceans and the solar energy flux was only 75 % of the present value which continually and very slowly increases. To ensure favorable stable conditions for the life on Earth during 3.8 Gyr the mean recession speed of the Earth from the Sun should be 5–10 meters per year. For a detailed calculation see [30, p. 212]. Such a large speed cannot be explained by tidal forces, magnetic fields, the reduction of the solar mass due to nuclear reactions, solar wind, ejections of plasma jets, etc., see [10], [29, p. 202], and also [33]. for the Faint Young Sun Paradox.

Example 2. The Moon moves away from the Earth more rapidly than would be gathered from classical Newtonian mechanics. Present laser technologies allows us to measure that the mean recession speed of the Moon from Earth is 3.84 cm per year, while tidal forces together with the law of conservation of energy yield only 2.13 cm/yr. The unexplained difference 1.71 cm/yr is of the same order as the Hubble-Lemaître constant. This could be explained by fictitious tiny repulsive forces due to gravitational aberration. For a detailed calculation we refer to [29, p. 182]. Dumin [8, 9] derived a similar recession speed of the Moon also from recorded ancient solar eclipses. The universe is thus expanding at almost the same rate as the Moon is moving away from Earth. This was already stated by King in [19] in 1970.

Example 3. The current mean temperature on Mars is approximately 63 °C. However, Mars had liquid water on its surface 4 Gyr ago when Sun's luminosity was only 75 % of its present value. Higher concentration of CO_2 surely contributed to a higher temperature, but cannot fully explain the existence of rivers, lakes and ocean on Mars, since Mars' albedo was much higher than at present, since there were water clouds. The solar flux on Mars was only one third if the actual solar flux on Earth, see e.g. [30, p. 192]. Hence, Mars must have been closer to the Sun 4 Gyr ago.

Example 4. According to two independent astrometric and radiometric measurements of the Cassini satellite, the mean recession speed of Titan from Saturn is 11.3 cm per year, whereas classical Newtonian models based on tidal forces postulated only 1 mm per year. Such a large recession speed cannot be explained by a resonance locking mechanism of five inner mid-sized moons of Saturn. In [28], we show that an essential part of the measured recession speed may come from a local Hubble expansion which is of the same order.

Example 5. Similar recession speeds can be found from orbital evolution of Mars' satellite Deimos, Jupiter's Galilean moons, the large inner moons of Neptune and also its moons Hippocamp and Proteus, large recession speed of orbital expansion of the Uranus moons Miranda and Ariel, etc., see e.g. [20, 21, 22].

Example 6. We know 11 rapidly-moving satellites that orbit Uranus below its stationary orbit. The average distance between neighboring satellites is only 2663 km, whereas distance of neighboring satellites above the stationary orbit is essentially larger. This can be explained by fictitious repulsive antigravity forces that act in opposite direction than gravity, and thus prevents these inner satellites from crashing onto Uranus, see [29, p. 226]. The antigravity force has a repulsive character even though very small. It is not a new fifth physical force but only a side effect of gravitational forces due to the finite speed of gravitational interaction.

Example 7. Large orbital momentum of the moon Triton of Neptune can also be explained by tiny repulsive forces. Its distance from Neptune is very large, namely 354 759 km. Since Triton is retrograde, it should slowly approach Neptune due to tidal forces. However, then it is not clear how such a large moon, which orbits in opposite direction along perfectly circular trajectory, was captured so far from its mother planet. Thus, Triton could be receding from Neptune due to fictitious repulsive antigravity forces which are slightly larger than tidal forces, see [30, p. 197].

Example 8. The Pluto-Charon system could also be subordinated by tiny repulsive forces. Since its orbital period and rotations of these two bodies are completely locked in the $1 : 1 : 1$ resonance and their retrograde orbits are almost circular, tidal forces are negligible. Their large orbital momentum can again be explained by tiny repulsive forces, see [30, p. 198]. The surrounding spacetime is continually deformed by the orbiting of these two large bodies. Thus, the Pluto-Charon system is best suited for testing the magnitude of antigravity forces in the Solar system if three lasers and three retroreflectors were placed on the surface of both bodies.

Example 9. Analysis of growth patterns on fossil corrals from solar data also indicates that the recession speed of the Earth from the Sun was several meters per year since the Devonian era [29, p. 195]. A similar analysis of lunar data shows that the mean speed of the Moon from Earth is several centimeters per year when tidal forces are subtracted from the total recession phenomenon. In this way, our Moon obtained its very large angular momentum. It is assumed that its initial distance from the Earth was only about 20 000 km.

Example 10. The rotation of Mercury is very slow (59 days) which could be caused by a very large impact in the early stage of its development. Another hypothesis is that this could also be caused by tidal forces, if Mercury was former much closer to the Sun 4.6 Gyr ago. The early tidal locking $1 : 1$ resonance could change into $3 : 2$ spin-orbit resonance due to local expansion. This could explain the absence of Mercury's moons whose orbits would be unstable near the Sun, see [30, p. 197].

There are many examples illustrating that single galaxies expand as well. For instance, by Martínez-Lombilla et al. [35, Sect. 8], the measured expansion rate of our Galaxy is 0.6–1 km/s which equivalent to the local expansion rate $H_0^{(\mathrm{loc})} \approx 2.1 \cdot 10^{-18}\ \mathrm{s}^{-1}$. It is again of the same order as (1), see also [29, p. 239], [46], etc.

According to Daisuke Taniguchi, the Sun and thousands of its twins (having a similar age and chemical composition) were born in the inner part of our Galaxy, roghly 10 000 ly closer to the center than their current positions, cf. [45]. Thus the corresponding expansion rate is also comparable to (1), namely,

$$H_0^{(\mathrm{loc})} \approx \frac{10\,000 \text{ ly}}{26\,000 \text{ ly} \cdot 4.6 \text{ Gyr}} = \frac{10}{26} \cdot \frac{1}{4.6 \cdot 31\,558\,150} \text{ s}^{-1} \approx 2.6 \cdot 10^{-18} \text{ s}^{-1}. \qquad (2)$$

If all galaxies would have the same size independently of time, then ancient protogalaxies would not fit into space, when it was much smaller, see [30, p. 202]. Tuomo Suntola [43, pp. 63, 262] made a similar conclusion by investigating angular sizes of distant galaxies. Hence, we again see that the law of conservation of energy has serious problems. Note that repulsive forces slightly, but continually generate energy in any system of free bodies that interact gravitationally. They increase its total (i.e. kinetic + potential) energy. These forces thus act locally in our Galaxy and in other star systems. Note that the above-mentioned arguments cannot be applied to Earth itself which is not a system of free bodies and therefore does not expand in time due to the finite speed of gravitational interaction.

Another theorem of Emmy Noether states that symmetry under rotation (i.e. rotational invariance) leads to the conservation of angular momentum. However, this symmetry and thus also the law of conservation of angular momentum are also slightly violated in the actual physical universe. For instance in the above Examples 1–10, we saw that the mean angular momentum of Solar system bodies slowly increases due to outward migrations. The paper [39, p. 4] even refers to an angular momentum catastrophe. How did galaxies and their (super)clusters acquire their extremely large angular momentum when the early universe was homogeneous and isotropic with some small random fluctuations? A natural answer is: From the local Hubble expansion, which arises due to the finite speed of gravitational interaction causing aberration effects, see [29].

In 1805, Pierre Laplace [34] decided on the basis of a detailed analysis of the motion of the Moon that the actual Newtonian speed of propagation of gravitational interaction must be at least $7 \cdot 10^6 c$ (see [34, Chap. VII, p. 642]). However, Laplace replaced Earth and Moon by point masses. If they were replaced by interacting gravitational potential wells, he would obtain smaller values of this speed. For instance, in 1905 Henri Poincaré in [41, p. 1507] conjectured that the speed of gravitational interaction is equal to the speed of light in the vacuum. Note that Albert Einstein presented the same conclusion later. In 2017, this was confirmed by the observation of two merging neutron stars [1].

3. Simple mathematical models should not be identified with reality

So we should ask a natural question:

What is the point of investigating simple mathematical models that do not meet basic assumptions and observations in the physical universe?

First recall that one second is defined by means of the constant speed of light in vacuum, but nothing is mentioned about gravity in this definition. Is the actual physical speed of light the same far away from all gravitational sources as on Earth's surface or in some strong gravitational field? Note that a high vacuum is nearby the Sun, where the space is supposed to be curved. So far we also cannot reliably verify whether the definition of 1 s is correct in systems moving at high relativistic velocities, see e.g. [4, 13, 24, 37].

An interesting idea is presented in [48], where speed of light depends on the gravitational energy. Vavryčuk claims that in a nonzero gravitational field the speed of light is less than c, just as the speed of light is in an optically dense medium. It this way, we can explain the bending of light near the Sun without assuming that the space near the Sun is curved. For example, suppose that light has a constant velocity $v < c$ inside the spherical region around the Sun, where the gravitational field is relatively strong, and outside this region light has the speed c. In Figure 1, we can see the corresponding bended trajectory of a photon. This example can be generalized to a more natural continuous dependence of v on the distance from Sun's surface, since the intensity of Sun's gravitational field continuously increases towards its surface.

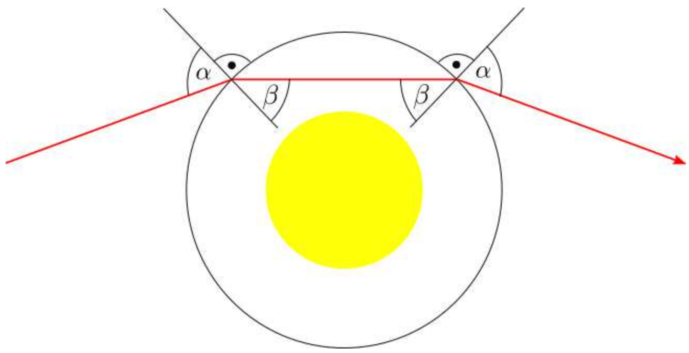

Figure 1. Schematic illustration of bending of light near the Sun for a flat space with two different isotropic media. This can be explained by Snell's law $c/v = \sin\alpha/\sin\beta$, which describes the relationship between the angle of incidence α and refraction angle β when referring to light of other waves passing through a boundary between the two isotropic media. In an optically denser environment, the photon slows down.

Serious doubts about the postulate of the constancy of speed of light are given in [5, 14, 18, 37, 44, 47, 48, 50, 51, 53], etc. Some other classical tests of general relativity (gravitational redshift and the Shapiro fourth effect of general relativity) can also be explained by the variable speed of light in a changing position inside Sun's gravitational field [30, p. 42]. For a detailed analysis of the ill-conditioned problem of Mercury's perihelion shift we refer to [25].

The contemporary cosmological model resembles the situation of article [23] with the title Lemma 1 which can be characterized as follows:

Assume that Lemma 1 implies Lemma 2, from which we further derive Lemma 3, etc. These auxiliary results lead to a new fascinating and beautiful theory such as the standard cosmological model. But after some time we find that Lemma 1 is wrong, and therefore the theory need not describe reality well. In contemporary cosmology such a hypothesis as Lemma 1 is the statement that the law of conservation of energy describes the actual physical universe very well. However, from Section 2 we know that this assumption is problematic. So what is the practical significance of studying mathematical models of our reality that are based on false assumptions?

It is often claimed that beauty is the main criterion for the validity of a physical theory, see [17]. Many misunderstandings, not only in relativity but in physics as a whole, arise because attractive mathematical models are identified with reality. Thus, it is very important to carefully distinguish between mathematical models (in our brains or on a piece of paper) and actual physical phenomena to be sure that the standard cosmological model represents a well based physical theory and is not only a mathematical construct.

For the time being, there is no known physical experiment that demonstrates length contraction. Concerning experiments with time dilation, the Hafele-Keating experiment with two portable cesium atomic clocks (eastward and westward) in commercial airplanes and one on the Earth is not too credible, since none of the corresponding three systems was inertial. The reason is that the Earth rotates and orbits the Sun which produces centrifugal forces. Vibrations of airplanes producing non-negligible accelerations and several other effects were not taken into account, too. Unfortunately, the associated zigzag trajectory is not specified in [16] and thus their measurements cannot be independently verified.

The experimental verification of time dilation by means of particles called muons is also not too credible. Their mean half-life time at rest is $\tau = 2.2 \cdot 10^6$ s. It is said that from observations of cosmic rays at high altitudes > 15 km we know that if muons move linearly at almost the speed of light, they will travel on average a much longer distance than $c\tau = 660$ m and thus are detected on Earth's surface. However, muons could arise in much lower altitudes, since Victor Franz Hess found already in 1912 that cosmic rays (producing muons) at the height 2500 m are stronger than the natural radioactivity produced by Earth, cf. also [37].

It is also questionable that, according to the well-known time dilation formula $t' = t\sqrt{1 - v^2/c^2}$ (see [11, p. 904]), any photon does not experience time t' in a vacuum for $v = c$, but can oscillate very rapidly. For instance, the frequency of CMB photons is about 160 GHz.

According to [2, p. 17], relativistic effects cannot be measured using the GPS system, since all its clocks are constantly synchronized with the U.S. Naval Observatory's reference ensemble. The proposed relativistic time dilation therefore has no effect on GPS time.

4. Main problems in contemporary cosmology

Einstein's field equations with time independent cosmological constant Λ (see [12]) are assembled so that the law of conservation of energy and momentum is valid absolutely exactly, see [30, p. 217]. Alexander Friedman in [15] applied these

equations to the whole universe and obtained a very simple 1st order differential equation with constant coefficients for the unknown expansion function describing the radius of a closed universe. It is modeled by a three-dimensional surface of an expanding four-dimensional ball. According to [7, 26, 43], etc., our universe is spatially closed. If the cosmological constant $\Lambda = \Lambda(t)$ would depend on time t, then the law of conservation of energy would be not valid. Let us note that Friedman did not assume that our universe could be modeled by a flat Euclidean space, as most cosmologists suggest today. In [30], we deal with mathematical explanations of various puzzles and paradoxes in contemporary cosmology which is based on the Friedman equation. Let us summarize the main problems of the standard generally accepted ΛCDM (Lambda-Cold Dark Matter) cosmological model:
1) The existence of dark matter
2) The existence of dark energy
3) Large angular momentum of all spiral galaxies
4) The problem of setting up accurate initial conditions
5) The problem of the cosmological constant Λ
6) The existence of stars that are older than the universe according to ΛCDM model
7) The ΛCDM model admits a division by zero in the curvature parameter
8) The flatness problem
9) The Big Bang problem
10) A 120-order-of-magnitude discrepancy between the measured and theoretically derived density of dark energy.

For other paradoxes in the actual physical universe see also [3, 26, 27, 32, 37, 50, 51], etc. According [30], all these problems appear due to excessive extrapolations, since various simple models, which are "tested" on scales of the Solar system, were applied to the entire universe. However, from Section 2 we know that the main source of these problems may also lie in the invalidity of the law of conservation of energy in the actual physical universe. Note that a popular modification of Newtonian theory called MOND (Modified Newtonian Dynamics) also contradicts the principle of causality (i.e., a consequence cannot precede its cause), since it assumes an infinite speed of gravitational interaction [36]. Hence, it can hardly be a good model of the physical universe on cosmological scales.

5. Conclusions

In Section 1, we presented Noether Theorem which states that the law of conservation of energy in each isolated system holds if it possesses symmetry with respect to time translations. However, in Section 2, we saw that the laws of conservation of energy and angular momentum are slightly violated in the actual physical universe, see [52]. Also the principle of relativity, which has been formulated by Poincaré [40] and three years later also by Einstein [11, p. 891], it is still under discussion if it holds or not in the physical universe, since all movements are absolute with respect to the CMB, see e.g. [3, p. 20], [6], [37]. For other preferred cosmic frames see [27].

According to [31], there is also a mathematical contradiction between the equivalence principle and time dilation in uniformly accelerated systems on arbitrarily short time intervals, cf. also [49]. The fact that some statement is called the Law or the Principle does not imply that it holds exactly in the physical universe.

Consequently, all fundamental laws of physics are valid only approximately in the physical universe. On small space-time scales the modeling error is usually very small, but on large cosmological distances the modeling error can be pretty large. Nature likes to gently disrupt also other symmetries like CP-symmetry, matter-antimatter symmetry, etc. Thus it seems that

the only law of physics is that there is no law describing the physical universe exactly.

We should never identify any mathematical model with physical reality, cf. Sect. 3. The standard cosmological ΛCDM model, has a large number of various problems as shown in Sect. 4, see also [7, 42, 43]. If energy is not conserved globally, then it is also not conserved locally. The reason is that we can always decompose the corresponding space manifold into smaller local parts and there must exist at least one such a part, where the energy is not conserved, cf. [30, p. 184]. Otherwise we would get a mathematical contradiction summing up the energy of all local parts.

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Conference *Cosmology on Small Scales 2026*
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GIORDANO BRUNO – THE FOUNDER OF MODERN COSMOLOGY

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Abstract: Giordano Bruno is undoubtedly one of the giants of European science and culture. Originally a monk of the Dominican Order, he was also a philosopher, mathematician, writer, poet, and theoretical cosmologist, and was also involved in mnemonics. He wrote approximately 30 scientific treatises in these fields. In astronomy, he became famous for his claim that neither the Earth nor the Sun are the center of the universe and that the universe is infinite.

Keywords: infinite universe, star, cosmology

PACS: 98.80-k

1. A brief curriculum vitae

Giordano Bruno was born in 1548 in the village of Nola near Naples, Italy. His original name was Filippo Bruno, but he later chose the nickname Nolanus or Nolan, after his place of birth. It is interesting to note that the first emperor of the Roman Empire, Julius Octavianus, also known as Augustus, died in the same place in 14 AD. After studying at the University of Naples (the first state university in Europe, founded in 1224 by Emperor Frederick II of Hohenstaufen, whose graduates included Giordano Bruno, St. Thomas Aquinas, Umberto Nobile, and current Italian astronauts Maurizio Cheli and Luca Parmitano) Bruno entered a Dominican monastery at the age of fifteen and later, in 1565, became a member of the Dominican Order, where he took his new name Giordano.

Seven years later, he was ordained as a priest. Even as a young man, Giordano Bruno understood the greatness of Copernicus' ideas and the limitations of the geocentric model, which was the officially recognized view at the time from the perspective of Aristotelian philosophy.

Later, he was accused of heresy, and in 1576 he fled from Naples to Rome and then to Geneva, thus beginning his life on the run. Because he was unwilling to adapt his views to the times, he was expelled from most of the places where he worked. From Geneva, he went to Toulouse, where he worked as a full professor. From there, however, he had to leave for Paris, and then, on the recommendation of the French king, he went to Oxford. There, however, he was labeled uneducated and a heretic. He was rejected because of his attacks on Aristotelian philosophy, which suited the church. At that time, the Aristotelian view of the world was practically untouchable in all corners of Europe.

Bruno spent his happiest years between 1583 and 1585 in London, where, at the age of 36, he published his important works *La Cena de le Ceneri* (The Supper on Ash Wednesday) and *De l'infinito, universo e mondi* (On the Infinite, the Universe, and the Worlds) in 1584, in which he corrected Copernicus' heliocentrism with his cosmological theory of the infinity of the universe, see [1]. From that time, opinions of the shape of the universe have often changed. Isaac Newton and many others understood the universe as the three-dimensional Euclidean space, see [3].

In 1585, Bruno returned to Paris, but had to leave again due to attacks on Aristotelian philosophy. He then stayed in Germany in the university towns of Marburg and Wittenberg. He then spent half a year in Prague in 1588 at the court of Emperor Rudolf II., where he found a rather welcoming environment. A commemorative plaque commemorating his stay in Prague is located above the stairs at the entrance to the Planetarium in Stromovka. Even then, the Czech-Moravian lands were renowned for their benevolent approach to religion and "free thinking."

Figure 1. In memory of the Renaissance philosopher and astronomer Giordano Bruno.

Giordano Bruno left Prague for the German town of Helmstedt (the first Protestant university in northern Germany was founded here in 1576, but in 1810 it was closed down as the largest Protestant university in Germany territory), then to Frankfurt am Main, and he also stayed in Zurich for a short time. He briefly headed the Department of Mathematics in Padua, but soon after his appointment, Galileo Galilei took over the position.

2. Bruno's struggle with the Church

In 1591, during his stay in Frankfurt, Bruno received an invitation from the Venetian Doge Giovanni Mocenigo to return to Italy. Having lost the opportunity to obtain a permanent chair in Padua, he accepted this invitation and returned to Venice. The Doge initially supported Giordano Bruno, but soon broke with him because he could not accept Bruno's radical philosophical and religious views. On his denunciation, Giordano Bruno was arrested in 1592, imprisoned by the Inquisition, and extradited to Rome in 1593.

He spent eight years in chains, during which time the Church attempted to re-educate him using regular instruments of torture. There are many theories and speculations about why Giovanni Mocenigo denounced Giordano Bruno, e.g., that Giovanni Mocenigo was an agent of the Inquisition from the beginning, that he suspected Giordano Bruno of not wanting to reveal everything he knew about black magic, etc. His extradition from Venice to Rome was not entirely a matter of course, as Venice prided itself on its independence; in his case, however, Rome was able to base its request on an agreement on the extradition of fugitive monks, and the republic's leadership at the time was trying to be accommodating towards Rome. Giordano Bruno thus became a victim of "high" politics. The Church was particularly bothered by the idea of multiple worlds, which stood in stark contrast to the Holy Scriptures and directly challenged the Church's supremacy, see [7].

After Giordano Bruno stubbornly refused to recant his views, despite physical and mental suffering, he was sentenced to death by burning at the stake. The death sentence was dated February 8, 1600, and was signed by nine cardinals. Part of the sentence was to include all of Giordano Bruno's writings in the "Index of Prohibited Books" the sentence was handed down, the condemned man said to his judges: *You who condemn me to death are more afraid than I who am undergoing death.*

The punishment was carried out on February 17, 1600, in Campo de' Fiori (Field of Flowers) in Rome. It was the height of summer, and the pilgrims who gathered to watch were enjoying their indulgences. Giordano Bruno was brought to the place of execution with a gag in his mouth so that he could not make inflammatory speeches. He was stripped naked, hung upside down by his feet over the stake, and burned alive. Even at the stake, Giordano Bruno turned away from the cross that was handed to him. His ashes were thrown into the Tiber river. The Church also banned all his writings and books. These writings were added to the "index."

3. Conclusions

Giordano Bruno expanded Copernicus' revolution in our view of the universe by arguing that, according to his opinions (even though they were only speculation without concrete scientific evidence and observation), the Sun was not the center of the universe, but only one of many stars. According to Giordano Bruno, the universe is infinite, there are an infinite number of suns with planets that may even be inhabited.

He claimed that God is good because he does good things, and since there are an infinite number of worlds that God has created, he is automatically infinitely good. The Catholic Church did not like the fact that this practically meant the infinite smallness and insignificance of man. Bruno was thus several centuries ahead of his time in this view.

It was not until 1889 (almost three centuries later) that a bronze statue was built at the site of the execution to honor the memory of the enlightened and courageous scholar Giordano Bruno (see Figure 2).

Figure 2. Monument to Giordano Bruno in Rome.

Bruno completely abandoned the idea of a hierarchical universe [6]. The universe is one, infinite, immobile... It is not capable of comprehension and therefore is endless and limitless, and to that extent infinite and indeterminable, and consequently immobile. Bruno's cosmology distinguishes between "suns" which produce their own light and heat, and have other bodies moving around them; and "earths" which move around suns and receive light and heat from them. Bruno suggested that some, if not all, of the objects classically known as fixed stars are in fact suns. According to astrophysicist Steven Soter, he was the first person to grasp that "stars are other suns with their own planets".

Bruno wrote that other worlds "have no less virtue nor a nature different from that of our Earth" and, like Earth, "contain animals and inhabitants". In *La Cena de le Ceneri* (1584) Bruno anticipates some of the arguments of Galileo on the relativity principle, cf. [2].

In conclusion, it can only be said that the fate of this man will forever remain a shining example of the constant struggle of a lonely individual against a state or other sophisticated structure that does not allow any interference with the established system. The hope seems to be that his views prevailed over time. For more information about the philosopher, mathematician, poet, astronomer, and cosmologist Giordano Bruno we refer, for instance, to monographs [4, 5].

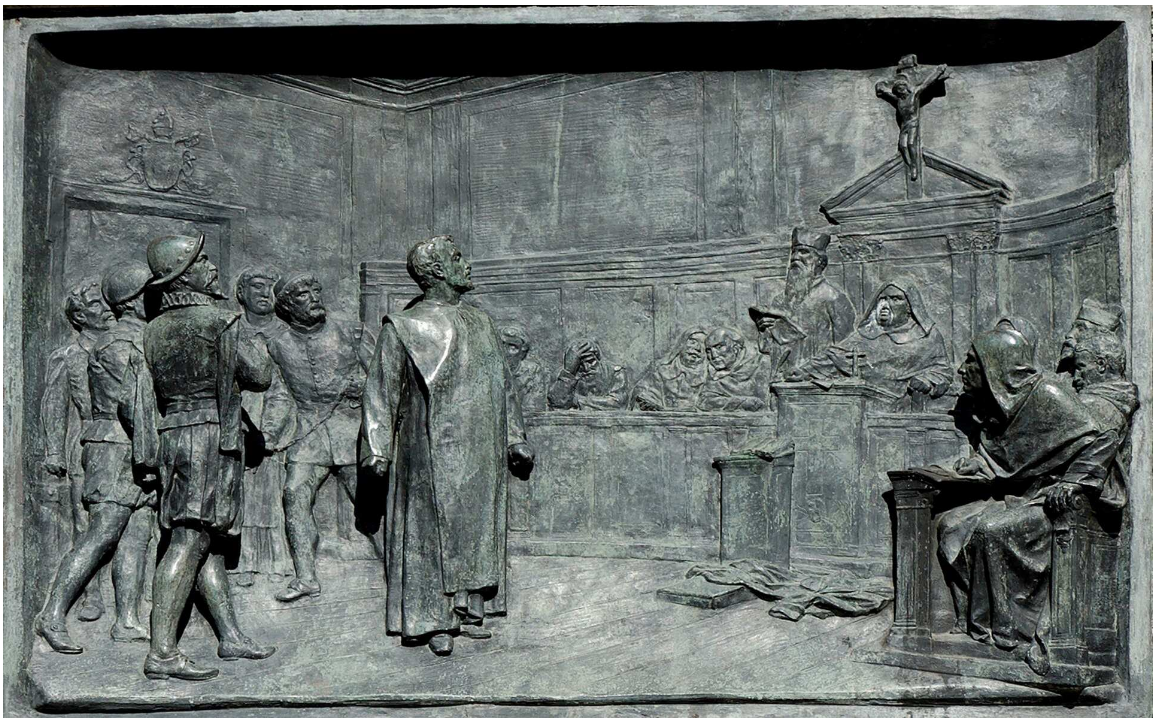

Figure 3. A bronze plaque depicting Giordano Bruno's trial with the Church is placed on the pedestal of the statue from Figure 2.

Acknowledgments

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VISUALIZATION OF A CLOSED 3-SPHERE UNIVERSE

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Abstract: We describe how to imagine a three-dimensional closed universe. To do this, we consider six orthogonally intersecting great circles of a three-dimensional sphere. Then we apply a stereographic projection of these circles into three-dimensional Euclidean space. This projection maps circles on circles and preserves angles. The resulting model can be printed on a standard 3D printer. This paper is an upgrade and large extension of our previous 2026 paper [4].

Keywords: 3-sphere, stereographic projection, cosmological principle, Copernicus principle, great circle, Ptolemy theorem

PACS: 98.80-k

1. Introduction

According to Copernicus' principle, people on Earth are not privileged observers of the universe. We are not in some special place – the center of the world, like the tip of an egg. According to Einstein's cosmological principle (as named by Milne [8]), the physical universe is homogeneous and isotropic on large scales at any fixed time instant. Roughly speaking, its curvature is constant at every point and in all directions.

More precisely, homogeneity is the assumed property of the universe that at any given moment in time and on large spatial scales, the universe appears the same to all observers, wherever they are. In other words, translational symmetry of the universe is required at all times, which is actually what Copernicus' principle asserts. However, this only applies to spatial coordinates. Homogeneity in time apparently does not apply, because the average density of the universe is gradually decreasing.

Similarly, isotropy is a presumed property of the universe, in which the universe appears the same in all directions to an observer at any point on large spatial scales, i.e., at any given time, rotational symmetry of the universe is required.

There are a number of tests that confirm the validity of isotropy, because the relevant astrophysical objects (microwave background radiation, active galactic nuclei, quasars, galaxies in the Hubble Deep Field, gamma-ray bursts, etc.) are distributed almost uniformly across the entire celestial sphere. The Hubble test is used to prove homogeneity, see [6, p. 108].

Denote by $\mathbb{E}^n$ the n-dimensional Euclidean space. Around 1900, the renowned German physicist and astronomer Karl Schwarzschild [9] came up with the idea that the geometry of the universe may not correspond to infinite three-dimensional Euclidean space $\mathbb{E}^3$, as Giordano Bruno had assumed at the end of the 16th century [7], but that instead, it could be described as a closed three-dimensional sphere for a given time, i.e., the three-dimensional surface of a four-dimensional ball of radius $r > 0$, see e.g. [1, 3, 11, 12]:

$$\mathbb{S}^3 = \{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 + w^2 = r^2\}.$$

This geometry cannot change over time, i.e., the bounded sphere $\mathbb{S}^3$ cannot suddenly change into the unbounded Euclidean space $\mathbb{E}^3$. For simplicity, we shall assume that the radius $r = 1$ in what follows.

Remark. The term "closed universe" has a completely different meaning in cosmology from what it has in mathematics. For example, the unbounded interval $[0, \infty)$ is a closed set in $\mathbb{E}^1$, while a closed universe is bounded (closed within itself). On the other hand, the bounded interval $(0, 1)$ is open in $\mathbb{E}^1$, but the open universe is unbounded.

Since $\mathbb{S}^3$ is a curved three-dimensional manifold, it is difficult to visualize it. In [6, p. 112], ten ways are presented to imagine it. One of them is a stereographic projection.

2. Stereographic projection

The one-dimensional sphere $\mathbb{S}^1$ and the two-dimensional sphere $\mathbb{S}^2$ are easy to imagine, because they can be embedded (see Fig. 1) in three-dimensional Euclidean space, whose geometry is familiar to us.

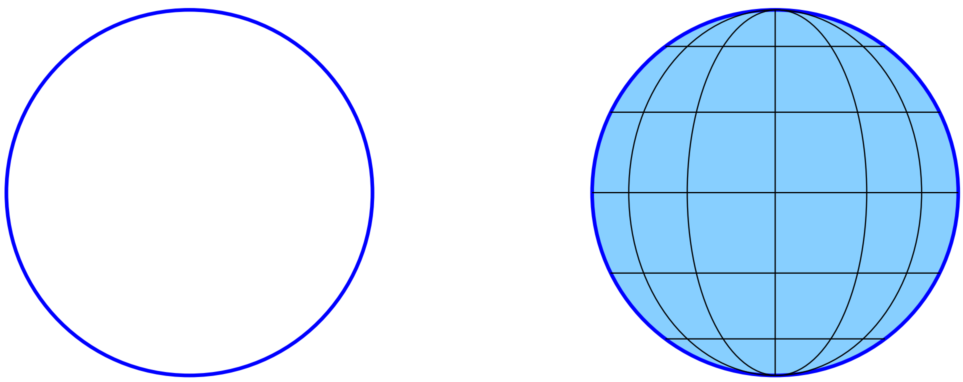

Fig. 1. Sphere $\mathbb{S}^1$ on the left and sphere $\mathbb{S}^2$ on the right can be embedded in three-dimensional Euclidean space in such a way that angles and distances are preserved.

However, sphere $\mathbb{S}^3$ cannot be embedded in $\mathbb{E}^3$ in order to preserve distances. Similarly, the spherical surface $\mathbb{S}^2$ (or its two-dimensional part) cannot be flattened onto the plane $\mathbb{E}^2$. One option is to use a stereographic projection, which preserves angles but not distances. Therefore, all planar maps of curved surfaces are necessarily distorted.

Stereographic projection from the North Pole N of the celestial sphere onto a plane tangent to the celestial sphere at the South Pole S was used, for example, in the construction of the astronomical dial of the Prague Astronomical Clock, see [5, p. 246]. It is defined as follows. Let $M \in \mathbb{S}^2$ be any point on a unit two-dimensional sphere such that $M \neq N$. Consider the meridian section MNS, where $N = (0, 1)$ is the North Pole and $S = (0, -1)$ is the South Pole, i.e., only two coordinates are enough. Let us denote (x, z) as the coordinates of point M in the meridian section. From the projection point N, let us draw a ray through point M. Let us denote its intersection with the projection plane $z = -1$ as $M' = (x', -1)$. Point M' is called the *stereographic projection* of point M. From the similarity of the right triangles in Fig. 2, we see that for the unknown coordinate x', the following applies: $x' : 2 = x : (1 - z)$. Consequently,

$$x' = \frac{2x}{1 - z} \quad \text{for } z \in [-1, 1).$$

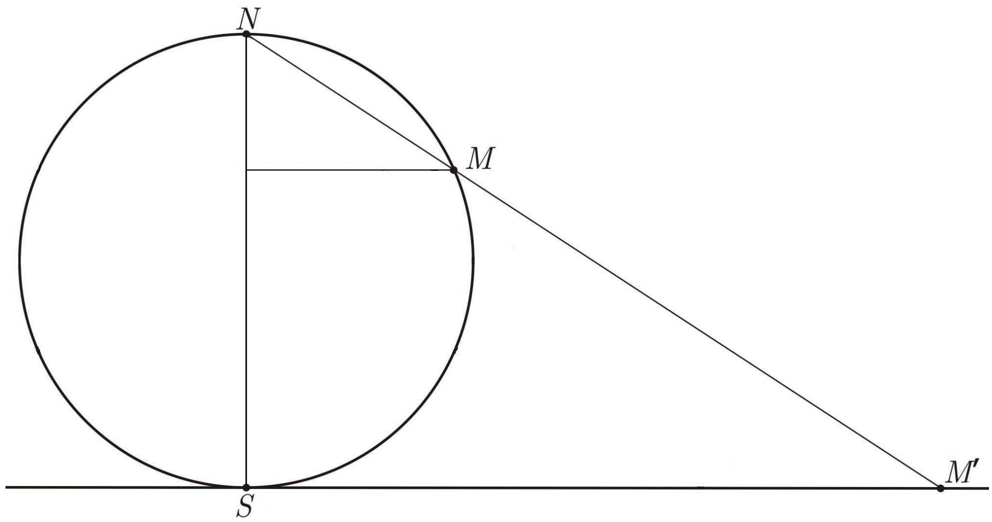

Fig. 2. Stereographic projection from the North Pole N of point M onto M' which lies in the plane tangent to the spherical surface at the South Pole S.

Let us recall the beautiful Ptolemy's theorem, almost 2000 years old, formulated by the famous ancient mathematician and astronomer Claudius Ptolemy. Its detailed proof is given, for example, in [5, p. 247].

Theorem 1 (Ptolemy's). *Every circle lying on the sphere $\mathbb{S}^2$ and not passing through the center of projection is again projected onto a circle in stereographic projection.*

For example, on the astronomical dial of the Prague Astronomical Clock, which is a projection plane of the celestial sphere, there are images of the celestial equator, the ecliptic (zodiac), the Prague horizon, and the tropics of Cancer and Capricorn are again circles. Another important property is given by the following statement, see [1, Chapt. 36].

Theorem 2. *A stereographic projection preserves angles.*

Let us recall that a *great circle* on the spherical surface $\mathbb{S}^2$ has the same radius and also the same center as $\mathbb{S}^2$. It is actually a geodesic that are the shortest connecting curves between two points on $\mathbb{S}^2$. Next, we will consider three great circles

$$x^2 + y^2 = 1, \quad x^2 + z^2 = 1 \text{ and } y^2 + z^2 = 1$$

of the sphere $\mathbb{S}^2$, which lie in mutually perpendicular planes $z = 0$, $y = 0$ and $x = 0$. Hence, they intersect at right angles at six distinct points

$$(\pm 1, 0, 0), \quad (0, \pm 1, 0), \text{ and } (0, 0, \pm 1),$$

where the last two are the North and South Poles. On the globe, these circles correspond to the equator and the meridians $(90°, 270°)$ and $(0°, 180°)$, see Fig. 3.

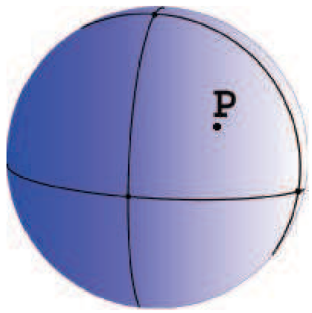

Fig. 3. Three great circles of the spherical sphere $\mathbb{S}^2$ intersect at right angles. The projection point is marked with the letter **P**.

Now we will show how these three great circles are displayed in a stereographic projection onto a plane. When projecting from the North Pole, the two circles formed by the meridians would be displayed as straight lines, because the conditions of Theorem 1 are not met. Consequently, this time we will not project from the North Pole (as in the case of the Prague Astronomical Clock) but from the point $\mathbf{P} = (p, p, p) \in \mathbb{S}^2$, where $p = \sqrt{3}/3$, which is symmetrically positioned with respect to the three great circles under consideration, see Fig. 3. We will project onto a plane that is tangent to $\mathbb{S}^2$ at the opposite point $(-p, -p, -p)$. According to Ptolemy's Theorem 1, the images of the circles under consideration will again be circles that intersect at six right angles, see Theorem 2. The stereographic image of the equator is located at the bottom of Fig. 4, the projections of the North and South poles are marked N' and S', the projection point **P** is actually "projected to infinity" and its opposite point is projected to the center of Fig. 4.

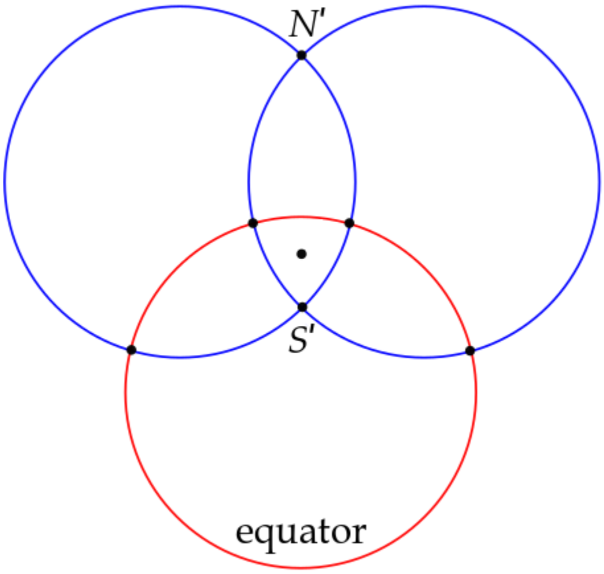

Fig. 4. The images of the three great circles of the sphere from Fig. 3 are again circles that intersect at right angles by Theorem 2.

3. Printing a rounded model of a closed universe on a 3D printer

Now consider six great circles on the unit sphere $\mathbb{S}^3$ that intersect perpendicularly at eight points with coordinates

$$(\pm 1, 0, 0, 0), \quad (0, \pm 1, 0, 0), \quad (0, 0, \pm 1, 0), \text{ and } (0, 0, 0, \pm 1):$$

$$x^2 + y^2 = 1, \quad x^2 + z^2 = 1, \quad y^2 + z^2 = 1, \tag{1}$$
$$x^2 + w^2 = 1, \quad y^2 + w^2 = 1, \quad z^2 + w^2 = 1. \tag{2}$$

For instance, let us denote by $N = (0, 0, 0, 1)$ the North Pole and by $S = (0, 0, 0, -1)$ the South Pole of the sphere $\mathbb{S}^3$. Then the equator of $\mathbb{S}^3$ (belonging to the hyperplane $w = 0$) is a two-dimensional sphere $\{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 = 1\}$ within which lie the first three circles (1) (see Fig. 3). The next three circles (2) intersect perpendicularly at the North Pole N and the South Pole S.

The stereographic projection of the above six circles into three-dimensional Euclidean space $\mathbb{E}^3$ can be constructed in the same way as in the case of two-dimensional case. This time, however, we will project from the point $\mathbf{P} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \in \mathbb{S}^3$ which is symmetrically positioned with respect to all six great circles (1)–(2) which do not contain $\mathbf{P}$. Their centers do not belong to the sphere $\mathbb{S}^3$ and therefore are not mapped anywhere in the stereographic projection.

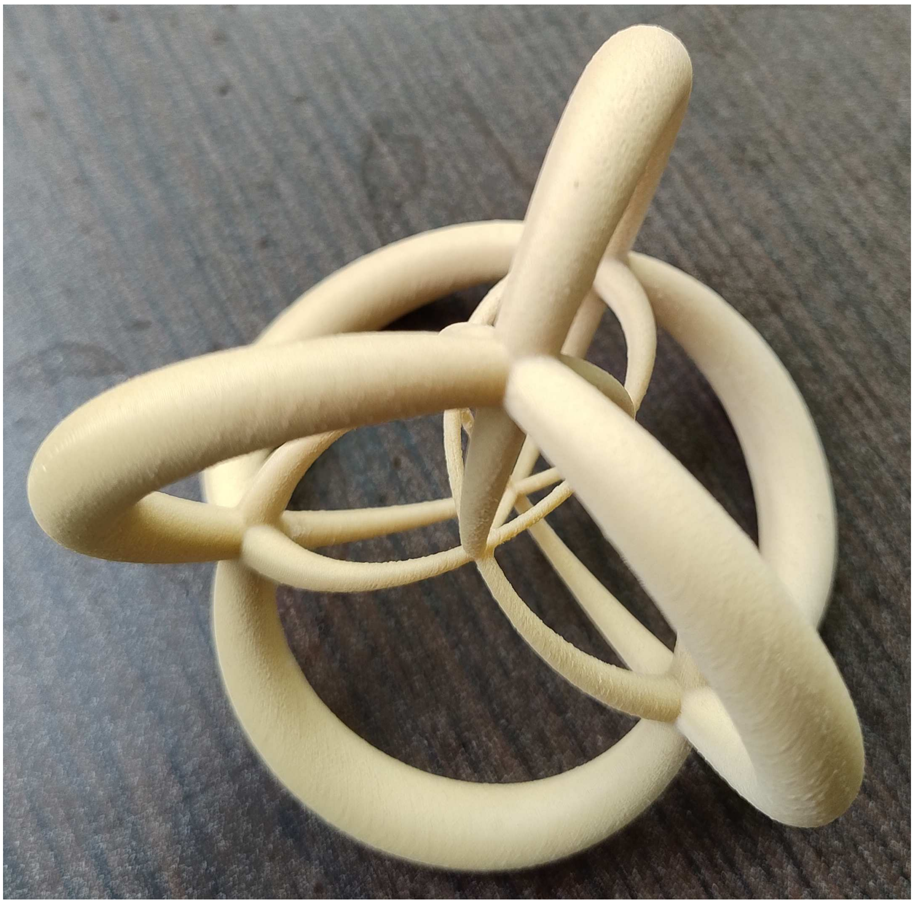

Fig. 5. A rounded model of a closed universe in which distances are not preserved, but angles remain the same.

It is known that Ptolemy's Theorem 1 and Theorem 2 can be generalized to any dimension, see e.g. [8]. Therefore, the six great circles (1)–(2) of $\mathbb{S}^3$ in stereographic projection into three-dimensional Euclidean space $\mathbb{E}^3$ are again mapped to circles that intersect at right angles, see Fig. 5. The resulting model can be printed on a standard 3D printer, see [10, p. 53]. The relevant program is available at the following web address:

`https://www.thingiverse.com/thing:1570212`

Using this model, we can demonstrate that if we start at the North Pole N and travel along any great circle in any direction, we will end up back at the North Pole N via the South Pole S. So, we will cross the equator twice. This situation is similar to what we encounter on Earth's surface, where the two main directions at the North Pole are perpendicular to each other, see Figs 3 and 4, However, at the North Pole N of the sphere $\mathbb{S}^3$, there are three directions mutually perpendicular: up–down, right–left, and forward–backward, see [14].

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ON LIFETIME OF COSMIC MUONS IN THE LOWER ATMOSPHERE

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Abstract: In this paper, we reexamine a decay of relativistic muons in the lower atmosphere using recent muon experiments that are more precise than those originally conducted by Rossi and Hall (Phys. Rev., 59, 223-228, 1941) or Frisch and Smith (Am. J. Phys., 31, 342-355, 1963). We utilize differential energy spectra of muons measured at the sea level and at mountain altitude using the ground-level experiment CAPRICE 1994/1997 and the balloon-borne experiment BESS 1999/2002. We show that the behavior of relativistic muons in the lower atmosphere is more complex than originally assumed. We point out significant discrepancies between theoretical predictions of the number of relativistic muons and the measured data. In particular, a puzzling deficit of muons in the momentum interval of 0.6-80 GeV/c is revealed.

Keywords: cosmic rays, cosmic muons, time dilation, Lorentz factor, light speed

PACS: 14.60.Ef, 13.35.Bv

1. Introduction

Relativistic cosmic muons are created when high-energy cosmic rays from galactic or extragalactic sources collide with particles in the Earth's atmosphere (1; 2; 3). During these collisions, energetic pions are produced and decay into muons within 26 ns (4). The lifetime of muons is about $\tau_0 = 2.2\,\mu$s (5), and they can travel at the relativistic speed towards the Earth's surface. The rest-mass energy of muons is about 106 MeV, but the mean energy of muons at sea level is about 4 GeV due to their high speed (6; 7; 8).

Since cosmic muons can travel with enormously high speeds, they are suitable for experimental testing of time dilation predicted by Special Relativity (SR) (9; 10; 11). Muons are assumed to be created in the Earth's atmosphere at an altitude of about 10 km or higher (12). Assuming that they travel close to the speed of light, they can reach a maximum distance of 660 m before decaying due to their short lifetime. However, due to relativistic effects, their lifetime τ_0 should be prolonged by the Lorentz factor γ, $\tau = \gamma\tau_0$. Consequently, they should be able to travel much longer distances and even reach the sea level before decaying when observed in the coordinate system connected with the Earth. In contrast, the proper muon lifetime τ_0 remains 2.2 μs in the muon's frame of reference. However, the large distance traveled in the Earth's atmosphere becomes shorter due to length contraction. Hence, measuring a correlation between the muon's lifetime and its velocity can serve as a probe of the time dilation as well as the length contraction effects predicted by SR.

In 1940, B. Rossi and D.B. Hall measured the relativistic time dilation of muons based on the idea of analyzing the occurrence of relativistic muons at two different altitudes (13; 14; 15): at Echo Lake (3240 m) and Denver (1616 m). They measured the dilated lifetime of muons traveling at speeds above 0.99 c (c being the speed of light) and found a qualitatively good agreement between the measured data and the formulas for relativistic momentum and time dilation. However, the measurements were not precise enough the results to be fully persuasive.

A more precise experiment was conducted by D.H. Frisch and J.H. Smith in 1962 (16), who counted muons on top of Mt. Washington (altitude 1910 m) and at Cambridge (altitude 3 m) in a narrow band of speeds ($0.9950 - 0.9954\,c$). They detected 563 ± 10 muons per hour at the upper site and about 408 ± 9 muons per hour at the lower site (16, their table I). Assuming that muons are produced in the upper atmosphere and no muons are created at altitudes between the upper and lower site, the number of muons N_L at the lower site can be calculated from the number of muons N_U at the upper site as follows

$$N_L = N_U e^{-\frac{t}{\tau_0}}\,, \tag{1}$$

where t is the time needed for muons to pass the distance between the upper and lower site, and τ_0 is the rest-frame muon lifetime. For $N_0 = 563$, $\tau_0 = 2.2\,\mu$s, and $t = 6.4\,\mu$s, we expect to detect about 30 muons at the lower site provided no relativistic effects are considered. The fact that actually 408 muons were observed instead of 30 muons at the lower site was interpreted as muons decaying more slowly because of time dilation. Eq. (1) predicts that elapsed time t must be 0.7 μs to observe 408 muons, and corresponding Lorentz factor is $6.4/0.7 \approx 9.0$. Assuming muons traveling at the top site with a speed of about 0.9952 c, the Lorentz factor should be about 10.2. This value is slightly higher than 9.0 calculated from the number of muons, but (16) argue that the average speed of muons should be slightly lower than that at the top site because muons are slowing down when traveling through the air. Hence, the authors concluded that their experiment proved the existence of time dilation and pointed out to the necessity of correcting measurements for this relativistic effect.

Although the experimental results of (16) seem to confirm the time dilation and length contraction effects according to SR with a reasonable accuracy, some difficulties and open questions arise. First, the authors measured the number of muons at only two altitudes and focused on muons with energy within only one narrow energy interval (e.g., corresponding to Lorentz factor $\gamma \approx 9.0$). It is unclear why the authors did not validate their results by repeating the experiment at other altitudes and/or with muons having other energy levels (corresponding to other Lorentz factors). Second, the authors assume that most of the primary cosmic protons have already been absorbed in the upper atmosphere, and thus no muons are produced at altitudes between the upper and lower site. The validity of this assumption is crucial because if some muons are created between the two sites of the measurement, the high number of muons at the lower site could be misinterpreted as an additional time dilation of muons. Consequently, this could lead the results of the experiment to be biased or even false. Even though the assumption of no production of muons at low atmosphere was critical for the validity of the test, the authors did not support it by any independent measurement. By contrast, more recent experiments indicate that some portion of high-energy cosmic protons can hit particles of the atmosphere even at low altitudes (2; 17; 18) and produce high-energy muons close to the sea level. Hence, we conclude that the experiments of (13; 14) or (16) might not be considered persuasive and need to be verified by other more accurate measurements.

In this paper, we reexamine the existence and properties of time dilation in relativistic muons using more advanced experiments than those originally conducted by (13) or (16). We employ precise differential energy spectra of muons measured at sea level and at several altitudes using the ground-level experiment CAPRICE 1994/1997 (19; 20; 21) and the balloon-borne experiment BESS 1999/2002 (22; 23; 8; 24). We show that the problem of time dilation in relativistic muons is more intricate than originally assumed. We point out significant discrepancies between theoretical predictions and the measured data. In particular, a puzzling deficit of muons in the muon momentum interval of 0.6-80 GeV/c is detected, and its possible origins are discussed.

2. Data

In contrast to the first simple muon experiments conducted using a stack of plates with Geiger-Muller counters (13; 14) or multiple scintillation counters (16), later, more advanced experimental setups based on magnetic spectrometers were designed for precise measurements of the energy spectrum of muons across a broad energy interval (25). We can cite, for example, the precise muon spectrometer of the L3 detector located at the LEP collider at CERN (26), ground-level experiments CAPRICE 1994/1997 (19; 20) and balloon-borne experiments with the MASS apparatus (27) or the BESS apparatus (22; 23; 8; 28; 24). These equipments included various sets of detectors (magnetic spectrometer, Cherenkov counter, and calorimeter) designed to study energy spectrum of cosmic rays with high resolution at various altitude levels. Hence, the measurements from these experiments can be used as an ideal tool for detailed studying the relativistic behavior of cosmic muons and for re-examining the validity of the first simple muon experiments (13; 14; 16).

Regarding the CAPRICE experiment (19; 29; 30), the muon momentum spectrum

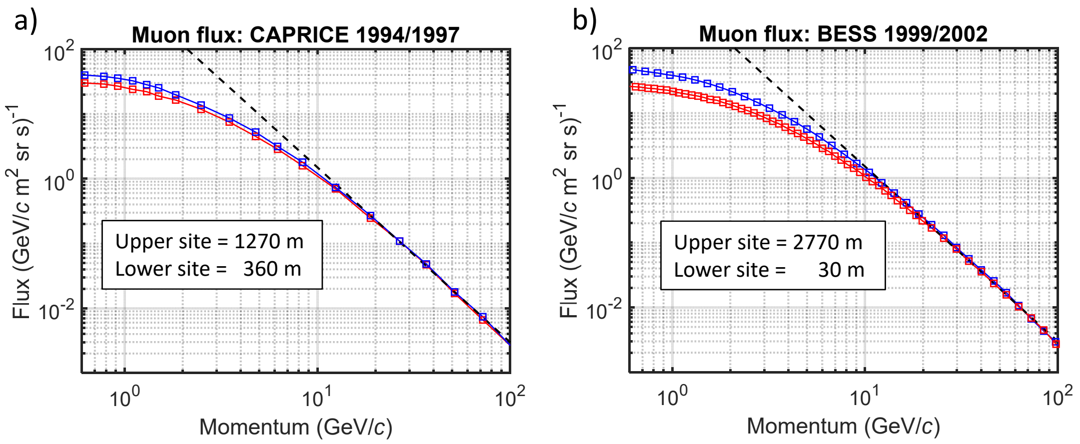

Figure 1: The muon differential momentum spectrum at two different altitude levels for the CAPRICE 1994, CAPRICE 1997, BESS 1999 and BESS 2002 experiments (19; 22; 8) in the log-log scale. The blue squares denote data for the upper site (CAPRICE - Fort Summer, New Mexico, 1270 m above sea level; BESS - Mt. Norikura, Japan, 2770 m above sea level), the red squares denote data for the lower site (CAPRICE - Lynn Lake, Manitoba, Canada, 360 m above sea level; BESS - Tsukuba, Japan, 30 m above sea level). The blue/red lines indicate the interpolation of the data. The black dashed line shows a decay of the differential power law with value of 2.7, which is a typical decay of a differential spectrum of primary cosmic protons.

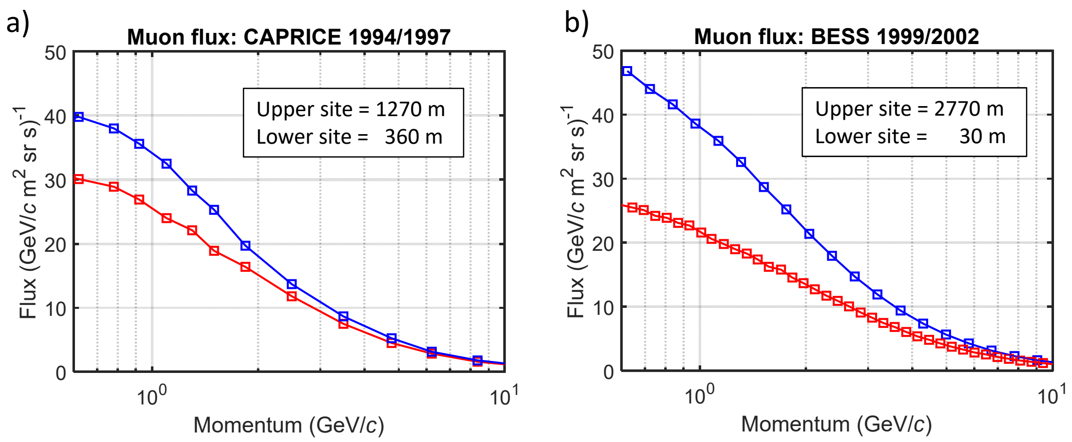

Figure 2: The muon differential momentum spectrum at two different altitude levels for the CAPRICE 1994, CAPRICE 1997, BESS 1999 and BESS 2002 experiments (19; 22; 8) in the lin-log scale. For details, see the caption of Fig. 1.

was measured from 200 MeV/c to 120 GeV/c with the NMSU-WIZARD/CAPRICE magnet spectrometer at two different geomagnetic locations: (1) Lynn Lake, Manitoba, Canada (altitude 360 m) and (2) Fort Summer, New Mexico (altitude 1270 m). The instrument included a ring imaging Cherenkov detector, a time-of-flight system, a superconducting magnet spectrometer with a tracking system and a 7 rad length silicon-tungsten imaging calorimeter. About 440 000 and 1200 000 events were recorded at Lynn Lake (July 19-20, 1994) and Fort Summer (April 26 - May 2, 1997). In both experiments, particles moving downward in near vertical directions were selected using the time-of-flight and tracking information. The absolute particle fluxes were calculated from the number of observed muons, taking into account the geometrical factor, spectrometer live time, and selection efficiencies. The CAPRICE94 measurements were taken at an atmospheric depth of 1000 g/cm^2, and the CAPRICE97 data at 886 g/cm^2. Extrapolated flux values to sea level at 1 GeV/c were 23.7 ± 0.3 (m^2 sr s GeV/c)$^{-1}$ for CAPRICE94 and 24.0 ± 0.3 (m^2 sr s GeV/c)$^{-1}$ for CAPRICE97. (19) concluded that a geomagnetic effect might be present in muon momenta only below 1 GeV/c. The observed muon differential momentum spectra at two different altitude levels in the momentum interval $0.6 - 100$ GeV/c measured in the CAPRICE 1994/1997 experiments are shown in Figs. 1a and 2a (19).

As for the BESS experiment (22; 8; 28; 24), the muon momentum spectrum was measured in the momentum range of 600 MeV/c through 106 GeV/c with the BESS/BESS-TeV spectrometer at Tsukuba, Japan (altitude 30 m, measurement period of October 1-6, 2002), and Mt. Norikura, Japan (altitude 2770 m, measurement period of September 17-19, 1999, and September 21-23, 1999). The BESS detector (31; 32; 33) is a high-resolution spectrometer suitable for the detection of rare cosmic-ray components and precise measurements of fluxes of various cosmic rays. It was equipped with a tracking system with a jet-type drift chamber and two inner-drift-chambers inside the magnetic field, time-of-flight hodoscopes, and an acrylic Cherenkov counter. The BESS 1999 measurements (22) were taken at Mt. Norikura at the mean atmospheric depth of 742.4 g/cm^2, and the BESS 2002 measurements (8) at Tsukuba at 1032.2 g/cm^2 . The observed muon differential momentum spectra at two different altitude levels in the momentum interval $0.6 - 100$ GeV/c measured in the BESS 1999/2002 experiments are shown in Figs. 1b and 2b (22; 8).

3. Results

Similarly to experiments by (13), (14) or (16), we can calculate the expected number of muons at sea level relative to the number of muons at the mountain altitude and compare this theoretical value with observations. According to the mentioned previous works, we adopt the assumption of no muons created in the lower atmosphere and apply relativistic formula for the time dilation of high-energy muons. Consequently, we get the theoretical ratio of survived relativistic muons $R = N_L/N_U$:

$$R = e^{-\frac{L}{\gamma v \tau_0}} , \qquad (2)$$

where $L = H_U - H_L$ is the difference in altitudes between the upper and lower site, γ is the Lorentz factor obtained from the muon momentum P and the muon rest-frame energy $E_0 = 105.658$ MeV as

$$\gamma = \sqrt{\frac{P^2 c^2}{E_0^2} + 1}\ , \tag{3}$$

v is the muon speed

$$v = c\sqrt{1 - \frac{1}{\gamma^2}}\ , \tag{4}$$

and $\tau_0 = 2.197\,\mu$s is the rest-frame lifetime of muon.

Figure 3 (black squares) shows the ratio between the observed flux densities at the lower and upper sites calculated in the momentum interval of $0.6-100$ GeV/c for both experiments. When comparing the data quality of both experiments, the scatter in the spectrum of the muon flux ratio is remarkably lower for the BESS experiment. Nevertheless, the muon flux ratio behaves consistently for both experiments, and it linearly increases with the muon momentum in the entire studied momentum interval (red lines in Fig. 3a,b).

However, the observed behavior of muons is unexpected from the relativistic point of view. If we consider the relativistic muon time dilation and calculate the theoretical ratio R using Eqs. (2)-(4), we obtain the blue lines in Fig. 3a,b. Hence, the theoretical muon flux ratio should steeply increase for low values of the muon momentum up to $2-3$ GeV/c. For higher values, the increase of the ratio R gradually slows down. Finally, the flux ratio should be close to 1 for the muon momentum of 100 GeV/c and higher. This expresses the fact that the muon lifetime becomes so long for high-energy muons that we observe almost no decay of muons between the two measurement sites.

Clearly, the differences between the measurements and the relativistic predictions are striking: (1) The observed flux ratio is roughly a linear function of the muon momentum, while the predicted ratio should be strongly nonlinear. (2) We observe a significant deficit of muons detected at the lower site almost in the whole interval of values of the muon momentum. The mentioned findings look robust because the discrepancy between theoretical predictions and observations is essentially the same for both experiments.

4. Discussion

The experiments of (13), (14) or (16), where muons were measured at two different altitudes within a narrow muon momentum interval, are commonly considered as robust tests of the existence of the relativistic time dilation. After applying relativistic corrections by these authors, the discrepancy between the number of muons observed at the mountain altitude and sea level disappeared, and theoretical predictions were reconciled with observations. However, the later, more precise experi-

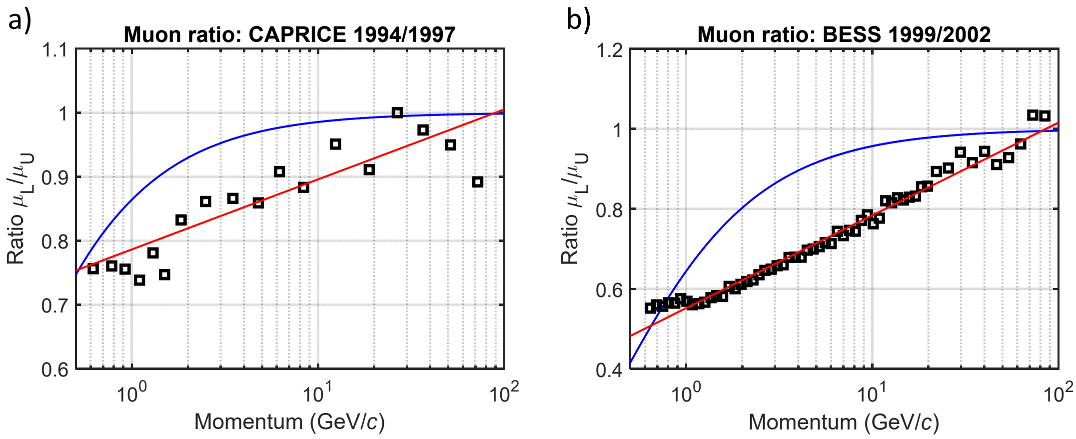

Figure 3: Ratio of the muon differential momentum spectra at two different altitude levels for the CAPRICE 1994, CAPRICE 1997, BESS 1999 and BESS 2002 experiments (19; 22; 8). The black squares are calculated from the measurements at the upper and lower sites shown in Figs. 1 and 2. The red line represents the linear regression of the data. The blue line indicates the relativistic prediction when time dilation is considered.

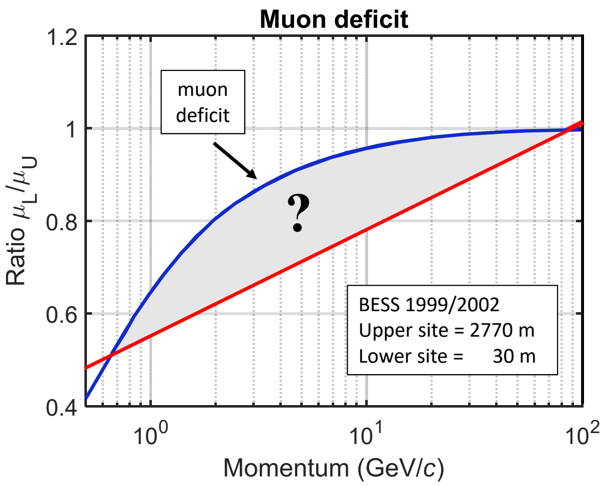

Figure 4: The muon deficit within the muon momentum range of 0.6-80 GeV/c (grey area) in the BESS 1999/2002 experiments (22; 8). Red line - function interpolated from data, blue line - function predicted by SR. The muon deficit marked by the grey area indicates a significant discrepancy between theoretical prediction and observations of muons at sea level.

ments CAPRICE 1994/1999 (19) and BESS 1999/2002 (22; 8) do not support these conclusions and indicate that the problem is more involved.

Measurements of muons in the broad interval of muon momentum $0.6 - 100$ GeV/c revealed that a discrepancy between predictions and observations of muons at different altitudes is still present. In fact, if we apply relativistic corrections, a significant deficit of muons at the lower site is detected in the momentum interval of $0.6 - 80$ GeV/c (see Fig. 4). Since muons are absorbed in the atmosphere very weakly (34; 1; 35), there is no physically plausible process that could appreciably reduce their number along the path between the two measurement sites. Moreover, if we take into account that muons can also be created at low altitudes by secondary cosmic rays (6; 1; 2; 17), the observed muon deficit at low altitudes becomes even more profound.

The discovered muon deficit indicates that (1) the relativistic correction to time dilation associated with high-speed muons is incorrect and must be avoided, (2) the relativistic correction to time dilation is simplistic and must be substantially modified, or (3) some unknown physical processes reduce significantly the number of muons in the lower atmosphere. To resolve the problem, more advanced models of the the behavior of relativistic muons and their interactions in the atmosphere will be needed.

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PREDICTION OF DARK MATTER WITHIN THE HYPOTHESIS OF UNIQUE INTERACTION

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Abstract: We briefly present the hypothesis of unique interaction (UI). It can explain a number of fundamental facts as in the macrocosm as microcosm. Within the UI, an alternative form of matter, that can be identified with the dark matter, occurs to be a natural analogue of normal, baryonic matter. We outline the fundamental properties of this alternative matter, which are predicted by the UI. In particular, it is explained why this matter does not act with a pressure on the baryonic matter. In addition, it is predicted that a DM gas is not effectively cooled via a bremsstrahlung radiation; hence the galactic DM halo is predicted to be extremely hot and no DM-object with a self-gravity weaker than the tremendous self-gravity of huge galactic halo can exist in the universe.

Keywords: dark matter, theory of unique interaction, primordial black hole, charged elementary particles

PACS: 12.10.-g, 14.20.-c, 95.35.+d, 04.70.Dy, 14.80.-j, 41.60.-m, 44.40.+a, 34.50.-s, 03.65.Xp, 98.62.Gq

1. Introduction

In this contribution, the dark matter is presented as an alternative form of matter that is predicted to exist by the hypothesis of unique interaction (UI). The hypothesis also predicts some of its fundamental properties.

The UI was outlined in our earlier work [1, 2]. Specifically, we supposed that there is only a single - unique - interaction in the universe and gravitational, electric, weak, and strong forces are only various aspects of this unique force.

Meanwhile, we propose to relate this interaction with the charged elementary particles which are supposed to be identified with the primordial black holes (PBHs). However, our concept of the PBH is different from the traditional concept of this object. We assume that the PBH is the fundamental object that is an *invariant of time arrow*. It means that the dynamical picture of such a PBH is the same in time elapsing forward as in that elapsing backward.

This is possible if we assume that the PBH emits the Hawking radiation (HR) in time forward within a half-period of its cyclic existence and in time backward within the next half period. In our one-way perception of time, we could observe the emission of the HR during the first half-period and absorption of this radiation during the second half-period. In the beginning of the period of its existence, the PBH has its maximum energy. Its energy is gradually lost during the first half-period. At the end of this half-period, the energy of the PBH is zero, the PBH disappears for a moment. In this moment, the PBH starts to absorb the energy from incoming HR. The energy increases up to the maximum at the end of the second half-period. Then, the cycle repeats. It is assumed to repeat eternally.

The emitted and absorbed HR is the agent mediating the interaction between objects. The HR also causes an inertia of material object. Some success to explain various phenomena can be achieved (see Sect. 3), when we assume that the HR is not a harmonic, but an evanescent wave that can give a reactive impulse to the PBH when is emitted and impulse when impacts the PBH. The magnitude of the impulse depends on the frequency of the HR wave.

If the PBH is in rest and isolated, the emission and absorption of the HR is spherically symmetric and the impulse given to the PBH in one direction is perfectly eliminated by the equal-size impulse given this PBH in the opposite direction. The PBH is the subject of a force when the symmetry is broken. It may happen in two ways.

At first, another object in a neighborhood of the given PBH can screen the HR and a part of the screened wave does not return to the PBH. The corresponding impulse from the opposite direction is no longer compensated and causes an acceleration of the PBH. At second, if the PBH accelerates because of any reason, the wave departing from and incoming to the PBH from the direction of acceleration is blue shifted and the wave departing or incoming from the direction opposite to the acceleration is red shifted. The impulse departed by the blue shifted wave is larger than that departed by the red shifted wave, which is no longer able to balance the former impulse. This breaking of the symmetry is the nature of the inertia force.

Since the HR is regarded as the agent mediating the unique interaction, therefore also electric interaction, it can be expected to be described by the classical Maxwell theory of electromagnetism supplemented with the Einstein's theory of general relativity (GR). In the next section, we give some further details.

2. Unification of gravitational and electric aspects of unique interaction

In the beginning of our further explanation, let us formally unify the elementary electric charge with mass. In the unification, we answer the question what a mass, M_o, must two equal-mass particles have to generate the Newton gravitational force which is the same as the electrostatic force due to two elementary electric charges, q_o, located in the same distance as the particles? This question can be answered with the equality of the relevant Newton and Coulomb laws, i.e.

$$\frac{GM_o^2}{r^2} = \frac{q_o^2}{4\pi\varepsilon_o r^2}, \tag{1}$$

from which

$$M_o = \frac{q_o}{4\pi\varepsilon_o G} = \sqrt{\alpha} M_P = 1.8594 \cdot 10^{-9}\,\mathrm{kg}. \tag{2}$$

In these relations, G is the Newton gravitational constant, r is the distance between the particles and charges, ε_o is the permittivity of vacuum, α is the fine structure constant, and M_P is the Planck mass.

The vector of electric intensity of the HR, $\vec{E}$, can be described with the help of the wave equation

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \vec{0}, \tag{3}$$

where Δ is the Laplace operator, c is the speed of light in vacuum, and t is time. The analogous equation is valid for the vector of magnetic induction, $\vec{B}$. Assuming the fields varying in time as $\mathrm{e}^{-\mathrm{i}\omega t}$ (i is the unit of imaginary numbers and ω is the angular frequency), the electromagnetic wave equation reduces to the Helmholtz equation for $\vec{E}$:

$$\Delta\vec{E} + k^2\vec{E} = \vec{0}. \tag{4}$$

In the last equation, k is the size of the wave vector.

Equation (4) is the vectorial equation. The solution for the radial part, for the radial component of the electric intensity, E_r, can be found in form

$$E_r = K_r \frac{\mathrm{e}^{\pm\mathrm{i}2\pi kr}}{r^2}\mathrm{e}^{-\mathrm{i}\omega t}, \tag{5}$$

where K_r is a constant of proportionality.[1]

Using relation (5) for the radial component of intensity, its value in the so-called "interaction radius" was used in the UI [1] to define the "complex mass", $\mathcal{M}$. Complex mass comprehends both common mass and electric charge. The interaction

[1]In relation (5), the argument of the first exponential contains the factor of 2π. This factor is a correction of the following misunderstanding. The atomic energy levels were first time explained, roughly, within the Bohr theory. In this theory, the electron in atom was assumed to circulate around the atom nucleus, whereby the product of its impulse and the length of the circular orbit was required to be only an integer multiple of the Planck constant, h. The length of the circular orbit was $2\pi r$. In the UI, the electron does not move around the nucleus, but stays in a specific distance r. If this concept reflects the reality, then the factor of 2π was redundant in the Bohr's theory. To satisfy the Bohr's condition, the Planck constant had to be also enlarged about the factor of 2π. With this enlarged constant, we can obtain a correct numerical result (correct atomic levels in hydrogen atom), when the argument of the exponential is also enlarged about the factor of 2π.

radius, R_I, was defined as the Compton wavelength corresponding to the elementary electromass divided by 2π, i.e.

$$R_I = \frac{\hbar}{2\pi M_o c}. \tag{6}$$

Obviously, the HR spreads symmetrically to all directions, therefore it is a spherical wave with the wave vector $\vec{k} = (k,\ 0,\ 0)$ in the spherical coordinates. Hence $\vec{k}.\vec{r} = kr$; this was used in the relation (5). The size, k, of the wave vector in the case of evanescent wave is

$$k = \frac{\sqrt{\omega^2 - \omega_o^2}}{c}, \tag{7}$$

where ω is - as already mentioned - the angular frequency of the wave in a curved spacetime (in distance r from the source of waving) and ω_o is the angular frequency in a flat spacetime (when $r \to \infty$). In a strong field, i.e. in a position not very distant from the gravitational radius, R_g,

$$\omega = \frac{\omega_o}{\sqrt{1 - R_g/r}}. \tag{8}$$

In the case of gravitational interaction, the gravitational radius can be given as the radius of a compact object, from which the escape speed of a test particle would be the speed of light, i.e. equation

$$mc^2 = m\Phi_{gR} = \frac{Gmm_a}{R_g}, \tag{9}$$

is relevant, where m_a is the mass of a particle that generates the gravitational potential, Φ_g, and acts on the test particle. The latter has mass m. In relation (9), gravitational potential in distance $r = R_g$ is denoted by Φ_{gR}. On the basis of equation (9), the gravitational radius can be given as

$$R_g = \frac{\Phi_{gR}}{c^2}. \tag{10}$$

If we consider two charged elementary particles, the electric interaction dominates and the gravitational potential, Φ_g, has to be replaced with its electric analog, Φ_e. The equation that is an analog to (9) can be used. The Newton potential on its right-hand side must be replaced with the Coulomb potential, which we write in terms of the elementary electromass. Specifically,

$$mc^2 = -q_o\Phi_{eR} = -\frac{q_o^2}{4\pi\varepsilon_o R_e} = -\frac{GM_o^2}{R_e}, \tag{11}$$

from which

$$R_e = -\frac{GM_o^2}{mc^2} = -\frac{M_o\Phi_{eR}}{mc^2}, \tag{12}$$

where the electric potential in distance $r = |R_e|$ is denoted by Φ_{eR}. In contrast to always positive mass, the charge of elementary particle and, hence, electromass M_o can also be negative. Therefore, electric potential can be either positive or negative. Consequently, quantity R_e is positive for two oppositely charged particles and negative for two equally charged particles.

In the GR, the angular frequency and flat-spacetime angular frequency are related according to relation (8). In the UI, the dominant potential is considered in the formulas like (8). In the case of two charged particles, the electric potential is dominant. Hence, equation (8) is modified to

$$\omega = \frac{\omega_o}{\sqrt{1 + R_e/r}}. \tag{13}$$

Now, using relation (7), the size of wave vector can be given as

$$k = \frac{\omega_o}{c}\sqrt{\frac{\frac{-R_e}{r}}{1 + \frac{R_e}{r}}}. \tag{14}$$

In distance $r = R_I$, the evanescent wave can be well-approximated with a harmonic wave and the size of the wave vector equals

$$k(R_I) = \frac{\omega_o}{c} = \frac{m_o c}{\hbar}. \tag{15}$$

Subsequently,

$$k(R_I) 2\pi R_I = \frac{m_o}{M_o}, \tag{16}$$

where m_o is the mass in a flat spacetime of the particle associated with the wave with the flat-spacetime angular frequency, ω_o, according to the de Broglie relation $m_o c^2 = \hbar\omega_o$.

The radial component of intensity in R_I is now proportional to

$$E_r(R_I) \propto \mathrm{e}^{\pm \mathrm{i}\frac{m_o}{M_o}}. \tag{17}$$

Based on this relation, the complex mass was defined, in the UI, as

$$\mathcal{M} = \mp \mathrm{i} M_o \mathrm{e}^{\pm \mathrm{i}\frac{m_o}{M_o}} = \mp \mathrm{i} M_o \left(1 \pm \mathrm{i}\frac{m_o}{M_o} - ...\right) = \mp \mathrm{i} M_o + m_o + ..., \tag{18}$$

where the upper (lower) sign is relevant to negatively (positively) charged particle. Since $m_o \ll M_o$ (unless the particle moves with a speed approaching the speed of light), we consider only two first terms of the series.

In the above definition, the first term of series, $\mp \mathrm{i}\, M_o$, corresponds to the negative or positive elementary electric charge and second term, m_o, to the always positive mass of particle in a flat spacetime. Since M_o is constant, there is no related blue shift or red shift and, therefore no inertia occurs at this, electric, term. On the other-hand side $m_o = \hbar\omega_o/c \propto \omega_o$, therefore the second term (mass) possesses and inertia.

3. Some consequences of unique force

There are a lot of observed fundamental phenomena that can be qualitatively and, sometimes, quantitatively explained with the help of the UI. There is not known a theory that would explain such a large number of phenomena at the same time and that would comprehend both macroscopic and miscoscopic phenomena at the same time. In the following, we note the most important implications of UI.

3.1. Electric neutrality and impossibility of gravitational neutrality

If a matter consists of elementary particles with the positive elementary electric charge and the same number of particles with the negative elementary electric charge, then the total electric charge of this matter is zero. Such a matter is electrically neutral. It means that it is not attracted or repelled by any electric charge in its vicinity.

The same effect is impossible in the case of the particle's mass. The mass can be only positive and a piece of matter consisting of non-zero set of particles is always attracted by a neighboring mass object.

The reason why the electric neutrality is possible, but gravitational neutrality is not possible, can be justified with the help of the definition of complex mass that is given, in terms of mathematics, by relation (18). If we have an object consisting of n positively charged elementary particles (e.g. protons with mass m_p) and the same number, n, of negatively charged elementary particles (e.g. electrons with mass m_e), then the sum of their complex masses is

$$\mathcal{M} = n(+\mathrm{i}M_o + m_p) + n(-\mathrm{i}M_o + m_e) = n(m_p + m_e). \quad (19)$$

While the first - electric - term can be as positive as negative, the second - gravitational - term can be only positive, since the second term of series $-\mathrm{i}\, M_o \mathrm{e}^{\mathrm{i}m_o/M_o} = -\mathrm{i}\, M_o + m_o + ...$ is the same as the second term of series $+\mathrm{i}\, M_o \mathrm{e}^{-\mathrm{i}m_o/M_o} = +\mathrm{i}\, M_o + m_o + ...$

3.2. Orientation of gravitational and Coulomb forces

Let us consider two small objects in a distance, r, much larger than their sizes. The complex mass of the first object can be given as $\mathcal{M}_1 = m_1 \pm \mathrm{i}\, M_1$ and that of the second object as $\mathcal{M}_2 = m_2 \pm \mathrm{i}\, M_2$, where m_1 and m_2 are the common masses of the objects and $\mathrm{i}M_1$ and $\mathrm{i}M_2$ are their electromasses corresponding to their electric charges.

The objects act each other with the complex force, $\mathcal{F}$. It can be calculated with the help of Newton gravitational law in which the masses of the objects are replaced with their complex masses. If both objects are charged with the charges of the same polarity, then

$$\mathcal{F}_{++} = -G\frac{(+\mathrm{i}M_1 + m_1)(+\mathrm{i}M_2 + m_2)}{r^2} = -G\frac{(-\mathrm{i}M_1 + m_1)(-\mathrm{i}M_2 + m_2)}{r^2} =$$

$$= +G\frac{M_1 M_2}{r^2} - G\frac{m_1 m_2}{r^2} \mp \mathrm{i}\frac{G}{r^2}(m_1 M_2 + m_2 M_1). \quad (20)$$

The first term of the real-valued part in the final result is the Coulomb law and the second term of this part is the Newton gravitational law. We can see that both terms are real-valued although the masses of the objects are complex-valued quantities. The sign of the first term is opposite than the sign of the second term. It means that the electric force is oriented in the opposite direction than the gravitational force in this case.

If both objects are charged with the charges of opposite polarity, then the complex force is

$$\mathcal{F}_{+-} = -G\frac{(+\mathrm{i}M_1 + m_1)(-\mathrm{i}M_2 + m_2)}{r^2} = -G\frac{(-\mathrm{i}M_1 + m_1)(+\mathrm{i}M_2 + m_2)}{r^2} = = -G\frac{M_1 M_2}{r^2} - G\frac{m_1 m_2}{r^2} \pm \mathrm{i}\frac{G}{r^2}\left(m_1 M_2 - m_2 M_1\right). \quad (21)$$

In this case, the sign of both electric and gravitational terms is the same. It means that the orientation of electric force is the same as that of the gravitational force. These orientations are actually observed.

3.3. Origin of the inertia force

In the beginning (Sect. 1), we claimed that the HR is the agent mediating any interaction between various material objects. This radiation is assumed to be emitted uniformly to all directions. Because of the spherical symmetry, the impulse delivered to a PBH from one direction is eliminated by that from the opposite direction. If there is no perturbation, therefore the spherical symmetry is not broken, the object remains in rest.

Beside a screening of the HR by a PBH located in a vicinity of given PBH, the symmetry can also be broken when the latter is isolated, but accelerates in a direction. Then, the HR in the direction of acceleration is blue-shifted and the HR in the opposite direction is red-shifted. The impulse delivered by the HR with a larger frequency, i.e. blue-shifted HR, is larger than the impulse delivered by the HR with a relatively smaller frequency (red-shifted HR). The impulse delivered to the given PBH from the direction of the PBH's acceleration is thus not completely eliminated by the impulse from the opposite direction and the excess is manifested as an inertia force.

When mass m_o is replaced with the flat-spacetime angular frequency ω_o in the definition (18) of complex mass, then the complex mass is

$$\mathcal{M} = \mp M_o\left(1 \pm \frac{\hbar\omega_o}{M_o c^2} + ...\right). \quad (22)$$

The further terms are negligible. In this context we remind that frequency ω_o is the frequency in a free, flat, spacetime, in contrast to angular frequency ω, which characterizes the HR in a curved spacetime. Quantity ω_o is also related to a moving object (but in the flat spacetime). So, ω_o can be blue-shifted or red-shifted.

In relation (22), M_o is a constant. The first term in the parentheses, i.e. unity, corresponds to the electric charge of the considered PBH and the second term to mass. Notice that the first term does not contain ω_o, therefore it cannot be changed due to an acceleration of PBH. It means that there cannot be any inertia force linked to this - electric - aspect of the PBH. The frequency ω_o occurs in the second term corresponding to mass. Hence, the mass has an inertia.

3.4. Tunneling effect

In Sect. 1, we described the PBH as an entity cyclically emitting the HR in time forward during a half-period and in time backward during the other half-period. In the beginning, the measure of the PBH's existence reaches its maximum and PBH disappears in the moment when the half-period elapses.

Let a given PBH reaches the maximum of its existence in time t_o. There is, however, no reason why all the PBHs in the universe should be synchronized. Other PBHs may reach their maximum existence in various times that are different than t_o. In other words, there are various phase shifts of the varying existence of the PBHs. The distribution of these shifts can be supposed to be random. The wave emitted by a PBH is, generally, shifted in phase about $\omega\Delta t$ in comparison with a wave of other PBH.

Two PBHs can likely interact only during a sub-period when the existence of one overlaps with the wave of returning to the other. The measure of the overlapping is illustrated in a scheme in Fig. 1 with a hatched area.[2] In panel (a) of this figure, two PBHs are synchronized and their interaction is maximum possible. In panel (b), there is a phase shift, $\omega\Delta t$. Two PBHs with this phase shift can partially interact.

In panel (c), $\omega\Delta t = \pi$. It means that two PBHs cannot perceive each other and interact. If these PBHs meet each other, we cannot calculate the force between them using a formula for a mean force, i.e. the force for an average phase shift. In other words, the force action, that is expected in the case of averaged force, does not occur in this case.

3.5. Comparison of uncertainty in the UI with the Heisenberg uncertainty principle

At the present, no experiment to determine the phase shift is known. Missing information about the shift causes an uncertainty in the evaluation of the interaction between the PBHs, i.e. between the elementary particles.

The phase shift can vary from zero to 2π. Hence the uncertainty due to absence the information about the shift can be expressed with inequality

$$\omega\Delta t \leq 2\pi. \tag{23}$$

[2]The overlapping below the horizontal axis is not hatched, since the outcoming wave already delivered a reaction impulse to the PBH. There can occur only an absence of the wave incoming to the PBH and just this absence results in an excess of the impulse and, therefore, interaction.

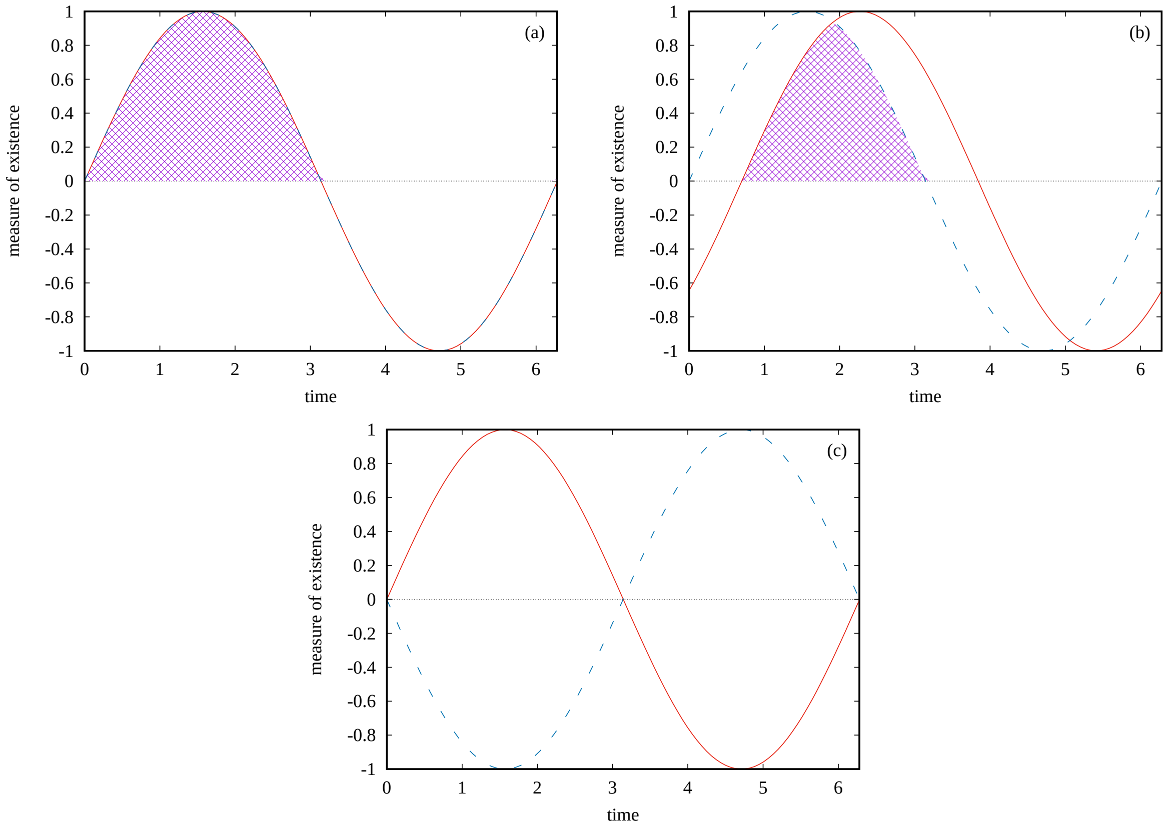

Figure 1: Measure of overlapping, in a given point, of the Hawking radiation emitted by two primordial black holes. In panels (a), (b), and (c), the phase shift between the waves is $\Delta t = 0,\ 0.7\,\text{rad}$, and π, respectively. The hatched area indicates the measure of the overlapping of the primordial black hole and wave moving in one direction that was emitted by other primordial black hole.

When this relation is multiplied with $\hbar$ (the Planck constant, h, divided by 2π), it is changed to

$$W\Delta t \leq h, \tag{24}$$

whereby de Broglie's relation $W = \hbar\omega$ between the energy, W, and angular frequency, ω, was used.

Inequality (24) can be compared with the Heisenberg uncertainty principle in form

$$\Delta W \Delta t \geq \hbar/2. \tag{25}$$

This inequality is represented in terms to explain the precision of measurement of the energy change, ΔW, during time interval Δt. If we know the time interval, the error of the determination of energy change must be equal or larger than $\hbar/(2\Delta t)$. However, the rigorously interpreted inequality also means that the error can be thousand $\hbar/(2\Delta t)$ or million $\hbar/(2\Delta t)$, etc. Thus, one can suspect that, rather, the opposite inequality is correct.

Anyway, the concept of uncertainty is also present in the UI, not only in the quantum physics.

3.6. Atomic energy levels

In Sect. 1, we explained that an interaction between two PBHs occurs when the HR from one PBH is screened by the other PBH. This screening happens in the distance r between both PBHs, where the density of the radiation carrying the impulse is smaller than the density of radiation in the distance of interaction radius R_I. The same is valid for the amplitude of the impulse carried by the wave.

However, the magnitude of impulse, that is not compensated by the absorbed impulse from the opposite direction and that is the agent mediating the interaction, is that in R_I. This was the reason why the complex mass was established on the basis of the amplitude of E_r in R_I. Anyway, the measure of the absorption of impulse is proportional to E_r in distance r.

It can be demonstrated that the size of the wave vector of evanescent wave can be approximated as $k = \omega_o/c$ in the distance $r \gg R_e$ and $r \ll R_e$. However, there is an interval of the distance where this approximation is impossible and one must use the rigorous formula for the radial component of the intensity of electric field, E_r. If we consider the system of particles consisting only of proton and electron, the formula reads

$$E_r = G\frac{M_o}{r^2}\mathrm{e}^{\mathrm{i}2\pi kr} = G\frac{M_o}{r^2}\left[\cos(2\pi kr) + \mathrm{i}\sin(2\pi kr)\right]. \tag{26}$$

Using this expression for E_r, the force between the proton and electron is

$$\mathcal{F} = -\mathrm{i}M_oE_r = G\frac{M_o^2}{r^2}\sin(2\pi kr) - \mathrm{i}G\frac{M_o^2}{r^2}\cos(2\pi kr). \tag{27}$$

Only the real-values part of the force is observable in real universe. We can see that this part is zero, if $\sin(2\pi kr) = 0$. This is satisfied, when

$$2\pi kr = 2\pi n, \tag{28}$$

where n is a natural number.

For the electron located in the force field of proton, the electric radius is negative; it equals $R_e = -GM_o^2/(m_e c^2)$. The mass of electron is denoted by m_e. If this R_e is supplied to relation (14) and, further, the result is supplied into condition $2\pi kr = 2\pi n$, then we obtain equation

$$\frac{\omega_o}{c}\sqrt{\frac{|R_e|/r}{1-|R_e|/r}}\,r = n. \tag{29}$$

Fraction ω_o/c can be given as $\omega_o/c = \alpha/|R_e|$. Using this and after a further handling, equation (29) can be re-written to quadratic equation for r:

$$\frac{\alpha^2}{n^2}r^2 - |R_e|r + |R_e|^2 = 0. \tag{30}$$

Its solution is

$$r_{1,2} = |R_e|\frac{n^2}{2\alpha^2}\left(1 \pm \sqrt{1-\frac{4\alpha^2}{n^2}}\right). \tag{31}$$

If we neglect term $4\alpha^2/n^2$ in the argument of square root in relation (31) and realize that the Bohr's radius $R_B = |R_e|/\alpha^2$, then the first solution equals $r_1 = R_B n^2$. The distances for n = 1, 2, 3,... are the distances, where no force acts on electron and the energy of this particle corresponds to its energetic levels in the Bohr's model of hydrogen atom.[3] In this way, the UI describes the quantum states of electron of atom shell.

However, there is also the second solution of the quadratic equation (30). The square root in solution (31) can be developed into the McLaurin series. If we constrain our attention to the three first terms of the series, we have $\sqrt{1-4\alpha^2/n^2} \doteq 1 - 2\alpha^2/n^2 - 2\alpha^4/n^4 - ....$ Then, $r_2 \doteq |R_e|(1+\alpha^2/n^2)$. We can see that the stable positions are located in a slightly larger distance than $|R_e|$.

Quantity $|R_e|$ for the proton electron system is numerically equal to the well-known "classical radius of electron", which equals $2.828 \cdot 10^{-15}$ m. It means that the stable positions corresponding with the second solution of equation (30) are in the distance scale of atomic nucleus. The UI is unique theory to predict the existence of quantum states, where these were actually detected, in atom shell and atom nucleus.

[3] The energy levels calculated within the UI with the precision comparable to their counterparts in the Dirac theory, for one set of quantum numbers ($n = 1, 2, 3, ...$, $l = n-1$, and $j = l + 1/2$), can be found in Table 2 our earlier paper [1].

In addition, these are predicted simultaneously, as the solutions of single quadratic equation.

In the UI, the concept of atom is different from its counterpart in the quantum physics. In the former theory, the electrons in the atom shell and nucleons in the atom nucleus are located in the distances, where the force acting on them is zero. In these distances, the particles can be in a stable-equilibrium and unstable-equilibrium position. While the former is that between the zone of repulsion and zone of attraction, the latter is between the zone of attraction and zone of repulsion. All zero-force distances could be gained, if we considered condition $2\pi kr = \pi n$ instead of condition (28). The factor of two on the right-hand side of (28) was inserted just to choose only the stable-equilibrium positions.

3.7. Electric nature of "strong force"

The energy of electron in the electrostatic field of proton can be calculated according to the well-known relation

$$W = m_e c^2 - \frac{q_o^2}{4\pi\varepsilon_o r} = \hbar\omega_e - \frac{GM_o^2}{r} = W_o \left(\frac{1}{\sqrt{1-|R_e|/r}} - \frac{|R_e|}{r} \right), \qquad (32)$$

where the electron's rest energy, $m_e c^2 = \hbar\omega_e$, is denoted by W_o.

Let us now evaluate the ratio of the free energy of electron, $W - W_o$, and magnitude of its Coulomb potential energy. We denote this ratio in distance r_1 (r_2) given by relation (31) as χ_1 (χ_2). We remind that the magnitude of the Coulomb potential energy is $| - q_o^2/(4\pi\varepsilon_o r)| = | - GM_o^2/r|$. Considering the approximative distances, we obtain $\chi_1 \approx 1/2$ and $\chi_2 \approx n/\alpha$. For a low n, $\chi_2 \sim 10^2$. One can see that the free energy is larger about two orders of magnitude in comparison with the energy, that would be calculated by using the Coulomb formula, in the region of atom nucleus. Within the UI, the so-called strong force seems to be only another aspect of electric force. The latter is regarded as an aspect of the unique force.

3.8. On no quantum states of the system of two protons

Finally, we mention a reason why there are no bound states of two electrically charged particles with the same polarity, e.g., two protons. The reason can be given with the help of formulas (12), (14), and (5). For the system of two protons, the relevant electric radius (see relation (12)) is $R_e = (+M_o)(+M_o)/(m_p c^2) > 0$. Hence, the size of wave vector given by relation (14) is $k = (\omega_o/c)\sqrt{(-R_e/r)/(1-R_e/r)} = \mathrm{i}(\omega_o/c)\sqrt{(R_e/r)/(1-R_e/r)}$. It means that the value of k is imaginary valued, therefore argument, $-\mathrm{i}2\pi(\mathrm{i}|k|)r = 2\pi kr$, of the exponential in relation (5) for the radial component of electric intensity is real-valued quantity.

The exponential of real argument can be given as $\mathrm{e}^{2\pi kr} = \cosh(2\pi kr) + \sinh(2\pi kr)$. In contrast to the goniometric cosine and sine, the hyperbolic cosine and hyperbolic sine are monotonous functions when their argument ($2\pi kr$ in this case) is positive.

Therefore, the force between two equally charged particles, like two protons, does not change from attractive in a range of distance to repulsive in other range. Thus, there are no distances where the force would be zero, between the regions of repulsion and attraction. The particle cannot be in rest, in a position where the force would be zero.

4. Basic properties of dark matter within the UI

The above-mentioned solution (5) of the Helmholtz equation (4) is likely not unique. We can expect some other forms of electric intensity. This expectation is supported by the following analogy.

A quadratic equation with real coefficients can have two real-valued roots or two complex-conjugate roots. This depends on the value of discriminant, which can be non-negative or negative. Analogously, we assume that there is possible to find a solution for the amplitude of radial component of electric intensity, which is not complex-valued (as that given by relation (5)), but real-valued, i.e. the proportionality

$$E_r \propto \frac{\mathrm{e}^{\pm 2\pi k r}}{r^2} \qquad \ldots \qquad E_r(R_I) \propto \mathrm{e}^{\pm m_o/M_o} \tag{33}$$

can be expected. In analogy with the definition (18) of the complex mass, which was derived from the radial component of intensity (5), we can define the dark-matter (DM) universal mass

$$\mathcal{M} = \pm M_o \mathrm{e}^{\pm \frac{m_o}{M_o}} = \pm M_o \left(1 \pm \frac{m_o}{M_o} + \ldots\right) = \pm M_o + m_o + \ldots, \tag{34}$$

which is derived on the basis of relation (33).

Using the de Broglie relation between the mass and angular frequency, the definition of DM universal mass can also be given as

$$\mathcal{M} = \pm M_o \left(1 \pm \frac{\hbar\omega_o}{M_o c^2} + \ldots\right) = \pm M_o + \frac{\hbar\omega_o}{c^2} + \ldots. \tag{35}$$

We name the first term of the series on the right-hand side of (34) as elementary "mass charge". We see that its size is the same as the size of elementary electric charge and neither any inertia is associated with this term. It is implied by the fact that it is a constant. No blue shift or red shift can be associated with this term, therefore there is no corresponding inertia. The inertia occurs at the second term that depends on the angular frequency as seen in relation (35). The second term can be regarded as a common mass.

Let us now compare the forces between two charged particles of baryonic matter and two particles one of which is baryonic and the other is a DM particle. The force between two baryonic particles is given by relations (20) or (21). Its real-valued part can be observed in our, real, universe. We see that the electric charge (mass) of one particle interacts with the electric charge (mass) of the other particle. There is no real-valued and, therefore, detectable force between the electric charge of one particle and mass of the other particle.

On contrary, the force between a baryonic particle and DM-particle is

$$\mathcal{F}_{bd} = -\frac{G}{r^2}(+\mathrm{i}M_o + m_b)(M_o + m_d) = -\frac{G}{r^2}m_b(M_o + m_d) - \mathrm{i}\frac{G}{r^2}M_o(M_o + m_d). \quad (36)$$

We considered the particles with a positive electric and mass charges. The mass of baryonic particle (DM-particle) is denoted by m_b (m_d). In this relation, we can see that the mass charge of the DM-particle as well as its mass interact with the common mass of baryonic matter. There is no real, detectable, interaction between the mass charge of the DM-particle and electric charge of the baryonic particle. This is the main difference between the interaction of two baryonic particles and particles of mixed kinds.

Further, let us investigate a typical cross-section of the collision of two particles of mixed kinds and compare it with the cross-section of the collision of two charged baryonic particles. When two charged baryonic elementary particles collide with a high energy, their cross-section, σ_{bb}, can be calculated by using the Rutherford formula

$$\sigma_{bb} = \frac{q_o^4}{W_1^2}\Theta, \quad (37)$$

where W_1 is the kinetic energy of incident particle and Θ is a function characterizing the scattering of the incident particles to various directions. Elementary electric charge can be replaced, in this relation, with the elementary electromass. The cross-section then is

$$\sigma_{bb} = C_\sigma \frac{M_o^4}{W_1^2}\Theta \quad (38)$$

with constant $C_\sigma = (4\pi\varepsilon_o G)^2$.

The fourth power of M_o in the last relation is the product of quadrates of the charge of the target and incident particles. If we consider the DM-particle instead of baryonic particle in the colliding pair, then M_o^2, representing the quadrate of electric charge of baryonic particle, should be replaced with m_b^2, i.e. the quadrate of the mass of baryonic particle. It is a consequence of the fact that the dominant interaction of two particles of mixed kinds is that between the mass of the baryonic particle, m_b, and mass charge, M_o, of the DM-particle. (We assume that $m_d \ll M_o$ in the analogy with $m_b \ll M_o$.) Realizing this circumstance, the cross-section of the collision of baryonic and DM-particles is

$$\sigma_{bd} = C_\sigma \frac{m_b^2 M_o^2}{W_2^2}\Theta. \quad (39)$$

Here W_2 is the kinetic energy of the incident particle in this case.

When we compare the cross-sections of the collisions of two pairs of particles, it is reasonable to compare it for the incident particles having the same energies.

Therefore, we put $W_1 = W_2$. The ratio of the cross-section of the particles of mixed kinds and cross-section of the baryonic particles is

$$\frac{\sigma_{bd}}{\sigma_{bb}} = \frac{m_b^2}{M_o^2} \approx \frac{(2 \cdot 10^{-27}\,\mathrm{kg})^2}{(2 \cdot 10^{-9}\,\mathrm{kg})^2} \approx 10^{-36}. \tag{40}$$

In this estimate, we considered the mass of the stable baryonic particle to be the mass of proton. In the case of electron, the ratio would be even smaller, about six orders of magnitude. We see, that the cross-section of the interaction between a DM-particle and typical baryonic particle with the large mass is about 36 orders of magnitude smaller than the cross-section of the interaction of two charged baryonic particles. The UI can explain the absence of interaction, other than gravitational, between the baryonic matter and DM.

Interestingly, when a mass-charged DM particle collides with other mass-charged DM-particle, the cross-section, σ_{dd}, is predicted, by the UI, to be essentially the same as in the case of two charged baryonic particles. Namely, the dominant interaction between two mass-charged DM-particles is that between their mass charges. Hence, σ_{dd} can be calculated by using relation (38), in fact.

Let us now deal with a cooling of a gas consisting of the DM particles. For this purpose, we consider a unit volume of gas in a galactic halo, in an arbitrary distance from the galactic center. Let the gravitational mass of this volume be M_h. If the gas in the volume consisted of baryonic matter, prevailingly hydrogen atoms, having mass $\sim m_b$, then their total number would be $N_b \sim M_h/m_b$. However, if the gas in the volume consisted of DM-particles, then their number would be $N_d = M_h/M_o$ if the total gravitational mass was the same. (We again assume that $m_d \ll M_o$.) Since $N_d/N_b = m_b/M_o \approx 10^{-18}$, number N_d would be about 18 orders of magnitude smaller than number N_b. This fact has a crucial impact on the main cooling mechanism of a gas constituting the halo (or other DM gas).

The main mechanism of the cooling of a gas situated in interstellar or intergalactic space is the emission of bremsstrahlung radiation. The photons of this radiation are emitted in the mutual collisions of the particles. In sake of simplicity, we roughly estimate the number of collision assuming a mono particle gas. The number of collisions per unit time (frequency) is $\nu_b = N_b/t_b$ in the baryonic gas and $\nu_d = N_d/t_d$ in the gas consisting of DM-particles, where t_b (t_d) is the average time between two collisions of the particle in baryonic (DM) gas, i.e. the time needed to go along the mean free path in the given kind of gas. This time can be calculated as the ratio of the mean free path, $l_b = (\sigma_{bb} N_b)^{-1}$ ($l_d = (\sigma_{dd} N_d)^{-1}$), and average velocity of the particles, $v_b = \sqrt{8 k_{Bb} T/(\pi m_b)}$ ($v_d = \sqrt{8 k_{Bd} T/(\pi m_d)}$), where T is the temperature of gas (we assume the same temperature for both kinds of gas) and k_{Bb} is the Boltzmann constant. Its counterpart for the DM is denoted k_{Bd}. Its value is unknown and can be different from k_{Bb}.

If we use all the above-mentioned formulas, the ratio of the frequencies of collisions in both kinds of gas is

$$\frac{\nu_d}{\nu_b} \sim \sqrt{\frac{k_{Bd} m_b}{k_{Bb} m_d}} \frac{N_d^2}{N_b^2} \sim 10^{-36} \sqrt{\frac{k_{Bd} m_b}{k_{Bb} m_d}}. \tag{41}$$

In the baryonic gas, the bremsstrahlung photons are emitted mainly by electrons with mass m_e, therefore we can here identify $m_b = m_e$. Mass of the DM-particles with the dominant cooling effect is unknown. If this mass is not many orders of magnitude smaller than the mass of electron and/or Boltzmann constant for DM is not many orders of magnitude larger than its counterpart for baryonic matter, then the cooling of a DM galactic halo is extremely less effective than would be the cooling of corresponding halo consisting of baryonic matter.

In our previous work [3], we presented an example of a hot galactic halo and estimated that it would have had an energy to emit a radiation only during a period shorter than about 1% of the current age of the universe, if the halo consisted exclusively of baryonic matter. Such the halo would collapse onto the center soon and we would not observe the old galaxies with halo. Our result presented in the previous paragraph however implies that the temperature of the DM halo has to be practically the same not only during the period from the Big Bang until the present, but during a much longer period.

It seems that the DM galactic halos retain their high temperature that they acquired at the Big Bang. Due to this circumstance, the halos are stable and we can observe them as well as the whole galaxies in any cosmological time.

The extremely high temperature of any DM gas, that is indicated in our deduction, is likely a reason why we observe the DM halos of huge, galactic-scale, sizes, but we do not observe some stellar-size, planetary-size, or even smaller objects consisting of the DM. Only the tremendous gravity of huge galactic halos is able to keep the very hot DM gas concentrated.

5. Summary and conclusions

In the case of common, baryonic matter, the UI hypothesis can explain: why there is an interaction between the masses of two particles and interaction between their charges, if they are electrically charged, but why there is no interaction between the mass and electric charge; why the interaction between the masses (gravity) is always attractive and electric interaction between two charges of the same polarity repulsive and charges of opposite polarity attractive; why there is an inertia of mass, but no inertia of electric charge; and why a matter can be electrically neutral, but a mass neutrality is impossible.

The UI is currently a unique hypothesis that can explain both phenomena in macrocosm and phenomena in microscosm. Within this hypothesis, it is possible to built a model of atom, with the electrons in atomic shell being in rest and residing in the borders between the zones of repulsion and attraction. The distance of given border is calculated as a root of quadratic equation, whereby there is a series of such equations, for various stable positions of electron that correspond to the energetic levels of this particle in the atom shell.

The second root of the quadratic equation gives the borders between the zones of repulsion and attraction and, thus, quantum levels (levels in the series of equations) close to atom nucleus. It means that the UI is the unique theory predicting the existence of two regions of quantum levels, in the atom shell and atom nucleus. In the other theories, these regions are regarded as the experimental facts. They are not necessary implications of theory.

Within the UI, further, the energy of particle in the nucleus region of stable levels is about two orders of magnitude larger than the corresponding energy calculated according to the Coulomb law. Hence, the UI has a potential to engulf the strong force within the unique description of interaction.

In the UI, the fundamental element of existence is an object that periodically emits and absorbs a radiation. This object can likely be identified with a primordial black hole and radiation with the Hawking radiation emitted alternately in time forward and backward. Within one period of existence, the object changes from the its maximum existence to zero existence and vice versa. In practice, the beginning of this period is unknown and there can be a various phase shift of the beginnings of the periods of different objects. Due to the unknown phase shift, there is an uncertainty in a measure that an object perceives other object. This uncertainty is probably related to the Heisenberg uncertainty principle in quantum physics.

The UI provides us with a possibility to predict an alternative form of matter, which seems to be possible to be identified with the DM. The essential difference between the common, baryonic, matter and DM is following. While the elementary electrically charged particle of baryonic matter possesses an electric charge, the analogous particle of DM possesses so-called mass charge. Its size is the same as the size of elementary electric charge and, at the same time, it possesses no inertia. However, the mass charge does not interact with any electric charge, but with a mass of particles of both kinds of matter.

The absence of the interaction with electric charge is the reason of extremely small cross-section when a DM-particle collides with a baryonic particle. In addition, there seems to be another serious consequence of the absence: unless the mass of a "standard" stable particle of DM is not extremely smaller than the mass of electron and/or the counterpart of the Boltzmann constant for a DM gas is not extremely larger than the Boltzmann constant, then the cooling efficiency of DM gas via bremsstrahlung radiation is many order of magnitude smaller than the efficiency of the cooling of baryonic gas via this mechanism. It means that the galactic DM halos have been cooled insignificantly during the whole age of the universe and their temperature should still be almost so large as at the Big Bang.

Because of the ineffective cooling of the DM, some concentrations of DM gas on scale smaller than the galactic scale are, most probably, impossible; the gravity of objects smaller than galaxy is not strong enough to keep a DM gas concentrated.

Currently, the UI is still a developing hypothesis, even in the case of phenomena related to baryonic matter. Its part concerning the DM is now only a "white paper". A lot of work is needed to confirm its relevancy to the real world. However, there are many promising partial results. One can reasonably hope that, once, the UI will become a successful theory.

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WHAT IS THE CURRENT VALUE OF THE ACCELERATION OF THE EXPANSION OF THE UNIVERSE?

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Abstract: We assume that the Friedmann equation for the expanding 3-sphere describes the evolution of our universe quite well. Under this assumption we derive that the current value of the acceleration of the radius of the 3-sphere is an extremely small number, namely, less than one nanometer per square second, even though the universe contains 70 % of dark energy responsible for this acceleration.

Keywords: expansion function, Friedmann equation, dark energy, 3-sphere

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1. Introduction

In 1975 James E. Gunn and Beatrice M. Tinsley published in Nature the paper: *An accelerating Universe?*, see [7]. Three years later Tinsley came with another paper: *Accelerating Universe revisited*, see [24]. This was followed by endless discussions about whether the observed expansion of the physical universe is really accelerating.

In 2011, the Nobel Prize for cosmology was awarded to three astronomers for their discovery of the accelerating expansion of the universe. The Swedish Royal Academy of Sciences has decided to reward the leaders of the two competing teams especially for their works [16], [19], and [15] from the years 1997–1999 and for the discovery that the expansion of our universe accelerates due to dark energy. These works contain various values of basic cosmological parameters.

The first laureate is the American Saul Perlmutter who won one half of the Nobel Prize. Perlmutter headed the *Supernova Cosmology Project* at the University of California at Berkeley.

The second Nobel Prize Winner is Adam Guy Riess, who is a professor of astronomy at Johns Hopkins University and the Space Telescope Science Institute in

Baltimore, Maryland. The third laureate is the American-Australian astronomer Brian Schmidt who was the team leader of the *High-z Supernova Search*. The latter two laureates shared the second half of the Nobel Prize.

In the above mentioned papers [15, 16, 19] it is not precisely defined what does it mean acceleration of the universe. Does it mean that

1) the cosmological constant is positive, or

2) the Hubble parameter is increasing in time, or

3) the deceleration parameter is negative, or

4) the scaling parameter and its derivative are increasing?

No two of these conditions are equivalent. Everyone could also propose their own non-equivalent condition 5). Moreover, the current value of the acceleration was not presented in [15, 16, 19]. Based on the Friedmann equation, we will derive in Section 2 that the current acceleration of the expansion of the universe is very roughly only 0.6 nm/s^2. Finally, note that Riess in [17] even claims that the expansion of the universe is faster than expected.

2. Calculation of the current expansion acceleration

The term "universe" in cosmology has various meanings: true spacetime, true space (i.e. a part of spacetime at a fixed time instant), and the observable universe, which is only seen as a projection on the celestial sphere. These are three different objects. Their mathematical models are also three completely different manifolds. To avoid ambiguities, we refer to Figure 1 to make these terms clearer for a universe with positive curvature. Thus altogether we have $6 = 3 + 3$ meanings of a general vague notion "universe" for which the terminology is not fixed yet. The first three contain real matter, whereas the other three are only abstract mathematical idealizations of reality.

All six above-mentioned objects have to be carefully distinguished, otherwise we may come to various confusions. For instance, the observable universe (represented by the yellow graph in Figure 1) and also the whole spacetime (red graph) are not homogeneous, since they have different mass densities at cosmological distances. On the other hand, the blue graph in Figure 1 corresponds to a homogeneous and isotropic universe for a fixed time, i.e., it satisfies the cosmological principle [13, p. 107].

Einstein's equations with cosmological constant Λ were developed in 1917 for a static universe [5] and not for a dynamical evolution of the universe. This was done later in 1922 by Alexander Friedmann [6] who derived from Einstein's equations a nonlinear ordinary differential equation (see (9) below) for the *expansion function* $a = a(t) > 0$ (sometimes called the scaling parameter). Here the symbol t stands for a *cosmic time* which is a measure of time by a physical clock with zero peculiar velocity in the absence of matter over-/under-densities (to prevent time dilation [11] due to relativistic effects or various confusions caused by expansion of the universe).

Friedmann in [6] assumed that the universe can be described by an expanding three-dimensional sphere

$$\mathbb{S}^3_a = \{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 + w^2 = a^2\}$$

which enabled him to avoid boundary conditions, see also Einstein [5, p. 152] for unchanging radius $a \equiv \text{const}$. Recent observations from JWST indicate that the physical universe can be described by an expanding three-dimensional sphere $\mathbb{S}^3_a$, see [10]. This means that the physical universe is bounded (i.e. closed) for every fixed time instant, see also [3, 12, 22, 23]. An unbounded infinite universe with almost the same temperature, density, pressure, curvature, etc., at every time instant on large scales is very unlikely. This would require an infinite speed of information transfer. For further arguments against an infinite universe see [13, p. 134].

For a given smooth positive expansion function $a = a(t)$ define the *Hubble parameter* in the standard way

$$H(t) := \frac{\dot{a}(t)}{a(t)}, \tag{1}$$

where $\dot{a}$ stands for the time derivative of a. Astronomical observations confirm that $a = a(t)$ is up to now an increasing function and therefore, $H(t) > 0$.

We say that the *expansion of the universe accelerates* on some time interval I, if the second derivative $\ddot{a} > 0$ on I, i.e., a is a strictly convex increasing function on I. In Figure 1, we can see that the increasing expansion function (red graph) is first strictly concave and then strictly convex. Its infection point is approximately in the middle.

Remark 1. The fact that $H = H(t)$ is at present a decreasing function (see e.g. [12, p. 118]) does not contradict the statement that the expansion of the universe could accelerate. To see this, consider, for example, $a(t) = t^2$ on $I = (0, \infty)$. Then we have $\dot{a}(t) = 2t$, $\ddot{a}(t) = 2 > 0$, and $H(t) = \dot{a}(t)/a(t) = 2/t$ is a decreasing function on I.

Next we define the dimensionless *deceleration parameter*

$$q := -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{a}H^{-2} = -\dot{H}H^{-2} - 1, \tag{2}$$

where the second equality follows from (1) and the last equality can be directly derived by differentiation of (1), namely, $\dot{H} = (\ddot{a}a - \dot{a}^2)/a^2$.

Remark 2. The term "deceleration parameter" q has its origin in classical cosmological models, where for some constant $C > 0$ it is assumed that

$$a(t) = Ct^{2/3}, \quad \text{i.e. by (1),} \quad H(t) = \frac{2}{3t}. \tag{3}$$

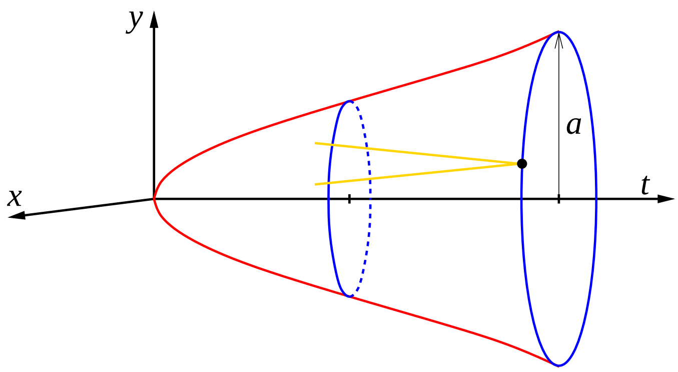

Figure 1: Schematic illustration of three different manifolds that are used in the Big Bang model our universe with positive curvature index. For simplicity, the space dimensions are reduced by two. Hence, the sphere $\mathbb{S}_a^3$ with radius $a = a(t) > 0$ at a fixed time instant t is replaced only by its great (blue) circle $\mathbb{S}_a^1$ for $z = w = 0$. This is a model of the universe (space) with positive constant curvature $1/a$. The spacetime model can be obtained by rotating the (red) graph of the expansion function $a = a(t)$ about the time axis t. The observable universe is marked by the (yellow) past light cone, the apex of which corresponds to the observer. This light cone is deformed near the origin, but we do not know exactly how. Each of these three models has a different center, see [13, p. 111].

We can derive that q is positive, since a is a strictly concave function. In particular, by (2) and (3) we get

$$q = -\dot{H}H^{-2} - 1 = -\Big(-\frac{2}{3t^2}\Big)\Big(\frac{3t}{2}\Big)^2 - 1 = \frac{3}{2} - 1 > 0.$$

Hence, in this special case the expansion decelerates.

However, Riess et al. in [19, p. 1026] obtained a negative value of the current deceleration parameter $q_0 \approx -1$ in the physical universe, since its expansion is accelerating, cf. also [15, p. 581]. Later this value was reduced to (see [20, p. 110])

$$q_0 = q(t_0) \approx -0.6, \tag{4}$$

where

$$t_0 \approx 13.8 \text{ Gyr}$$

is the age of the universe according to the standard ΛCDM (Lambda Cold Dark Matter) model, see e.g. [12, p. 208]. This means that a is strictly convex in a neighborhood of t_0. By recent JWST data the physical universe is probably much older than t_0, see e.g. [10].

From (2) we can express the second derivative as follows

$$\ddot{a} = -qaH^2. \tag{5}$$

The current value of the Hubble parameter reads $H_0 = H(t_0) \approx 70$ km/(s Mpc). In the literature we can find many other admissible values ranging in the interval 70 ± 5 km/(s Mpc), see e.g. [17, 18] for a faster expansion of the physical universe. Taking into account that 1 pc $= 3.086 \cdot 10^{16}$ m, we find that

$$H_0 \approx \frac{70 \cdot 10^3}{3.086 \cdot 10^{16}} \text{ s}^{-1} = 2.27 \cdot 10^{-18} \text{ s}^{-1} \tag{6}$$

in the SI units.

The mean density of the physical universe in our neighborhood is about $\rho \approx 10^{-26}$ kg/m^3, i.e., approximately 6 protons per m^3. Moreover, over $2 \cdot 10^{12}$ galaxies are currently observed by JWST each having on average about $4 \cdot 10^{11}$ stars like the Milky Way. Thus, the total mass M of the entire universe can be roughly estimated as follows $M \approx 2 \cdot 10^{12} \cdot 4 \cdot 10^{11} \cdot M_\odot = 16 \cdot 10^{53}$ kg. Since the volume of 3-sphere is $V = 2\pi^2 a^3 = M/\rho$ (see [13, p. 114]), we get $\pi^2 a^3 \approx 8 \cdot 10^{53+26}$ m^3. Hence, for the present value of the radius of $\mathbb{S}^3_a$ we obtain the following estimate

$$a = a(t_0) \approx \sqrt[3]{8 \cdot 10^{78}} \text{ m} = 2 \cdot 10^{26} \text{ m}, \tag{7}$$

see the blue circle on the right of Figure 1. Substituting (4), (6), and (7) into (5), we get for $t = t_0$ that

$$\boxed{\ddot{a}(t_0) \approx 0.6 \cdot 10^{-9} \text{ m/s}^2} \tag{8}$$

even though the universe contains 70 % of dark energy responsible for its accelerated expansion.

Taking into account that one sidereal year has 31 558 149.54 s and that 1 pc $=$ $3.086 \cdot 10^{16}$ m, we find from (8) that the current expansion rate of the radius of the 3-sphere is

$$a(t_0) \approx 0.6 \cdot 10^{-9} \cdot (3.155814954 \cdot 10^{13})^2/(3.086 \cdot 10^{16}) \text{ pc/Myr}^2 \approx 19 \text{ pc/Myr}^2.$$

3. Concluding remarks

We often hear that the universe has no center. This is similar to the statement that a circle has no center. The circle, of course, has its center even though it does not belong to it. In Figure 1, the centers of the blue circles lie on the axis t. Therefore, also the model of the universe $\mathbb{S}^3_a$ has its center at the origin $(0, 0, 0, 0)$ of coordinates of the space $\mathbb{E}^4$ although $(0, 0, 0, 0) \notin \mathbb{S}^3_a$. On the other hand, the Earth is at the center (see the yellow vertex in Figure 1) of the observable universe. Its horizon can be modeled by a two-dimensional sphere. The center of the red model of the expanding universe corresponds to the Big Bang at an initial time. An enormous artificial magnification is connected with the Big Bang itself, which appeared roughly 13.8 Gyr ago. Although it happened in a minimal volume, its present position is on the possibly greatest 2-sphere (the so-called *horizon*) with an unimaginable large radius.

Remark 3. We showed that the acceleration of the radius of the universe is less than one nanometer per square second, see (8). This seems to be a completely negligible value, but we must not forget that this tiny value has been acting for about 7 billion years according to the standard ΛCDM model. From this cumulative effect, we obtain measurable values of expansion at large cosmological distances. Moreover, all numerical values given in Section 2 should be taken "with a grain of salt," as only approximate models burdened by a number of various errors were employed, see [25].

Remark 4. From (2) we see that the current value (4) of the deceleration parameter $q_0 = q(t_0)$ appears at the quadratic term in the Taylor expansion at the time instant t_0,

$$\begin{aligned} a(t) &= a(t_0) + \dot{a}(t_0)(t - t_0) + \tfrac{1}{2}\ddot{a}(t_0)(t - t_0)^2 + \dots \\ &= a(t_0)(1 + H_0(t - t_0) - \tfrac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots), \end{aligned}$$

Therefore, it was necessary to obtain spectra of type Ia supernovae, which are very distant (with redshifts $z > 0.4$). According to [12, p. 207], [13, p. 162], we have

$$|H_0(t - t_0)| \gg \frac{1}{2}|q_0| H_0^2 (t - t_0)^2,$$

which means that the linear term in the Taylor expansion is much larger than the quadratic term during the last 7 Gyr. Neglecting higher order terms, we find that the expansion function is almost linear in this time interval.

Remark 5. Friedmann equation can be written in the form (see [13, p. 153] for details)

$$\dot{a}^2 = Aa^2 + B + \frac{C}{a} \tag{9}$$

with time independent constant coefficients A, B, C, where $A = \frac{1}{3}\Lambda c^2$, c is the speed of light in a vacuum, and the terminal condition in t_0 can be taken from (7) or derived from the Hubble constant H_0, see [13, p. 154]. We observe that the cosmological constant Λ stands at the major quadratic term a^2 which dominates for large t's.

Remark 6. To date, we do not know any significant digit of Λ, even though many authors [15, 16, 17, 19] have attempted to measure it. We do not even know whether Λ is a fundamental physical constant that actually exists, or whether it is a mere parameter in the current standard cosmological model.

Remark 7. Dividing the both sides of (9) by $\dot{a}^2$, we get the so-called *normalized Friedmann equation*. Suppose that $\dot{a}(t_1) = 0$ for some t_1 which can happen for oscillating or loitering universe. Then all three terms on the right-hand side of (9) are divided by zero, i.e., we would have an infinite density of dark energy $Aa^2/\dot{a}^2$ even though the expansion stops: $\dot{H}(t_1) = 0$. This shows that the model (9) is not a good approximation of the physical universe.

Remark 8. A negative value of the deceleration parameter $q < 0$ was already predicted by the New Zealander Beatrice Tinsley (1941–1981) at the end of the seventies, see [24]. Indeed, if the expansion function $a = a(t)$ were to be concave everywhere (see the red graph in Figure 2) and $\dot{a}(t_0) > 0$, then by (6) and (1) the following estimate would hold for the truth age t_0 of the universe,

$$t_0 \leq T_0 = \frac{a(t_0)}{\dot{a}(t_0)} = H_0^{-1} \approx 4.41 \cdot 10^{17} \text{ s} \approx 13.97 \text{ Gyr}. \tag{10}$$

However, the inequality (10) contradicts observations, because there are stars which were judged to be older than 14.5 Gyr independently of cosmological models. Consequently, already in 1978, Tinsley found [24] that the expansion function must be somewhere strictly convex. Finally note that the strict convexity of the expansion function at present time has already been predicted by Georges Lemaître in 1934, see [8, p. 30] and [14].

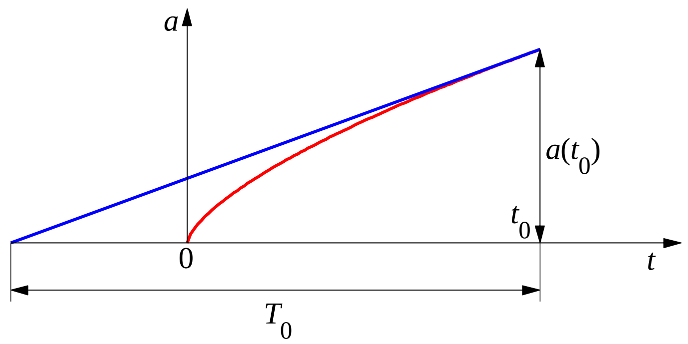

Figure 2: If the expansion function were concave everywhere, then its (red) graph would be below the tangent line passing through the point $(t_0, a(t_0))$. Therefore, for $\dot{a}(t_0) > 0$ by (10) the age of the universe t_0 would not exceed the *Hubble age* $T_0 := 1/H_0 = 13.97$ billion years, which is not in accordance with the measured data.

Remark 9. According to [1, 21], we should consider the extinction of light by host galaxies. The measured luminosity depends considerably on whether the supernova lies inside or on the edge of the galaxy, the direction of its rotation axis, etc. Thus, type Ia supernovae are standard candles only very roughly.

Remark 10. The second author of this article met Brian Schmidt at the IAU General Assembly in Beijing in 2012 and asked him: *Do you cite Beatrice Tinsley's 1975 article in your works?* He answered: *No, since her data were not relevant.* Note that Perlmutter et al. in [15, 16] also do not cite Tinsley's 1975 article.

Remark 11. M. Carrera and D. Giulini [2, p. 175] derive that at the distance of Pluto (i.e. about 40 au) the acceleration of the expansion of the universe is only $\ddot{a}_{\text{Pluto}} \approx 2 \cdot 10^{-23}$ m/s^2 which is indeed an entirely negligible quantity. According to (7), this value nicely fits to (8), namely,

$$\ddot{a}_{\text{Pluto}} \approx \frac{40 \cdot 150 \cdot 10^9}{2 \cdot 10^{26}} 0.6 \cdot 10^{-9} \text{ m/s}^2 = 1.8 \cdot 10^{-23} \text{ m/s}^2.$$

However, Carrera and Giulini did not consider a relatively large value of the Hubble constant H_0 which stands at the linear term in the above Taylor expansion. In other words, an accelerated expansion does not manifest itself on scales of the Solar system, but the expansion itself is observable, see e.g. [4, 12, 13, 23].

Remark 12. The commonly used term *Big Rip* contradicts the Friedmann equation (9), since

$$\dot{a} = \sqrt{Aa^2 + B + \frac{C}{a}} < 2\sqrt{A}a$$

holds for sufficiently large a. The function $b(t) = \exp(2\sqrt{A}t)$ is the solution of the upper bound equation $\dot{b} = 2\sqrt{A}b$ and thus

$$0 < a(t) < b(t) \quad \text{for } t \to \infty.$$

So we see that the radius $a(t)$ is a finite number for any time instant and thus, no Big Rip can be described by (9).

Remark 13. According to [3, 10, 23] the global geometry of the physical universe can be described by an expanding sphere $\mathbb{S}^3_a$. One of the main reasons is that the JWST recently discovered several very distant and bright galaxies at a distance of ≥ 13 Gyr. This fact can be explained by the spacetime-lens principle valid on $\mathbb{S}^3_a$ (see [9]), which causes three artificial magnification effects (see [10] and also Figures 1 and 3). In the case of Euclidean or hyperbolic geometry, we would observe many more galaxies at distance of ≥ 13 Gyr. The spectime-lens principle can also explain the existence of the massive galaxy cluster JADES-ID1 in the early universe at a redshift of $z = 5.68$. From a statistical standpoint, such a massive structure could not have formed. However, this cluster is not actually that large if we accept $\mathbb{S}^3$ geometry at each time instant.

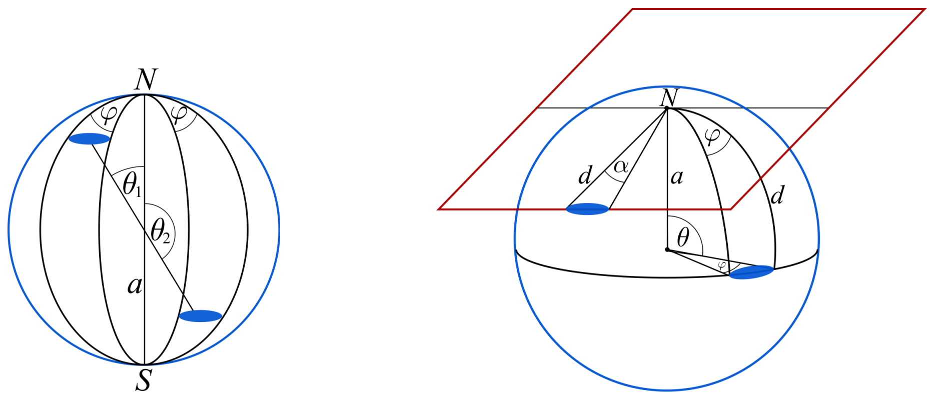

Figure 3: Left: Two galaxies having the same diameter and the same bolometric luminosity are seen from the North Pole N of the sphere $\mathbb{S}^2_a$ under the same angle φ when the radius a is fixed. Their observed luminosity flux will also be the same although their comoving distances expressed in the spherical coordinates $\theta_1 = 30°$ and $\theta_2 = 150°$ differ five times. The viewing angle $\varphi > 0$ can be arbitrarily small. The magnification takes place along the whole curved trajectory, i.e. at any point.

Right: Angular size of a galaxy (except for MW) is indirectly proportional to its distance d if the space is modeled by $\mathbb{E}^3$ for a small viewing angle α measured in radians. This dependency is completely different for the 3-sphere with the viewing angle φ, where $\alpha = \frac{D}{d} = \frac{D}{a\theta} < \frac{D}{a \sin\theta} =: \varphi$, where D is the diameter of a galaxy. The space dimensions are reduced to two.

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AN UNCERTAINTY RELATION FOR RETARDED GRAVITY

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Abstract: In recent years retarded gravity has explained many of the mysteries surrounding the "missing mass" related to galactic rotation curves, the Tully-Fisher relations, and gravitational lensing phenomena. Indeed 143 galaxies analyzed, has demonstrated that retarded gravity will suffice to explain the galaxies rotation curves without the need to postulate dark matter for multiple types of galaxies. Moreover, it also demystified the "missing mass" related to galactic clusters and elliptic galaxies in which excess matter wad derived through the virial theorem. Here we give a mathematical criterion which specifies the cases for which retardation is important for gravity (and when it is not). The criterion takes the form of an "uncertainty" relation.

Keywords: galactic dynamics, retarded gravity, "dark matter"

PACS: 98.62.Dm; 95.35.+d; 04.20.-q

1. Introduction

From a practical standpoint, general relativity (GR) has been validated by numerous observations spanning various fields. However, its current status presents challenges. While supported by substantial observational evidence, GR faces significant hurdles. Its verifications in cosmology and astrophysics are under scrutiny, primarily because it relies on unproven concepts like dark matter and energy to explain phenomena on a large scale, such as galaxies and the universe. Often, these unconfirmed elements are employed while simultaneously overlooking a crucial aspect of GR: retardation, which contradicts Newtonian principles of action at a distance. This discrepancy may be connected to the problems we have understanding gravitational interactions on cosmic scales.

The mystery surrounding dark matter has long been a topic of discussion within the astronomical community, dating back to the 1930s, and possibly even earlier in the 1920s when it was referred to as the "question of missing mass." Over time, this enigma has only grown more prominent, particularly as the need for dark matter (and the tendency to overlook retardation) has increased on larger scales under examination. Despite extensive and costly efforts, including a forty-year search conducted underground and using accelerators, dark matter's existence remains unproven. Recent years have only added to the challenge, as the Large Hadron Collider's failure to detect any supersymmetric particlesa favored form of dark matter among astroparticle physicistsposes further complications. These particles are not only crucial for understanding dark matter but also play a pivotal role in string theory, which is anticipated to provide insights into the quantization of gravity.

As far back as 1933, Zwicky observed anomalies in the velocities of galaxies within the Coma Cluster that exceeded predictions based on Newtonian theory. He calculated [1, 2] that the required amount of matter to explain these velocities could be 400 times greater than that of visible matter, although later adjustments mitigated this discrepancy to some extent. Had Zwicky utilized the concept of retarded gravity in his calculations, this issue might have been resolved without significant complication [3]. In 1959, Volders [4] noted similar discrepancies on a smaller scale within the outer rims of the nearby spiral galaxy M33, where velocities did not follow the expected $1/\sqrt{r}$ pattern. This observation was corroborated in subsequent years by Rubin and Ford [5, 6], who demonstrated that velocities at the outer rim of many spiral galaxies either plateaued or continued to increase, each at a different velocity. Previous studies have indicated that such velocity patterns can be directly inferred from general relativity (GR) if retardation effects are considered [7, 8, 9, 10, 11, 12, 13, 14]. The mechanism underlying retardation is intricately linked to the dynamics of matter density within galaxies, specifically to the second derivative of density. Changes in density may result from various factors, including gas depletion in surrounding intergalactic gas [12] or dynamic processes such as star formation and supernova explosions [10, 11]. These processes can be characterized by three different length scales: the density gradient length, velocity field gradient length, and dynamical length scale. The importance of retardation is determined by the shortest among these length scales [15].

The well-known Tully-Fisher relation [16], which links the baryonic mass of a galaxy to the fourth power of its rotational velocity at the outer rim, can also be derived from the principles of retarded gravity [17]. It was shown that the effects of retarded gravity extend beyond just slowly moving particles and also apply to photons. While there may be some differences in the mathematical analysis for each case, it is ultimately concluded that the observed "dark mass" inferred from galactic rotation curves must be identical in both the lensing and rotation curves scenarios [15, 18].

While the prevailing notion of dark matter remains prominent, the current circumstances warrant consideration of the possibility that this prevailing paradigm may need to be reevaluated. Several challenges cast doubt on this common idea:

Firstly, in order to align with observed phenomena and structure formation simulations, a set of properties has been assigned to dark matter (DM) [2]. However, despite being over 50 years since its inception, dark matter has yet to be directly observed, nor have any known particles been identified that match its purported properties.

Secondly, simulations involving dark matter often encounter what is known as the core-cusp problem. The Navarro-Frenk-White (NFW) [19] profile, derived from Cold Dark Matter (CDM) simulations and commonly used to fit rotation curves, is a prime example. However, this profile faces challenges, particularly when applied to Low Surface Brightness galaxies (LSBs). Predictions made by the NFW profile regarding rotational velocities frequently diverge from actual observations, leading to discrepancies. Specifically, while the NFW profile anticipates a "cuspy" inner region for a dark halo (where density changes rapidly), observations tend to favor a "core-like" behavior (where density remains approximately constant). Efforts to address this issue have often relied on specific and somewhat contrived adjustments, raising doubts about whether these solutions were devised primarily to maintain the current paradigm.

Thirdly, Sancisi's Law [20] presents a significant and broadly applicable observation. It suggests that changes in the luminosity profile of a galaxy correspond to changes in its rotation curve, and vice versa. This phenomenon applies to various types of dark halos. However, from a dark matter perspective, this relationship is unexpected: the dark halo is typically assumed to be much more massive than the baryonic matter. Consequently, fluctuations in the distribution of baryonic matter should not significantly affect the velocity distribution, contrary to what is observed. This discrepancy is particularly pronounced in LSBs, where the dark halo is believed to dominate at every radius, yet the velocity distribution exhibits fluctuations corresponding to each "baryonic bump". This suggests that, somehow, the overall velocity distribution is influenced by small fluctuations in baryonic matter.

Hence, the current retarded gravity proposition offers a unique perspective. Unlike alternative theories that propose modifications to general relativity, such as Milgroms Modified Newtonian Dynamics (MOND) [21], Mannheims Conformal Gravity [22, 23], or Moffats Modified Gravity (MOG) [24], our approach does not seek to alter the fundamental framework of general relativity. Instead, we adhere strictly to the principle of Occams razor, as advocated by both Newton and Einstein (but was also recommended by Aristotle which preceded even Occam himself). Our objective is to replace the need for dark matter with phenomena inherent within standard General Relativity itself (retardation). It's worth noting that recent research has shed light on the relationship between retardation and MOND [25], demonstrating how criteria for low acceleration MOND can be derived from retardation theory, and how the MOND interpolation function can approximate retarded gravity effectively.

It is essential to highlight that significant retardation effects are not contingent upon high velocities of matter within galaxies, although higher velocities may enhance these effects. In reality, the majority of galactic constituents, such as stars and gas, move relatively slowly compared to the speed of light. This is indicated by the ratio of the velocity v to the speed of light in vacuum c, denoted as v/c, which is much smaller than 1. For instance, typical velocities within galaxies are around 100 km/s, resulting in a ratio of 0.001 or smaller. This will be discussed in more details in the sections that follow.

In contrast to the Solar system, where retardation effects are considered negligible [26, 27], observations of galaxies' velocity curves suggest that these effects become significant beyond a certain distance [12, 10, 11]. Recent research [29] has expanded the empirical basis for the theory of retarded gravity. Building upon previous studies that analyzed eleven galaxies [10, 11, 28], the latest research extends its scope to a larger sample of 143 galaxies sourced from the SPARC Galaxy collection. These galaxies vary in type, size, and luminosity. The analysis indicates that in most cases, an excellent fit to the observed data is achieved without the need to postulate dark matter or modify general relativity (see Figure 1 for some examples).

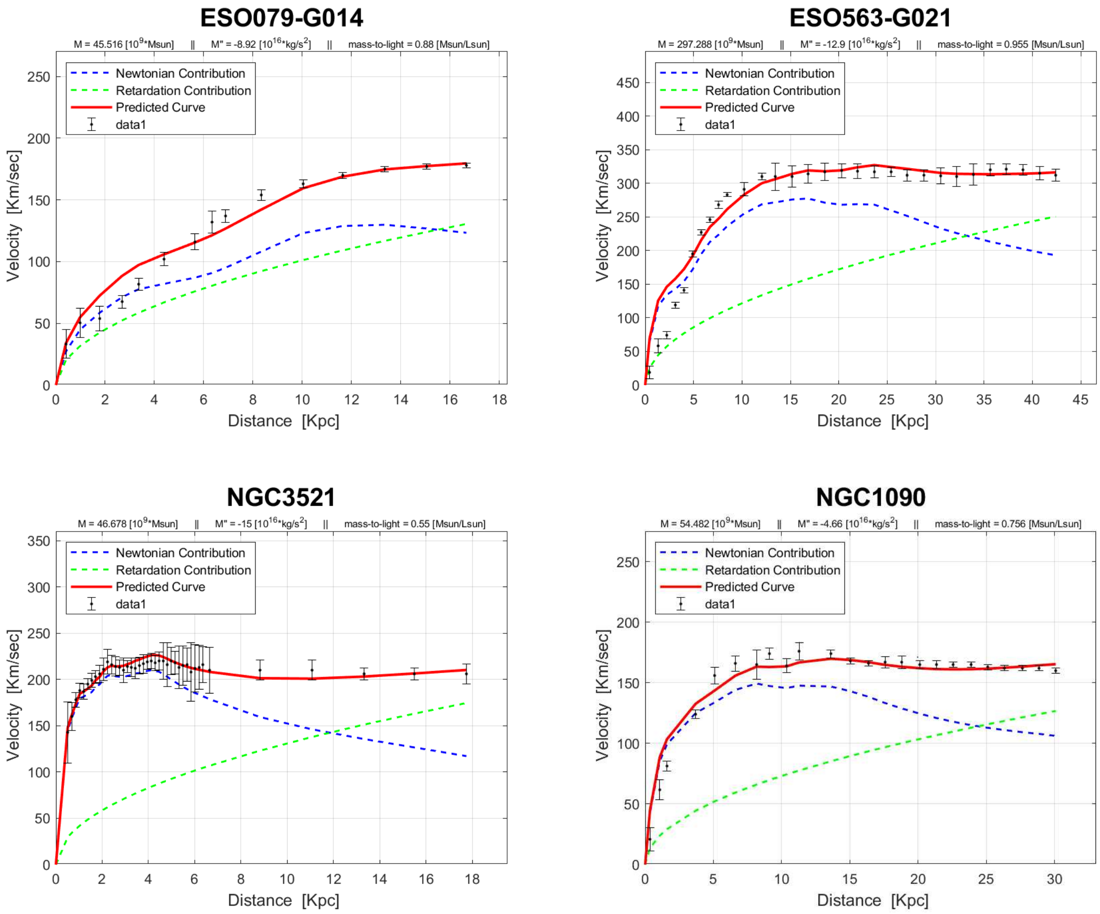

Figure 1: The rotation curves of 'Sbc' type galaxies

As we shall show below this is not an accident but is rather dictated by general relativity.

2. A discrete model

Retarded gravity emerges from the weak field approximation of general relativity [12]. In this framework, the metric perturbation h_{00} can be expressed in terms of a retarded potential ϕ as follows [12, 15]:

$$\phi = -G \int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x', \quad \phi \equiv \frac{c^2}{2} h_{00}, \quad h_{00} = \frac{2}{c^2}\phi, \tag{1}$$

where G represents the gravitational constant, $\vec{x}$ denotes the location where the potential is measured, $\vec{x}'$ signifies the location of the mass element generating the potential, $\vec{R} \equiv \vec{x} - \vec{x}'$, $R \equiv |\vec{R}|$, and ρ represents the mass density. The characteristic duration R/c for galaxies might span a few tens of thousands of years but can be considered short relative to the timescale over which galactic density significantly changes. Therefore, we can express the density using a Taylor series expansion:

$$\rho(\vec{x}', t - \frac{R}{c}) = \sum_{n=0}^{\infty} \frac{1}{n!} \rho^{(n)}(\vec{x}', t)(-\frac{R}{c})^n, \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}. \tag{2}$$

By substituting equation (2) into equation (1) and retaining the first three terms, we can derive:

$$\phi = -G \int \frac{\rho(\vec{x}', t)}{R} d^3x' + \frac{G}{c} \int \rho^{(1)}(\vec{x}', t) d^3x' - \frac{G}{2c^2} \int R \rho^{(2)}(\vec{x}', t) d^3x'. \tag{3}$$

The initial term in the series is referred to as the Newtonian potential:

$$\phi_N = -G \int \frac{\rho(\vec{x}', t)}{R} d^3x'. \tag{4}$$

The second term does not influence the force acting on subluminal particles, since its gradient is zero. As for the third term, it serves as a lower-order correction to the Newtonian potential,

$$\phi_r = -\frac{G}{2c^2} \int R \rho^{(2)}(\vec{x}', t) d^3x'. \tag{5}$$

The geodesic equation governing the motion of a "slow" test particle in the given space-time metric can be approximated by utilizing the force per unit mass, as described in [12]:

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi. \tag{6}$$

The total acceleration is thus:

$$\begin{aligned} \vec{a} &= \vec{a}_N + \vec{a}_r, \\ \vec{a}_N &\equiv -\vec{\nabla}\phi_N = -G \int \frac{\rho(\vec{x}', t)}{R^2} \hat{R} d^3x', \quad \hat{R} \equiv \frac{\vec{R}}{R}, \\ \vec{a}_r &\equiv -\vec{\nabla}\phi_r = -\frac{G}{2c^2} \int \rho^{(2)}(\vec{x}', t) \hat{R} d^3x'. \end{aligned} \tag{7}$$

Now, let us examine a point particle with a mass m_j positioned at $\vec{r}_j(t)$. Such a particle will possess a mass density given by:

$$\rho_j = m_j \delta^{(3)}(\vec{x}' - \vec{r}_j(t)). \tag{8}$$

Here, $\delta^{(3)}$ represents a three-dimensional Dirac delta function. This particle induces a Newtonian potential given by:

$$\phi_{Nj} = -G\frac{m_j}{R_j(t)}, \qquad \vec{R}_j(t) = \vec{x} - \vec{r}_j(t), \qquad R_j(t) = |\vec{R}_j(t)| \tag{9}$$

and a retardation potential in the following form:

$$\begin{aligned} \phi_{rj} &= -\frac{Gm_j}{2c^2}\frac{\partial^2}{\partial t^2}R_j(t) = \frac{Gm_j}{2c^2}\left(\hat{R}_j \cdot \vec{a}_j - \frac{\vec{v}_j^2 - (\vec{v}_j \cdot \hat{R}_j)^2}{R_j(t)}\right), \\ \hat{R}_j &\equiv \frac{\vec{R}_j}{R_j}, \ \vec{v}_j \equiv \frac{d\vec{r}_j}{dt}, \ \vec{a}_j \equiv \frac{d\vec{v}_j}{dt}. \end{aligned} \tag{10}$$

Thus any point particle moving at the vicinity of particle j will be accelerated as follows:

$$\begin{aligned} \vec{a}_{Tj} &= \vec{a}_{Nj} + \vec{a}_{rj}, \\ \vec{a}_{Nj} &= -\vec{\nabla}\phi_{Nj} = -G\frac{m_j}{R_j^2}\hat{R}_j, \ \vec{a}_{rj} = -\vec{\nabla}\phi_r = \frac{Gm_j}{2R_j^2c^2}\left(R_j\vec{a}_{\perp j} + \hat{R}_j\vec{v}_{\perp j}^2 - 2(\vec{v}_j \cdot \hat{R}_j)\vec{v}_{\perp j}\right) \\ \vec{a}_{\perp j} &\equiv \vec{a}_j - (\vec{a}_j \cdot \hat{R}_j)\hat{R}_j, \qquad \vec{v}_{\perp j} \equiv \vec{v}_j - (\vec{v}_j \cdot \hat{R}_j)\hat{R}_j. \end{aligned} \tag{11}$$

in which the reader should not confuse the acceleration of the point particle j denoted by $\vec{a}_j$ and the acceleration caused by particle on a test particle located at point $\vec{x}$ denoted by $\vec{a}_{Tj}$.

3. The uncertainty relation of retarded gravity

First we notice that:

$$a_{Nj} = |\vec{a}_{Nj}| = G\frac{m_j}{R_j^2} \quad \Rightarrow \quad \vec{a}_{rj} = a_{Nj}\frac{\left(R_j\vec{a}_{\perp j} + \hat{R}_j\vec{v}_{\perp j}^2 - 2(\vec{v}_j \cdot \hat{R}_j)\vec{v}_{\perp j}\right)}{2c^2}. \tag{12}$$

For non relativistic matter:

$$\beta \equiv \frac{v}{c} \ll 1, \tag{13}$$

hence we may write approximately:

$$\vec{a}_{rj} \simeq a_{Nj}\frac{R_j\vec{a}_{\perp j}}{2c^2}, \qquad \vec{a}_{Tj} = \vec{a}_{Nj} + \vec{a}_{rj} \simeq a_{Nj}\left(-\hat{R}_j + \frac{R_j\vec{a}_{\perp j}}{2c^2}\right). \tag{14}$$

Thus in order for retarded gravity to have a significant effect $R_j \vec{a}_{\perp j}/(2c^2)$ must be of the order of unity ($|\hat{R}_j| = 1$) or larger, this leads to the retarded gravity "uncertainty" type relation [30] (this is only a superficial similarity no connection is intended to the statistical content of Heisenberg uncertainty relation of quantum mechanics):

$$\frac{R_j a_{\perp j}}{2c^2} > 1 \quad \Rightarrow \quad R_j a_{\perp j} > 2c^2 \quad \Rightarrow \quad a_{\perp j} > a_c = \frac{2c^2}{R_j}. \tag{15}$$

Consider a point mass located on a circle which serves as border of the M33 galaxy, then we may ask what will be the amount of acceleration suffered by the point mass that will cause a retardation effect on a test particle located across the diameter of the galaxy (which is the furthest point on the imaginary circle from the point mass). Now the radius of the galaxy M33 is $Rs \simeq 30\,000$ light years $= 2.8 \cdot 10^{20}$ meters. Hence, we will need an acceleration of about:

$$a_c = \frac{2c^2}{R_j} = \frac{2c^2}{2Rs} = \frac{c^2}{Rs} \simeq 0.00032 \text{ m/s}^2 \tag{16}$$

to observe the effect of retarded gravity. This does not seem to be such a huge acceleration and many point masses (atoms, molecules etc.) in the galaxy may have accelerations that need to be considered in the total galactic balance of gravitational forces. From this point of view we may partition the galactic point masses in to two classes: Newtonian gravity particles and retarded (+Newtonian) gravity particles the difference depends on how big is the Newtonian radius (which depends on the particle's acceleration):

$$\frac{R_j a_{\perp j}}{2c^2} < 1, \qquad R_j < R_{Nj} \equiv \frac{2c^2}{a_{\perp j}}. \tag{17}$$

This reality is depicted in Figure 2. Of course each test particle in not affected by just one massive particle but by all N_p massive particles this leads to the equation:

$$\vec{a}_T = \sum_{j=1}^{N_p} \vec{a}_{Tj} = \sum_{j=1}^{N_p} (\vec{a}_{Nj} + \vec{a}_{rj}) = \sum_{j=1}^{N_p} \vec{a}_{Nj} + \sum_{j=1}^{N_p} \vec{a}_{rj} \simeq \sum_{j=1}^{N_p} \vec{a}_{Nj} + \sum_{j=1, j \in \text{RG}}^{N_p} a_{Nj} \frac{R_j \vec{a}_{\perp j}}{2c^2} \tag{18}$$

in which RG means particles that have a retarded influence in point $\vec{x}$, that is particles in which point $\vec{x}$ is outside their Newtonian radius. We point out that the stellar component of disk galaxies is not responsible for the retardation effects. To see this look at the velocity & acceleration curves of the M33 galaxy depicted in Figure 3: Assuming stars (and gas) moving in circles, we obtain accelerations which do not exceed $1.4 \cdot 10^{-10}$ m/s^2 yielding a Newtonian Radius not smaller $R_N \simeq 4 \cdot 10^7$ kpc, hence the effects of those stars and rotating gas is completely Newtonian within the galaxy. Thus to obtain retardation corrections one must look at other processes taking place in galaxy like stellar winds, supernovae explosions [11] and the depletion of gas in the intergalactic medium [12] leading to the deceleration of the rate of mass accretion by the galaxy and to an attractive gravitational retardation effect.

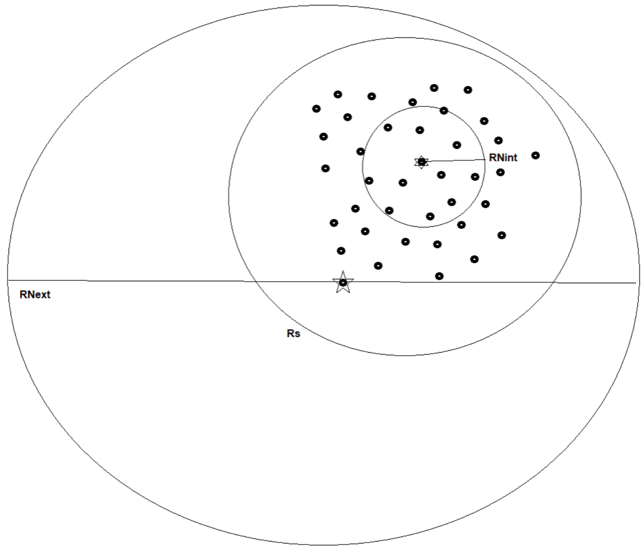

Figure 2: Point particles in the galaxy and their Newtonian spheres of influence. For some point particles $RNint$ is within the galaxy of radius Rs, hence retarded gravity due to those particles does effect the galaxy dynamics, while for other particles $RNext$ contains the entire galaxy hence those particle cause only gravitational Newtonian effects.

4. Evidence for the gas depletion model

The paper [32] (see also [33, 34, 35]) highlights that recent measurements of gas velocity in the outer regions of galaxies at high redshifts indicate a prevalence of steeply declining rotation curves, contrasting with the nearly universal flat rotation curves observed in nearby galaxies. This aligns with the proposition put forth by [12], suggesting that gas depletion plays a role in generating significant $\ddot{M}$ and implies that older galaxies should not exhibit significantly smaller $\ddot{M}$, resulting in steep rather than flat rotation curves. Furthermore, the paper suggests that once a smooth stellar disk forms within the baryonic matter, resembling properties of high-redshift galaxies, the computed rotation curves consistently remain relatively flat at large radii in the gas disk. Strikingly, only simulations devoid of a dark matter halo successfully replicate observed rotation curves. This finding supports our theory, which dismisses the presence of dark matter. Moreover, the paper implies that the flat rotation curves observed in low-redshift galaxies may either result from dark matter falling into the galactic potential well or necessitate an alternative explanation apart from dark matter. Indeed, an alternative explanation, considering retardation, is plausible.

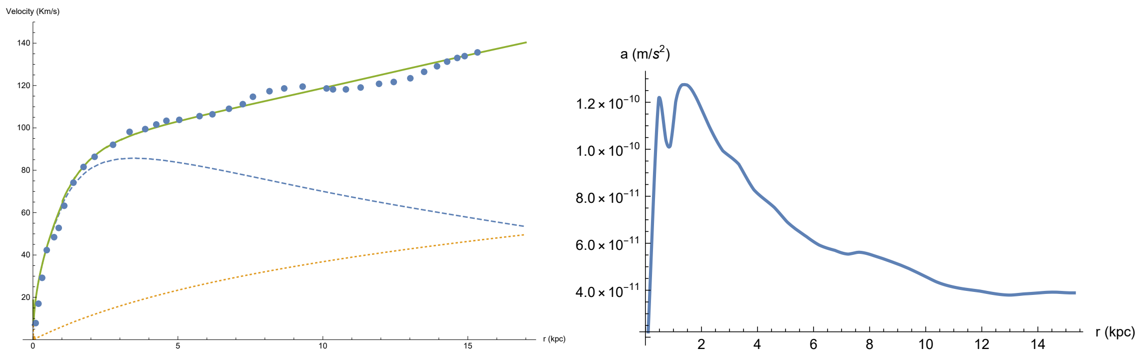

Figure 3: Rotation & Acceleration curves for M33 [31]. In the left-hand side we have the complete rotation curve, depicted by the solid line, represents the combined effect of two contributions: the dotted line represents the contribution from retardation, while the dashed line represents the contribution from Newtonian gravity [12]. In the right-hand side the acceleration assuming that the stars and gas move in circles around the galactic center and thus have an acceleration of $\frac{v^2}{r}$, where r is the distance from the center of the galaxy.

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ON THE VALIDITY OF THE PRINCIPLE OF EQUIVALENCE

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Abstract: We investigate the well-known twin paradox. We present several specific examples showing that relativistic time dilation during uniformly accelerated motion contradicts the equivalence principle over any length of time.

Keywords: time dilation, constant acceleration, equivalence principle, twin paradox.

PACS: 04.20-q; 95.30.Sf; 98.80-k

1. Introduction

At the beginning of the last century, Albert Einstein [1] formulated the principle of equivalence (PE), according to which "inertial mass and gravitational mass are the same", and therefore it is impossible to distinguish between the effects of a homogeneous gravitational field and the constant acceleration of an observer. Einstein proposed that an observer in a closed box is unable to tell whether he stays on Earth or in a rocket traveling with constant acceleration equal to Earth's gravitational acceleration g, i.e.,

$$a = g = 9.81 \text{ m/s}^2.$$

In other words, no physical experiment can demonstrate a difference between these two cases, because they are completely equivalent. In this article, however, we will show that if we measure time over any time interval, no matter how short or long (i.e., locally or globally), we can distinguish between these two cases.

2. The twin paradox in accelerated reference frames

For simplicity, let us assume that the Earth, with time variable $\tilde{t}$, is sufficiently isolated from other gravitational sources, such as a freely floating, non-rotating, completely isolated (rogue) planet. The gravitational field is weak at the Earth's surface, and clocks on Earth tick at a rate similar to that of clocks in a zero-gravity field with time variable t. According to [3, p. 659], the difference in their times after one year will be only

$$\delta = (1 - \sqrt{1 - r/R})T = 6.5 \times 10^{-10} T \approx 0.02 \text{ s}, \tag{1}$$

where $R = 6373$ km is the mean radius of the Earth, $r = 8.87$ mm is its Schwarzschild radius, and

$$T = 31\,558\,149.45 \text{ s} \tag{2}$$

is the sidereal year. The difference between the times $\tilde{t}$ and t after one year can therefore be considered negligible in our analysis, since, according to equation (1), clocks on Earth will lag by approximately 1 second after 50 years.

Next, we will consider the following time-dependent problem, which is spatially one-dimensional. Let us consider twins Adam and Bob, or alternatively, two precise clocks equipped with transmitters and receivers. Throughout the experiment, Adam remains at point x_0 in the reference frame (x, t) fixed to Earth, see Figure 1.

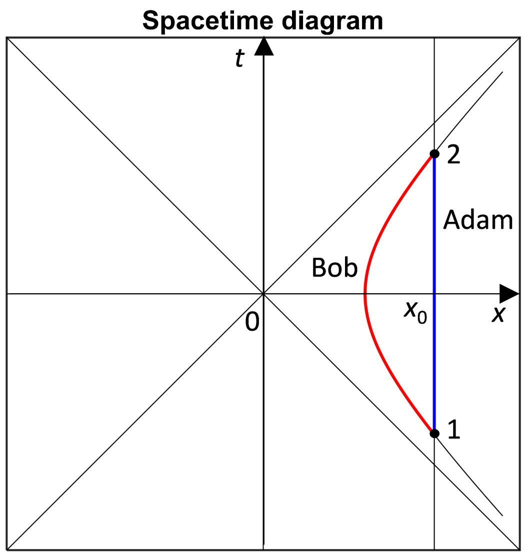

Figure 1: The spacetime diagram for $c = 1$ (ly/yr). The world line of Adam is vertical, whereas Bob's word line is a hyperbola defined by (6).

Let us further assume that Bob is flying in a rocket from the right along the x-axis in the negative direction $\longleftarrow$ toward Earth. Throughout the flight, he will be subjected to a positive constant acceleration $a = g$, just like Adam on the Earth's surface; that is, Bob will not rotate his rocket at any point during the entire experiment. At the moment of their first encounter (see event 1 in Figure 1), Adam and Bob will exchange the time data on their clocks. After this encounter, Bob will decelerate until he comes to a complete stop and then he will accelerate in the opposite direction $\longrightarrow$ toward Earth, where he will again exchange his time data with Adam (see event 2 in Figure 1).

Without loss of generality, we can assume that Adam's spatial coordinate is a fixed constant $x_0 > c^2/a$, where $c = 299\,792\,458$ m/s is the speed of light in a vacuum. This strict inequality ensures that Adam's and Bob's light rays will intersect twice, as will be seen from the relation (4).

Since Bob's constant acceleration is the same as Adam's on the Earth's surface, we can expect their two reference frames to be equivalent under the principle of equivalence. It follows that time should pass at the same rate for both Adam and Bob. Now we will verify whether this statement is true.

Let us denote Bob's proper time by τ and recall that $\tilde{t}$ and t represent Adam's proper coordinate time on Earth and in a weightless state sufficiently far from all sources of gravity. These time variables can be set such that

$$t = \tilde{t} = \tau = 0$$

on the x-axis, see Figure 1, i.e., exactly halfway between events 1 and 2. Here, Bob's velocity is zero for $\tau = 0$, and Bob's world line in Adam's coordinate system satisfies the so-called *Rindler coordinates* (see [3, p. 166], [4]):

$$t = t(\tau) = \frac{c}{a} \sinh \frac{a\tau}{c}, \tag{3}$$

$$x = x(\tau) = \frac{c^2}{a} \cosh \frac{a\tau}{c}. \tag{4}$$

From the relativistic time dilation equation (3), we can calculate by how many seconds Bob's clock will lag behind Adam's clock. Specifically, Adam would be

$$\Delta = 2(t - \tau) \tag{5}$$

older than Bob, which is the time difference between events 1 and 2. From (3) and the Taylor expansion series

$$\sin z = \frac{z}{1!} + \frac{z^3}{3!} + \frac{z^5}{5!} + \dots$$

for $z > 0$, we find that

$$t > \tau.$$

The coefficient 2 in (5) follows from the symmetry of both world lines about the x-axis. By multiplying (3) by the speed c, then squaring equations (3) and (4), and subtracting them, we find that

$$x^2 - c^2t^2 = \frac{c^4}{a^2}. \tag{6}$$

Bob's world line is therefore a hyperbola in the Cartesian coordinate system (x, t), in which Adam has a fixed coordinate x_0, see Figure 1.

Bob's proper time passes more slowly than Adam's, because according to (5), $\Delta > 0$. Adam and Bob can thus easily tell who flew in the rocket and who stayed on Earth. Therefore, the reference frames associated with Adam and Bob are not physically equivalent, because proper time passes at different rates in them, which contradicts the principle of equivalence. Let us illustrate this with three specific examples that lead to an invalid mathematical relation

$$\tau < t = \tau,$$

where the first inequality follows from time dilation and the second equality follows from the equivalence principle, compare with [5].

3. Illustrative examples

Example 1. For $a = g$ according to (4), we have

$$x_0 = x(0) = \frac{c^2}{a} = 9.16 \times 10^{15} \text{ m},$$

which is approximately 1 light year $= 9.46 \times 10^{15}$ m. Let us assume that Bob's half-life time between events 1 and 2 is $\tau = 1$ year, see (2). The argument of the hyperbolic functions in (3)–(4) corresponding to event 2 is equal to $a\tau/c = 1.032$. From equation (4) we then obtain Adam's fixed position $x_0 = x(\tau) = 14.49 \times 10^{15}$ m, and from (3) the corresponding proper time of Adam

$$t = 3.747 \times 10^7 \text{ s} = 1.187 \text{ yr}. \tag{7}$$

On the other hand, from PE we obtain

$$\tilde{t} = \tau = 1 \text{ yr}, \tag{8}$$

which differs from (7) by more than 2 months, since the difference $t - \tilde{t} = 0.02$ s $\approx 6 \times 10^{-10}$ years is, according to (1), clearly a negligible value. This can be written in more detail using (7)–(8):

$$0.187 = t - \tau \le (t - \tilde{t}) + (\tilde{t} - \tau) = t - \tilde{t} = 6 \times 10^{-10}$$

in years, which is a contradiction. What is the main cause of this disagreement? Bob's total flight time 2τ increases linearly, whereas according to (3), Adam's total time $2t$ increases almost exponentially, since $\sinh z$ approaches $\frac{1}{2}\mathrm{e}^z$ as $z \to \infty$.

Example 2. For $a = g = 20.69$ m/s^2 and $\tau = 1$ year, it follows from equation (3) that $t = 2$ years. From the equivalence principle (8), we then obtain the truly absurd equality

$$2 = t = \tau = 1 \tag{9}$$

in years, where the first equality follows from time dilation (3), the second equality follows from the equivalence principle, and the third equality is the initial data. However, according to the equivalence principle, Adam and Bob should age at the same rate, which is not the case.

Furthermore, from (9) one can easily derive that "$1 + 1 = 3$", see [6].

Example 3. We will now consider only the upper half of Bob's hyperbolic world line from Figure 1. According to equation (3), Bob, traveling this time from Earth with constant acceleration g, would need only $\tau = 15$ years to reach a distance of

$$d = x(\tau) - x(0) = \frac{c^2}{g}\Big(\cosh\frac{g\tau}{c} - 1\Big) = 2\,583\,607 \text{ ly}.$$

This is roughly the distance to the Andromeda Galaxy (M31), from which light takes 2.5 million years to reach us, which again contradicts PE. We present further contradictory examples in [2].

4. Conclusions

Examples 1–3 show that neither special nor general relativity is internally consistent:

- either the principle of equivalence does not hold universally,
- or the twin paradox under consideration requires a different interpretation,
- or both of these concepts fail to accurately describe physical reality.

If the phenomena of time dilation and length contraction do not exist in the real universe, then our optimistic plans for interstellar or even intergalactic travel will be significantly limited.

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DIMENSIONLESS FIELD EQUATIONS AND POSSIBLE COSMOLOGICAL EVOLUTION OF FUNDAMENTAL CONSTANTS

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Abstract: We try to reveal a deeper physical meaning of the fundamental physical constants, like the gravitational constant, permittivity of vacuum, speed of light, etc. In the purpose of our goal, we consider the Einstein's field equations for a static, spherically symmetric compact object as well as the equation of motion of a test particle situated in the Newtonian and/or Coulomb force field. All quantities in these equations are usually given in the units defined by humans (e.g. SI units). We transform them into the natural physical units, whereby the Planck units are regarded as the latter. After the transformation was made, all fundamental constants disappeared in the equations. In addition, the equations acquired the dimensionless form. This result implies that the fundamental physical constants are, in fact, the transformation constants. A possible variation of these constants on the cosmological time scale should be considered in this context; an example of possible variation is presented.

Keywords: fundamental physical constants, Planck units, artificial physical units, natural physical units

PACS: 01.55.+b, 06.20.Fn

1. Introduction

A gradual change of some fundamental constants, if existed, would seriously influence the cosmological models of the universe. In this contribution, we discuss the problem of the change from an unusual point of view.

Physical quantities are usually given in the units defined by humans. The length was given in yards, foots, miles, meters, etc. and the mass in, e.g., pounds, grams, kilograms, etc. These units were established historically, in the context of practical needs of common human life. If there was an extraterrestrial civilization in the Andromeda galaxy (M31), we would not be able to communicate the numerical values of the physical quantities expressed in these units with the scientists in the Andromeda.

However, the extraterrestrial scientists would likely know the Planck length, Planck mass, and Planck time. Therefore, they would likely understand the values of the quantities given as the multiples of these Planck's constants. The constants can thus be regarded as the "natural physical units".

In our contribution, we re-write the quantities in the Einstein's field equations (EFEs) given in the SI units to their counterparts given in the natural, Planck, units. We will consider the EFEs applied in a static case and for few well-defined equations of state (EoSs). In addition, we will make such a re-writing also in the classical equation of motion describing an acceleration of a test particle which is initially in rest and is attracted by the forces described by the Newton gravitational and Coulomb electrostatic laws. The purpose of this transformation of the units is to see its impact on the fundamental physical constants.

2. Field equations

In sake of simplicity of our explanation, let us firstly deal with the static, spherically symmetric compact gaseous object. In the case of the spherical symmetry only the diagonal components of the tensors in the EFEs are non-zero and these non-trivial EFEs acquire the form (e.g. [9])

$$\kappa T_1^1 = -\mathrm{e}^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)+\frac{1}{r^2}, \tag{1}$$

$$\kappa T_2^2 = -\mathrm{e}^{-\lambda}\left(\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}+\frac{\nu'-\lambda'}{2r}\right), \tag{2}$$

$$\kappa T_3^3 = -\mathrm{e}^{-\lambda}\left(\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}+\frac{\nu'-\lambda'}{2r}\right), \tag{3}$$

$$\kappa T_4^4 = \mathrm{e}^{-\lambda}\left(\frac{\lambda'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}, \tag{4}$$

where T_μ^σ is the stress-energy tensor, r is the radial distance from the center of the object, λ and ν are the auxiliary quantities related to g_{rr} and g_{tt} components of metric tensor as $g_{rr} = -\mathrm{e}^{\lambda}$ and $g_{tt} = \mathrm{e}^{\nu}$, and κ is the Einstein gravitational constant. With the help of the common, Newton, gravitational constant, G, and speed of light in vacuum, c, it can be given as $\kappa = 8\pi G/c^4$. The prime (double prime) indicates the derivative (second derivative) in respect to r.

If one considers the gaseous object, the stress-energy tensor

$$T_1^1 = T_2^2 = T_3^3 = -P, \qquad T_4^4 = \tilde{\rho}, \qquad T_\mu^\sigma = 0 \text{ for } \mu \neq \sigma \tag{5}$$

is relevant. With this tensor, the EFEs (1)−(4) contain four unknown quantities, P, $\tilde{\rho}$, λ, and ν, which are the functions of r in the static case. Since $T_2^2 = T_3^3$, EFEs (2) and (3) become to be identical. Thus, we have only three equations, in fact, but four unknown quantities remain. The fourth equation must be supplied from the outside of general relativity (GR) to complete the system of equations.

If we establish, in accord with Oppenheimer and Volkoff [8], another auxiliary metric function, $u = u(r)$, defined by relation

$$u = \frac{1}{2} r \left(1 - \mathrm{e}^{-\lambda}\right), \tag{6}$$

With the help of u, EFEs (1), (2) that is identical with (3), and (4) can be re-written to form

$$\frac{du}{dr} = \frac{1}{2} \kappa \tilde{\rho} r^2, \tag{7}$$

$$\frac{d\nu}{dr} = \frac{2}{r^2 - 2ru} \left(\frac{1}{2} \kappa P r^3 + u\right), \tag{8}$$

$$\frac{dP}{dr} = -\frac{\tilde{\rho} + P}{r^2 - 2ur} \left(\frac{1}{2} \kappa P r^3 + u\right) = -\frac{\tilde{\rho} + P}{2} \frac{d\nu}{dr}. \tag{9}$$

In the equations introduced until now, all quantities and constants were implicitly assumed to be given in the SI units. In the following sections, we transform the quantities to the Planck units, i.e. the Planck length

$$L_P = \sqrt{\frac{G\hbar}{c^3}}, \tag{10}$$

Planck mass

$$M_P = \sqrt{\frac{\hbar c}{G}}, \tag{11}$$

and Planck time

$$t_P = \sqrt{\frac{G\hbar}{c^5}} = \frac{L_P}{c}. \tag{12}$$

In these relations, $\hbar = h/(2\pi)$ where h is the Planck constant.

3. Field equations in the dimensionless form

3.1. Equation of state in form of polytrope

Since we intend to deal with a gaseous object, let us consider the fourth equation to complete the system of the EFEs with the EoS in form of polytrope relating pressure, P, with the material density, ρ, as

$$P = K_P \rho^\gamma, \tag{13}$$

where K_P is a constant of proportionality and γ is related to the polytrope index, N, as $\gamma = 1 + 1/N$. In the case of the polytrope, the energy density, $\tilde{\rho}$, is given as [10]

$$\tilde{\rho} = \frac{1}{\gamma - 1} K_P \rho^\gamma + c^2 \rho = NP + c^2 \rho. \tag{14}$$

When a model of the internal structure of object is created, some initial conditions must be defined to determine the initial values of the state quantities. In accord with our earlier discovery, the maximum pressure, P_{max}, and maximum energy density, $\tilde{\rho}_{max}$, occur in so-called zero-gravity distance, r_o, whereby $r_o > 0$. The maximum material density is correlated with the maximum pressure, i.e. it occurs at the distance r_o.

The condition for the local maximum of pressure is $dP/dr = 0$. Seeing relation (9), this condition is satisfied when

$$\frac{1}{2}\kappa P_{max} r_o^3 + u_o = 0. \tag{15}$$

In the last relation, we denoted $u(r_o) = u_o$. If we choose the values of P_{max} and r_o, relation (15) can be used to calculate u_o.

The material density can be calculated as the mass of the particles, constituting the object, in an unit volume. To keep our explanation simple, we consider an object consisting of only one kind of the particles. We denote their mass by symbol m. Since we assume the Planck units as final, let the unit volume is L_P^3 in a flat spacetime. Inside the spherically symmetric object, the radial distance is contracted due to strong gravity, therefore the unit volume is

$$V_1 = L_P^3 \mathrm{e}^{\lambda/2} = \frac{L_P^3}{\sqrt{1 - 2u/r}}. \tag{16}$$

We denote the number of particles in unit volume located in distance r from the object's center by symbol χ. The mass of all particles in the unit volume is $m\chi$ and material density can be given as

$$\rho = \frac{m\chi}{V_1} = \frac{m\chi}{L_P^3}\sqrt{1 - \frac{2u}{r}}. \tag{17}$$

Further, we use the well-known de Broglie relation $mc^2 = \hbar\omega$ between the mass and angular frequency, ω, of the wave corresponding with the particle. This angular frequency is related with the wavelength, l_m, of the wave as $\omega = 2\pi c/l_m$. With the help of these relations, relation (17) can be re-written to

$$\rho = \frac{2\pi\hbar\chi}{cL_P^3 l_m}\sqrt{1 - \frac{2u}{r}}. \tag{18}$$

Denoting the number of the particles in the unit volume located in zero-gravity distance r_o by symbol χ_o, the maximum material density can be given as

$$\rho_{max} = \frac{2\pi\hbar\chi_o}{cL_P^3 l_m}\sqrt{1 - \frac{2u_o}{r_o}} \tag{19}$$

and ratio of the density in distance r and maximum density is

$$\frac{\rho}{\rho_o} = \sqrt{\frac{1-2u/r}{1-2u_o/r_o}\frac{\chi}{\chi_o}}. \tag{20}$$

Using relations (13), (15), and (19), the constant K_P can be expressed as

$$K_P = -\frac{2u_o}{\kappa r_o^3}\rho_{max}^{-\gamma}. \tag{21}$$

Now, the pressure can be given as

$$P = -\frac{u_o}{r_o}\frac{L_P^2}{r_o^2}\left(\sqrt{\frac{1-2u/r}{1-2u_o/r_o}\frac{\chi}{\chi_o}}\right)^{\gamma}\frac{c^7}{4\pi G^2\hbar} \tag{22}$$

and energy density as

$$\tilde{\rho} = -N\frac{u_o}{r_o}\frac{L_P^2}{r_o^2}\left(\sqrt{\frac{1-2u/r}{1-2u_o/r_o}\frac{\chi}{\chi_o}}\right)^{\gamma}\frac{c^7}{4\pi G^2\hbar} + 2\pi\chi\sqrt{1-\frac{2u}{r}}\frac{L_P}{l_m}\frac{c^7}{G^2\hbar}. \tag{23}$$

Supplying the last two relations into the EFEs (7), (8), (9), and after some algebraic handling, the EFEs acquire form

$$\frac{du}{dr} = \left[8\pi^2\chi\sqrt{1-\frac{2u}{r}}\frac{L_P}{l_m} - \frac{u_o}{r_o}\left(\frac{L_P}{r_o}\right)^2\left(\sqrt{\frac{1-2u/r}{1-2u_o/r_o}\frac{\chi}{\chi_o}}\right)^{\gamma}\right]\left(\frac{r}{L_P}\right)^2, \tag{24}$$

$$\frac{d(L_P\nu)}{dr} = \frac{2}{\frac{r}{L_P}-\frac{2u}{L_P}}\left[\frac{u}{r} - \frac{u_o}{r_o}\left(\frac{r}{r_o}\right)^2\left(\sqrt{\frac{1-2u/r}{1-2u_o/r_o}\frac{\chi}{\chi_o}}\right)^{\gamma}\right], \tag{25}$$

$$\frac{1}{\chi}\frac{d(L_P\chi)}{dr} = \frac{\frac{L_P}{r}}{1-\frac{2u}{r}}\frac{du}{dr} + \left(\frac{4\pi^2}{\gamma}\frac{L_P}{l_m}\sqrt{1-\frac{2u}{r}}\chi - \frac{N}{2\gamma}\right)\frac{d(L_P\nu)}{dr} - \frac{\frac{L_P}{r}\frac{u}{r}}{1-\frac{2u}{r}}, \tag{26}$$

respectively.

We can see that the EFEs in the form (24), (25), and (26) are dimensionless. They consist of the dimensionless constants, like π or numbers of particles, χ and χ_o, and fractions with both nominator and denominator consisting of form that has the same unit, meter, or its power.

3.2. EoS of cool, degenerated, Fermi-Dirac neutron gas

Now, let us consider the fourth equation that it the EoS of cool, Fermi-Dirac neutron gas. This EoS was also considered by Oppenheimer and Volkoff [8] when they created the first model of neutron star.

In this case, the EoS is

$$P = \frac{m_n^4 c^5}{96\pi^2\hbar^3}\left(\sinh\tau - 8\sinh\frac{\tau}{2} + 3\tau\right), \tag{27}$$

$$\tilde{\rho} = \frac{m_n^4 c^5}{32\pi^2\hbar^3}\left(\sinh\tau - \tau\right), \tag{28}$$

where the dimensionless auxiliary quantity τ related to the Fermi impulse, p_f, is defined as

$$\tau = 4\ln\left[\frac{p_f}{m_n c} + \sqrt{1 + \left(\frac{p_f}{m_n c}\right)^2}\right]. \tag{29}$$

For this EoS, the EFEs (7), (8), and (9) can be re-written to the dimensionless form [6]

$$\frac{du}{dr} = 2\pi^3\left(\frac{L_P}{l_n}\right)^4\left(\frac{r}{L_P}\right)^2(\sinh\tau - \tau), \tag{30}$$

$$\frac{d(L_P\nu)}{dr} = \frac{1}{1-2u/r}\frac{L_P}{r}\left[\frac{4\pi^3}{3}\left(\frac{L_P}{l_n}\right)^4\left(\frac{r}{L_P}\right)^2\left(\sinh\tau - 8\sinh\frac{\tau}{2} + 3\tau\right) + \frac{2u}{r}\right], \tag{31}$$

$$\frac{d(L_P\tau)}{dr} = -\frac{2}{1-2u/r}\frac{L_P}{r}\frac{\sinh\tau - 2\sinh(\tau/2)}{\cosh\tau - 4\cosh(\tau/2) + 3}\cdot$$
$$\cdot\left[\frac{4\pi^3}{3}\left(\frac{L_P}{l_n}\right)^4\left(\frac{r}{L_P}\right)^2\left(\sinh\tau - 8\sinh\frac{\tau}{2} + 3\tau\right) + \frac{2u}{r}\right], \tag{32}$$

respectively. In these equations, l_n is the wavelength associated with neutron according to well-known de Broglie's relation $m_n c^2 = \hbar\omega_n$ between its mass, m_n, and associated angular frequency, ω_n, whereby $l_n = 2\pi c/\omega_n$.

Equations (30), (31), and (32) also contain only the dimensionless constants and quantity τ, and fractions with both nominator and denominator in the same unit, unit of length or its power, specifically.

3.3. EoS for a radiation fluid

In our earlier paper [7], we demonstrated that, within the GR, it is possible to create a model of compact objects which exclusively consists of photons. Finally, let us introduce the dimensionless EFEs with the fourth equation being the EoS for radiation. This EoS can be written in the form

$$\tilde{\rho} = 3P = aT^4, \tag{33}$$

where $\tilde{\rho}$ is the energy density of radiation fluid, a is the radiation constant, and T is the temperature. Considering a static fluid, with a constant temperature, the dimensionless EFEs are [7]

$$\frac{du}{dr} = \frac{128\pi^3}{27}\frac{\beta_3^4}{\beta_4^3}\left(\frac{L_P}{L_m}\right)^4\left(\frac{r}{L_P}\right)^2, \tag{34}$$

$$\frac{d(L_P\nu)}{dr} = \frac{L_P}{r}\frac{1}{1-2u/r}\left[\frac{256\pi^3}{81}\frac{\beta_3^4}{\beta_4^3}\left(\frac{L_P}{L_m}\right)^4\left(\frac{r}{L_P}\right)^2 + \frac{2u}{r}\right], \tag{35}$$

$$\frac{dL_m}{dr} = \frac{1}{2}\frac{L_m}{L_P}\frac{d(L_P\nu)}{dr}, \tag{36}$$

where $\beta_3 = \sum_{j=1}^{\infty} j^{-3} = 1.202056898$ and $\beta_4 = \sum_{j=1}^{\infty} j^{-4} = \pi^4/90 = 1.082323234$ are dimensionless constants, and L_m is the defined mean wavelength of photons at temperature T. It can be calculated as

$$L_m = \frac{\beta_3 hc}{3\beta_4 kT}. \tag{37}$$

In the last relation, k is the Boltzmann constant.

3.4. Dynamical case: a simple equation of motion

In the previous three subsections, we dealt with the EFEs describing a static object. Let us now demonstrate a dimensionless equation, without the fundamental constants, also in a dynamical case, i.e. in an equation containing an acceleration. Specifically, we are going to deal with the equation giving the acceleration of a charged test particle in the gravitational and electrostatic force fields generated by a charged massive particle (acting particle). We assume that both test particle and acting particle are the point-like particles, which are initially in rest.

The mass of the test particle, m_{tp}, is the sum masses of elementary particles of several kinds, whereby the number of particles of j-th kind is n_j. The mass of the particle of the j-th kind is hereafter denoted by m_j. Similarly, the mass of the acting particle, m_{ap}, also consists of several kinds of elementary particles and there are N_j particles of j-th kind. If the s is number of kinds of the particles, mass of the test particle (acting particle) can be given as $m_{tp} = \sum_{j=1}^{s} n_j m_j$ ($m_{ap} = \sum_{j=1}^{s} N_j m_j$).

In the test particle (acting particle), η_{t+} (η_{a+}) particles have the positive and η_{t-} (η_{a-}) have the negative charge. Hence, the total charge of the test particle (acting particle) is $Q_{tp} = (\eta_{t+} - \eta_{t-})q_o$ ($Q_{ap} = (\eta_{a+} - \eta_{a-})q_o$), where q_o is the elementary electric charge.

Using the well-known de Broglie relation, the mass of the particle of j-th kind can be given with the help of angular frequency, ω_j, of the wave associated with this particle,

$$m_j = \frac{\hbar\omega_j}{c^2}, \tag{38}$$

and further, the angular frequency is related to the wavelength, l_j, of the associated wave as $\omega_j = 2\pi c/l_j$. Hence, mass m_j can be given as

$$m_j = \frac{2\pi\hbar}{cl_j}. \tag{39}$$

The equation of motion of the considered test particle is

$$m_{tp}\frac{\Delta v}{\Delta t} = -G\frac{m_{tp}m_{ap}}{r^2} + \frac{Q_{tp}Q_a}{2/pi\varepsilon_o r^2}, \tag{40}$$

where ε_o is the permittivity of vacuum. After we supply the previously found masses and charges into this equation and also assume $\Delta t = t_P$, it acquires form

$$\left(\sum_{j=1}^{k} n_j \frac{2\pi\hbar}{cl_j}\right) \sqrt{\frac{c^5}{G\hbar}} \Delta v = -\frac{G}{r^2} \left(\sum_{j=1}^{k} n_j \frac{2\pi\hbar}{cl_j}\right) \left(\sum_{j=1}^{k} N_j \frac{2\pi\hbar}{cl_j}\right) +$$
$$+\frac{q_o^2}{4\pi\varepsilon_o r^2} \left(\eta_{t+} - \eta_{t+}\right) \left(\eta_{a+} - \eta_{a-}\right). \tag{41}$$

When this equation is multiplied with $L_P/(2\pi c)$ and after some handling, it is changed to

$$\left(\sum_{j=1}^{k} n_j \frac{L_P}{l_j}\right) \frac{\Delta v}{c} = -2\pi \left(\frac{L_P}{r}\right)^2 \left(\sum_{j=1}^{k} n_j \frac{L_P}{l_j}\right) \left(\sum_{j=1}^{k} N_j \frac{L_P}{l_j}\right) +$$
$$+\frac{\alpha}{2\pi} \left(\frac{L_P}{r}\right)^2 \left(\eta_{t+} - \eta_{t-}\right) \left(\eta_{a+} - \eta_{a-}\right). \tag{42}$$

In the last equation, $\alpha = q_o^2/(4\pi\varepsilon_o\hbar c)$ is the dimensionless fine structure constant.

Further, let us analyze the ratio $\Delta v/c$ on the left-hand side of equation (42). Speed of light, c, says about the length of trajectory of photon during a time interval. With the help of the Planck units, $c = L_P/t_P$. It means that the photon travels the distance of L_P during the Planck time t_P.

Speed of the test particle, v, is a distance traveled by this particle during a unit time interval and Δv is change of this distance during the interval. We assume that this time interval is t_P and, further, we consider the differential corresponding to the second derivative to give an explicit explanation. In one-dimensional space, the test particle is assumed to travel from point A in distance r_1 from the origin of coordinate frame to point B which is located in distance r_2 during time interval equal to t_P. During the subsequent time interval t_P, the particle travels from point B to point C in distance r_3. Hence, the differential is

$$\Delta v = \frac{r_1 - r_2}{t_P} - \frac{r_2 - r_3}{t_P}. \tag{43}$$

In other words, when a photon passes the Planck length, the length of the trajectory passed by the test particle changes from $r_1 - r_2$ to $r_2 - r_3$.

In term of mathematics, ratio $\Delta v/c$ can be expressed as

$$\frac{\Delta v}{c} = \left(\frac{r_1 - r_2}{t_P} - \frac{r_2 - r_3}{t_P}\right) \left(\frac{L_P}{t_P}\right)^{-1} = \left(\frac{r_1}{L_P} - \frac{r_2}{L_P}\right) - \left(\frac{r_2}{L_P} - \frac{r_3}{L_P}\right). \tag{44}$$

Using the latter, equation (42) can be re-written as

$$\left(\sum_{j=1}^{k} n_j \frac{L_P}{l_j}\right) \left[\left(\frac{r_1}{L_P} - \frac{r_2}{L_P}\right) - \left(\frac{r_2}{L_P} - \frac{r_3}{L_P}\right)\right] =$$

$$= -2\pi \left(\frac{L_P}{r}\right)^2 \left(\sum_{j=1}^{k} n_j \frac{L_P}{l_j}\right) \left(\sum_{j=1}^{k} N_j \frac{L_P}{l_j}\right) +$$

$$+\frac{\alpha}{2\pi} \left(\frac{L_P}{r}\right)^2 (\eta_{t+} - \eta_{t-}) (\eta_{a+} - \eta_{a-}) . \tag{45}$$

After all these handling, we see that the equation of motion is dimensionless.

4. Representation of fundamental constants

The initial EFEs (7), (8), and (9) as well as the initial equation of motion (40) contained quantities like pressure, energy density, metric function u, mass, and electric charge. These quantities were implicitly assumed to be expressed in a system of units defined by humans, e.g. in the SI units.

In addition, the initial equations contained the fundamental physical constants like speed of light, Einstein and Newton gravitational constants, Planck constant, and permittivity of vacuum. The values of these constants are also assumed to be given in a human-defined units.

We demonstrated that the quantities figuring in the equations can be expressed only with the help of the Planck length, which can be regarded as a natural unit. Interestingly, all fundamental physical constants disappeared in the final, dimensionless equations. It happened in the case of the EFEs with polytrope EoS (Sect. 3.1), Eqs. (24), (25), and (26)), EoS of cool, degenerated, Fermi-Dirac neutron gas (Sect. 3.2, Eqs. (30), (31), and (32)), EoS of radiation (Sect. 3.3, Eqs. (34), (35), and (36)), as well as in the case of the equation of motion of test particle in the Newton and Coulomb force fields (Sect. 3.4, Eq. (45)).

In addition, the quantity in all final equations, that is not dimensionless, is exclusively the length and it is given as the multiple of the Planck length. The EFEs in this form satisfy the Einstein's demand on his theory of gravity to be exclusively a geometric theory.

Because the fundamental constants are no longer present in the final equations, we can regard these constants as the transformation constants between the artificial system of units defined by humans (e.g. the SI units) and natural physical units, i.e. the Planck units. If the physical quantities are given in the natural units, no transformation is needed and, hence, no transformation constants are needed.

5. Cosmological variability of fundamental constants?

Are the fundamental physical constants actually constant during the history of the universe? Physicists have pondered whether the fine-structure constant, α, is in fact constant, or whether its value differs over time. A varying α has been proposed as a way of solving some problems in cosmology and astrophysics [5, 1, 2, 3, 4]. The fine-structure constant can be given with the help of the elementary electric charge, permittivity of vacuum, Planck constant, and speed of light as

$$\alpha = \frac{q_o^2}{4\pi\varepsilon_o\hbar c}. \tag{46}$$

If α was variable, q_o, ε_o, h, c, and other constants could be variable as well.

In the context of our conclusion at the end of Sect. 4, only a variability of the Planck units is reasonable. Of course, in practice, we measure the fundamental constants and calculate the Planck units using them. A variability of one or more Planck units would then be reflected in the change of measured constants. However, the change of the constants cannot be independent. The definitions of the Planck units expressed with relations (10), (11), and (12) must be satisfied.

Until the present, no change of any constant has been reliably detected, therefore we can make only some speculative assumptions. For example, we can assume that the Planck length and Planck time change in cosmological time scale as

$$L_P = L_o\phi, \tag{47}$$

$$t_P = t_o\phi, \tag{48}$$

where L_o and t_o are true constants and $\phi = \phi(t)$ is a function of time, whereby $\phi(t_{now}) = 1$ at the present time, t_{now}. Under this assumption, speed of light, $c = L_P/t_P = L_o/t_o$ is also true constant.

Since the energy conservation law is expected to be always valid, the Planck energy

$$E_P = M_P c^2 = \sqrt{\frac{\hbar c^5}{G}} \tag{49}$$

can be expected to be constant and the Planck mass must also be true constant because of the constancy of c.

Relation (47) implies that the Newton gravitational constant can be given as

$$G = \frac{L_o c^3 \phi^2}{\hbar}. \tag{50}$$

From relation (49) squared, we can obtain

$$\hbar = \frac{E_P^2 G}{c^5} \tag{51}$$

and supplying this into (50), the Newton gravitational constant equals

$$G = \frac{L_o c^4}{E_P}\phi. \tag{52}$$

We see that G is the same function of time as L_P, since L_o, c, and E_P are the true constants. The same function of time is also $\hbar$, since it can be expressed as

$$\hbar = \frac{E_P L_o}{c}\phi \tag{53}$$

when G, given by relation (52), is supplied into relation (51).

Let us now deal with the fine-structure constant. If the right-hand side of relation (46) is multiplied with form $\hbar c/(M_P^2 G)$, i.e. with unity in fact, we obtain

$$\alpha = \frac{q_o^2}{4\pi\varepsilon_o M_P^2 G}. \tag{54}$$

Form $4\pi\varepsilon_o$ can be expressed, using the well-known relation $c = 1/\sqrt{\varepsilon_o\mu_o}$ between the speed of light, permittivity of vacuum, and permeability of vacuum, $\mu_o = 4\pi/10^7$. It equals $4\pi\varepsilon_o = 10^7/c^2$. Using the latter, relation (52), and after a handling, the fine-structure constant can be re-written to

$$\alpha = \frac{q_o^2 c^2}{10^7 E_P L_o}\frac{1}{\phi}. \tag{55}$$

In (55), we see that α could be the true constant if the elementary charge would be the function of time as

$$q_o = q_{oo}\sqrt{\phi}, \tag{56}$$

where q_{oo} is true constant. Or, we can speculate that the elementary charge is true constant, then the fine structure constant would be the function of time as $\alpha(t) \propto \phi^{-1}$.

We could make the similar deductions for several combinations of the initial assumptions. These deductions would predict various function of the dependence of measured fundamental constants on time. However, one should remember that any dependence is bound by the definition of the Planck units.

6. Conclusion

The fundamental physical constants occur to be the transformation constants when we transform the physical quantities given in the artificial, human-established, units to the corresponding quantities given in the natural, Planck, units.

When we speak about a cosmological variability of the fundamental constants, it makes a sense to consider a variability of the Planck units and take into account that the fundamental constants are only the transformation ("technical") constants.

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TESTING THE LOCAL HUBBLE EXPANSION IN THE SUN–EARTH SYSTEM

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Abstract: Although the concept of Hubble expansion is one of cornerstones of the modern cosmology, the crucial question of the scale at which it begins to operate and, more specifically, how it behaves with decrease in the characteristic distance still remains unclear. A promising approach to answer this question is testing dynamics of the solar-system bodies, *e.g.*, the Sun–Earth system. Here, we present the particular kind of analysis, which is based on thermal evolution of the Earth throughout its entire lifetime (about 4 Gyr) as can be inferred from the geological, geochemical, and paleobiological data. The estimated value of the local Hubble parameter varies from the standard intergalactic number to twice that quantity, depending on the method of processing used. This is a rather reasonable result, taking into account a lot of uncertainties in the observational data.

Keywords: local Hubble expansion, faint young Sun problem, Křížek–Somer hypothesis

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1. Introduction

The concept of Hubble–Lemaître (or, shortly, Hubble) expansion is a cornerstone of cosmology for almost a century. However, it is still unclear what is the spatial scale at which such an expansion begins to operate. This question was posed by McVittie already in 1933 [19], and subsequently it was addressed by Einstein & Straus [11] and a number of other researchers; for the review, see [1, 7]. It was found in the most of these studies that the Hubble expansion is strongly suppressed or disappears completely in the vicinity of a compact gravitating mass (*e.g.*, a star). Unfortunately, the exact criteria of such suppression strongly depend on the particular models and differ from each other very much. Most probably, just this uncertainty was the main reason why the most of astronomers in the middle of 20th century began to rely on the trivial Newtonian argument: a system of gravitating bodies can experience the expansion if its kinetic energy is greater than the potential one, and it is constrained otherwise. So, this point of view was commonly accepted in the astronomical community and became the major paradigm.

However, from out point of view, the above-mentioned criterion is just the fact "accepted by repetition", because it does nor reflect the crucial feature of Hubble–Lemaître expansion as the phenomenon of General Relativity, namely, that this is expansion of the space itself rather than motion of any bodies in a fixed space. The need for taking this fact into account was clearly emphasized, for example, already in paper by Bonnor [1] in 2000. In a more pictorial language, one can mention that the Hubble expansion exists even in the space filled with a perfectly uniform matter, where there are no any gradients and, therefore, the gravitational forces. Consequently, it is meaningless to speak about the Newtonian potential energy of interaction between the gravitating bodies and, therefore, to compare it with their kinetic energy. In other words, the cosmological expansion corresponds to the additional "degree of freedom", which is present only in the General Relativity and has no analogue in the Newtonian mechanics.

Next, the problem of local Hubble expansion became even more topical in the last quarter of century, when the concept of dark-energy-dominated Universe became the commonly-accepted paradigm in cosmology. Really, since the dark energy (or the cosmological Λ-term) is present everywhere, one can expect that at least a fraction of Hubble expansion, produced by the Λ-term, should be present at any scale. (Particularly, the well-known Einstein–Straus proof of absence of the expansion in an empty cavity about a point-like mass [11] becomes inapplicable, because it is meaningless to speak about an empty cavity in the vacuum energy distribution). Therefore, there is a strong theoretical evidence to believe that the local Hubble effect should exist, but its expected magnitude (*i.e.*, the value of local Hubble parameter $H_0^{(\mathrm{loc})}$) is not so clear.

In paper [8], we tried to accurately calculate the rate of local Hubble expansion by treating "the restricted two-body problem" (*i.e.*, motion of a test particle about the central point-like mass) against the expanding cosmological background formed by the Λ-term, which is described by the well-known Kottler (or Schwarzschild–de Sitter) metric reduced to the Robertson–Walker coordinates [5]. As a result, it was found that such a system really exhibits a kind of the local Hubble expansion, *i.e.*, the trajectory of a test particle becomes spiral; but this effect (namely, dependence of the radial velocity on radius) is substantially non-linear, as distinct from the standard Hubble relation,

$$v_r = H r \, . \tag{1}$$

Of course, it is possible to linearise the calculated dependence at some (not so long) time interval, thereby getting the formula (1). However, such a linearised relation (and the respective value of Hubble parameter $H_0^{(\mathrm{loc})}$) will be strongly dependent of the starting point of the time interval under consideration (*i.e.*, for example, on the instant of formation of a planetary system or a binary star at the cosmological time scale). In other words, our attempt to derive theoretically the value of local Hubble parameter led us to the conclusion that this quantity depends on the initial conditions for the particular set of differential equations [8]. In general, it hardly possible to chose the adequate initial conditions based on the purely theoretical arguments, so that they should to be derived mostly from observations of the particular astronomical systems. This emphasizes once again the crucial importance of observational tests of the local Hubble expansion at the sufficiently small scales, particularly, within the Solar system.

2. Dynamics of the Sun–Earth System

By now, one can identify two major approaches to testing the local Hubble expansion in the Solar system. One of them stems from the old idea by King [14] that the observable rate of recession of the Moon from the Earth—although it is usually attributed to the tidal friction—closely resembles (within a factor about unity) the rate of the universal cosmological expansion. The observational basis for testing this idea are the high-accuracy measurements of the Earth–Moon distance by the the method of lunar laser ranging (LLR) [3], which became available due to deployment of a few retroreflectors on the lunar surface in the course of US and USSR missions in the very late 1960s and early 1970s.

In brief, subtracting the tidal effect from the experimentally measured rate of increase of the lunar orbital radius, one can get a probable cosmological contribution to the above-mentioned recession rate. The first quantitative analysis of this kind was undertaken about two decades ago in our papers [4, 6], and it was substantially improved in a few subsequent publications, particularly, by Maeder & Gueorguiev [18]. It was found that the local Hubble parameter is usually somewhat less than the global one but, in general, these two quantities coincide up to a factor close to unity.

Yet another approach to testing the local Hubble expansion was suggested in 2012 by Křížek [16] and subsequently developed by him in collaboration with Somer [17]. Their analysis was aimed originally at resolution of the so-called faint young Sun paradox but, as shown in our recent publications [9, 10], can serve also as a tool for finding the value of the local Hubble parameter. While testing the Earth–Moon dynamics is based on the high-accuracy measurements of distance for a rather short time interval (a few decades), the thermal analysis of the Sun–Earth system involves the data on temperature that are rather crude but collected over a huge time period (a few Gyr).

In fact, the main idea by Křížek and Somer was to employ the local Hubble expansion—and the resulting increase in the Earth's orbital radius—for compensation of the increasing solar luminosity so that a solar irradiation of the Earth to be approximately constant in the course of time. It was found that at a certain value of the local Hubble parameter, namely, $H_0^{(\mathrm{loc})} \approx 0.5 H_0$ (where H_0 is its global value) the solar irradiation remains the same with accuracy about 1% for the entire period of biological evolution of the Earth (∼3.5 Gyr) although the solar luminosity increases by 25–30 %.

Unfortunately, the above-written value of the local Hubble parameter looks somewhat suspicious, because the dark energy alone (*i.e.*, even without a contribution from the ordinary—both dark and visible—matter) should give a higher expansion rate. Furthermore, and more importantly, the employed assumption of the constant irradiation—and, therefore, constant temperature—of the Earth looks oversimplified: according to the contemporary paleochemical and paleobiological data, which will be discussed in more detail below, the temperature on the early Earth might be much greater than today, up to 70−80°C. In principle, this fact was mentioned already in paper [16], but its significance was underestimated. So, it was the aim of our recent papers [9, 10] to incorporate into analysis the more realistic data on the temperature; and the corresponding results will be outlined below.

Finally, one should mention that already in the late 1970s Canuto & Hsieh [2] tried to employ the data on enhanced ancient temperatures of the Earth, which became available by that time, to test the so-called scale-invariant extensions of General Relativity. Unfortunately, since the corresponding models involved a number of additional parameters (particularly, a variable gravitational constant, the rate of matter creation, and the specific scale factor), the resulting constraints on these parameters were rather uncertain. From our point of view, the local Hubble expansion can exist even in the standard General Relativity, so that such tests should constrain just the well-known parameters of the cosmological models. By the way, the most reliable fits of the Earth's temperature were obtained in the above-cited work by Canuto & Hsieh for the case when the gravitational constant decayed as t^{-1}, so that the Sun–Earth distance increased linearly with time. On the other hand, the same approximately linear dependence can be obtained just by linearising the standard Hubble expansion over some interval in the past.

2.1. Thermal balance of the Earth

A simplest model of the thermal balance of a planet can be written as (*e.g.*, review [21]):

$$\pi R^2 (1{-}A) K = 4\pi R^2 (1{-}\alpha)\, \sigma\, T_{\rm s}^4 \,, \qquad (2)$$

where R is the planetary radius, $T_{\rm s}$ is its surface temperature, A is the albedo (reflective capability) of the planet in the visible range of spectrum, which contains the major part of incident solar energy); α is the effective atmospheric albedo for the infrared radiation, which is emitted from the planetary surface to the outer space; K is the flux of solar energy per unitary area of the planetary surface, which is commonly called the "solar constant"; and σ is the standard Stefan–Boltzmann constant, characterizing the black-body radiation. In fact, the left-hand side of this equation represents the flux of solar energy absorbed by the planetary cross-section πR^2, while the right-hand side describes the infrared thermal flux from the entire planetary surface $4\pi R^2$ outgoing to the outer space.

Next, if we admit a temporal variability of both the solar luminosity L and distance from the Sun to the planet r, then the above-mentioned solar constant will be actually the function of time:

$$K(t) = \frac{L(t)}{4\pi r^2(t)}\,. \tag{3}$$

Strictly speaking, relation between the orbital radius r and the cosmological scale factor a might be very non-trivial [8]; but in the subsequent semi-empirical analysis we shall assume for simplicity that these two quantities are simply proportional to each other.

Besides, we shall take both the coefficient A and α in formula (2) to be independent of time, which is a rather crude assumption from the viewpoint of climatology. Then, taking a ratio of the equations (2) at the arbitrary instant of time in the past t and at the present time t_0, we can easily get temporal variation on the relative temperature:

$$T_{\mathrm{s}}(t)/T_{\mathrm{s0}} = [L(t)/L_0]^{1/4}\,[a(t)/a_0]^{-1/2}\,. \tag{4}$$

The luminosity $L(t)$ is usually obtained by a numerical integration of the equations of stellar evolution and subsequent interpolation by the sufficiently simple formulas, such as [13]:

$$L(t) = L_0 \left(1 - \frac{2}{5}\,\frac{t}{t_{\mathrm{S}}}\right)^{-1}. \tag{5}$$

Here, $t_{\mathrm{S}} \approx 4.7{\cdot}10^9$ yr is the age of Sun, so that the luminosity becomes smaller at the earlier times.

Next, variation of the cosmological scale factor $a(t)$ with time can be found from the standard Friedmann equation, *e.g*, review [20]. It can be conveniently rewritten for our purposes as

$$\left(\frac{a'}{a}\right)^2 = \Omega_\Lambda + \Omega_{\mathrm{D}} \left(\frac{a_0}{a}\right)^3, \tag{6}$$

where Ω_Λ and Ω_{D} are the relative energy densities of the dark energy (Λ-term) and non-relativistic (dust-like) matter, both dark and visible. Here, the prime denotes a differentiation with respect to the dimensionless time τ, which is normalized to the age of Universe (*i.e.*, the inverse Hubble constant):

$$\tau = H_0\, t\,. \tag{7}$$

A contribution from the relativistic matter (radiation), which is proportional to $1/a^4$, was not taken here into account, because we are interested only in the sufficiently late stage of the cosmological evolution. Besides, in accordance with the commonly-accepted paradigm, we consider the spatially flat Universe, where the curvature term, proportional to $1/a^2$, is absent and $\Omega_\Lambda + \Omega_{\mathrm{D}} = 1$.

Next, equation (6) can be rewritten as

$$a' \equiv \frac{da}{d\tau} = a \sqrt{(1 - \Omega_{\mathrm{D0}}) + \Omega_{\mathrm{D0}} \left(\frac{a_0}{a} \right)^3} \tag{8}$$

and integrated numerically for the decreasing τ. Thereby, we obtain the function $a(t)$, which should be substituted into formula (4). Let us remind that, according to the modern cosmological data, $\Omega_{\mathrm{D0}} \approx 0.25-0.3$.

As a result, equation (4) gives us a tool to predict a relative variation of the planetary temperature in the past. It is interesting that this variation turns out to be universal (*i.e.*, the same for any planet) provided that the contemporary value of temperature T_{s0} is taken appropriately. For example, according to the review [21], $T_{\mathrm{s0}} = 288\,\mathrm{K}$ for the Earth and 218 K for Mars; but other works can give slightly different values, depending on the particular methods of averaging and the employed sets of observational data. Finally, by plotting a series of theoretical curves $T_{\mathrm{s}}(t)/T_{\mathrm{s0}}$ at various cosmological parameters and comparing them with the available experimental data on planetary temperature, it will be possible to estimate a magnitude of the local Hubble expansion within the Sun–Earth system.

2.2. Observational data on the ancient temperature

We have now two major sets of observational data on the Earth's temperature in the past: paloechemical and paleobiological. The first of them is based on the ratio of concentrations of various isotopes of the same chemical element (*e.g.*, ^{18}O and ^{16}O) deposited at the bottom of terrestrial oceans. Since these ratios depend on the temperature of the environmental water from which the minerals were deposited, they can be used to derive the temperature at the time of their formation. Such investigations were initiated as early as 1960's and 1970's, and it was claimed already in one of the first publications [15] that "a few data for the Precambrian suggest the possibility that Earth surface temperatures may have reached about 52°C at 1.3 billion years and about 70°C at 3 billion years [ago]".

A modern set of the isotopic data [22] is plotted in the top panel of Figure 1: blue broken line represents a relative variation in the isotopes of oxygen, which is commonly designated as δ^{18}O; and the cyan polygon, in the isotopes of silicon, δ^{30}Si. It is seen that the scatter of data is rather large, but general tendency for the increase of temperature in the past is evident. Let us mention that a few other isotopes were also used in such analyses. For example, the early studies often employed a deuterium-to-hydrogen ratio; but the corresponding data are considered now as less reliable and not presented here.

The second set of experimental data on the Earth's temperature was taken from the absolutely different branch of science—molecular biology and genetics. They are based on studying the temperatures of existence of the reconstructed protein sequences in the ancient bacteria [12]. The corresponding constraints are shown in the same figure by green circles with error bars. They show the same tendency for increasing temperature in the past.

At last, taking into account universality of the equation (4), we tried to incorporate into analysis also the data on temperature of the Mars—yet another terrestrial-type planet, for which the most detailed information is available. Namely, reasonable constraints on the temperature could be inferred from the existence of an extensive ocean of liquid water on its surface soon after formation of this planet. There are somewhat different estimates in the literature for the respective span of time, but the most frequently cited interval is about 4.1 to 3.7 Gyr ago. So, the temperature in that period should be above the freezing point of water.

However, the following two subtle points should be taken into account in such an analysis. Firstly, it is commonly recognized that the martian atmosphere substantially disappeared during its lifetime. So, the atmospheric pressure on the ancient Mars would be much greater than its contemporary value, which is about 600 Pa. Fortunately, as follows from the phase diagram of water, a boundary between its solid and liquid phases is almost fixed at the same temperature within a considerable pressure range, from the triple point (611 Pa) up to 10 MPa. Consequently, even if the martian atmospheric pressure in the past would be four orders of magnitude greater than today, the freezing point of water remained almost unchanged. The second, much more unfavourable fact is that the freezing point strongly depends on the salinity of water. For example, the freezing temperature of the terrestrial oceans is reduced by only 2°C; but a larger salinity could lead to the much greater reduction. Particularly, sodium chloride (NaCl) in high concentrations can lower the freezing temperature of water down to −21°C; and calcium chloride ($CaCl_2$), down to −55°C. Because of the absence of any reliable information about chemical composition of the martian ocean, we can impose only a set of rather weak constraints on the martian temperature for the liquid ocean to exist. These constraints are shown in Figure 1 by a column of magenta bars with upward arrows, and the respective numbers designate the freezing temperatures.

To avoid misunderstanding, let us mention that there is ample evidence of the existence of liquid water or brines on Mars at the much later times, *i.e.*, closer to the present epoch. These are, particularly, the dry river beds on the martian surface that are dated by a few hundred million years ago or even younger. However, they are most probably fed by the underground thermal sources and, therefore, are irrelevant to the global climatic system represented by equation (2).

2.3. Confronting the theory with observations

The bottom panel in Figure 1 represents fitting of the theoretical curves $T_s(t)/T_{s0}$ to the observational data. (It should be kept in mind that values for the Earth and Mars are normalized to the different contemporary temperatures: $T_{s0} = 288\,\mathrm{K}$ and $218\,\mathrm{K}$, respectively.) The set of black dotted curves correspond to various values of the present-day Hubble parameter $H_0 = (\dot{a}/a)_0$, which is expressed for conciseness in the units of $h = 100$ (km/s)/Mpc. Solid curve corresponds to the standard intergalactic value of the Hubble parameter, approximately $0.7h$; and dashed curve, to the original Křížek–Somer model [16, 17], where the local Hubble parameter was about half of the global one, and the resulting Earth's temperature remained approximately constant for a very long time.

As is seen, the best agreement between the theory and observations for the Earth is achieved at $H_0 = 1.2-1.5\ h = 120-150$ (km/s)/Mpc, which is about two times greater than the standard intergalactic value of the Hubble parameter. On the other hand, the "standard" curve with $H_0 = 0.7\ h = 70$ (km/s)/Mpc lies evidently below the observational data, while the Křížek–Somer curve deviates even further. The martian constraints are very weak, since the theoretical curves with $1.2-1.5\ h$ are consistent with almost any freezing temperature of the martian ocean. However, if $H_0 = 1.2\ h$, this temperature cannot be above -10°C, since otherwise a part of the theoretical curve will lie below the admissible region. Then, the martian ocean should be assumed to be moderately salted.

A remarkable feature of this figure is that all theoretical curves substantially deviate from the observational data in the sufficiently late, the so-called Cambrian epoch ($t > -0.6{\cdot}10^9$ yr). In general, this is not surprising because both atmospheric and surface properties of the Earth—characterized by the coefficients α and A, respectively—could experience substantial changes in the Cambrian due to the active development of life. As a result, the simplifying assumptions employed in the derivation of formula (4) will no longer be valid.

Therefore, it might be reasonable to exclude this interval from the analysis and to perform fitting the observational data only in the Precambrian. All temperatures in this case—both theoretical and experimental—should be normalized to the temperature in the end of Precambrian T_{s1} rather than to its contemporary value T_{s0}. Unfortunately, T_{s1} is not known exactly and can be estimated to lie in the interval 35°C to 45°C. So, this is actually an additional fitting parameter, and the respective results are presented in Figure 2 for two different values of T_{s1}.

As one can see in the upper panel, corresponding to $T_{s1} = 45$°C, the best fit is achieved at the *contemporary* values of the Hubble parameter $H_0 = 0.7-0.9\ h = 70-90$ (km/s)/Mpc, which is in much better agreement with the intergalactic value (about 70 (km/s)/Mpc). On the other hand, at the low temperature in the end of Precambrian, $T_{s1} = 35$°C (bottom panel), the Hubble parameter is estimated to be $H_0 = 0.95-1.15\ h = 95-115$ (km/s)/Mpc, *i.e.*, deviates more substantially from the standard intergalactic value.

3. Conclusions

1. In the present work we refined the earlier analysis by Křížek and Somer [16, 17] on a probable influence of the local Hubble expansion on thermal evolution of the Earth throughout the most part of its lifetime (about 4 Gyr). Namely, we employed the realistic observational data on the Earth's surface temperature in the past—both paleochemical and paleobiological—instead of postulating its approximate constancy.

2. As a result, it was found that the theoretical curves are best fit to the observational data if only the Precambrian period is used for the analysis, when influence

of biological evolution on the Earth's surface and atmosphere was not so much.

3. The corresponding values of the local Hubble parameter turned out to be in agreement with the standard intergalactic value up to a factor about unity. However, it is interesting to mention that the best fits tend to be shifted towards greater values as compared to the intergalactic one; while just the opposite situation takes place in the analysis of the Earth-Moon system [4, 6], where they are shifted downwards.

4. As distinct from the old work by Canuto & Hsieh [2], which was based on the scale-invariant generalization of General Relativity, we did not employ in our analysis any kind of the "new physics" and rely solely on a more careful treatment of the commonly-established concepts of General Relativity and the standard cosmological model. It is interesting to mention that the best fits were obtained by Canuto & Hsieh for the case when the Earth–Sun distance increased linearly with time due to the variation of the gravitational constant as t^{-1}. On the other hand, the same linear dependence can be trivially obtained by a linearisation of the Hubble expansion in the past 4 Gyr.

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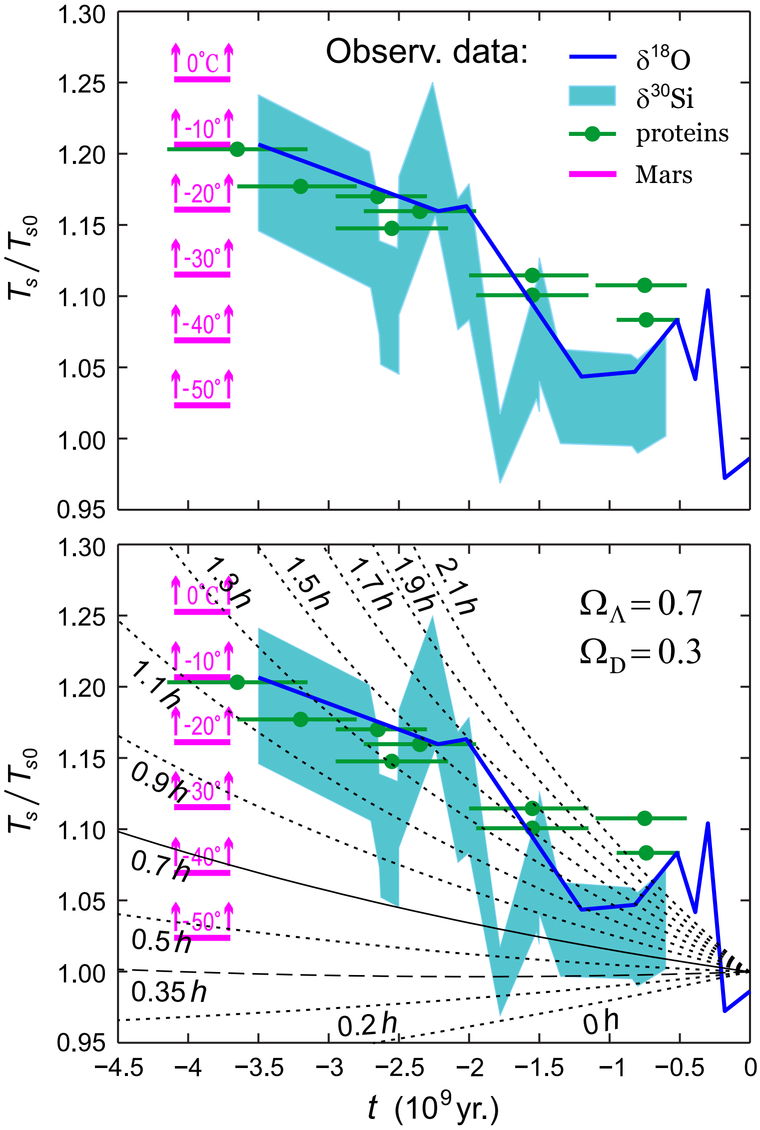

Figure 1: Temporal evolution of the Earth's surface temperature in the past: observational data (top panel) *vs.* the theoretical predictions (bottom panel) in the framework of standard cosmological model at various values of the present-day Hubble parameter, expressed in units of $h = 100$ (km/s)/Mpc (numbers near the curves).

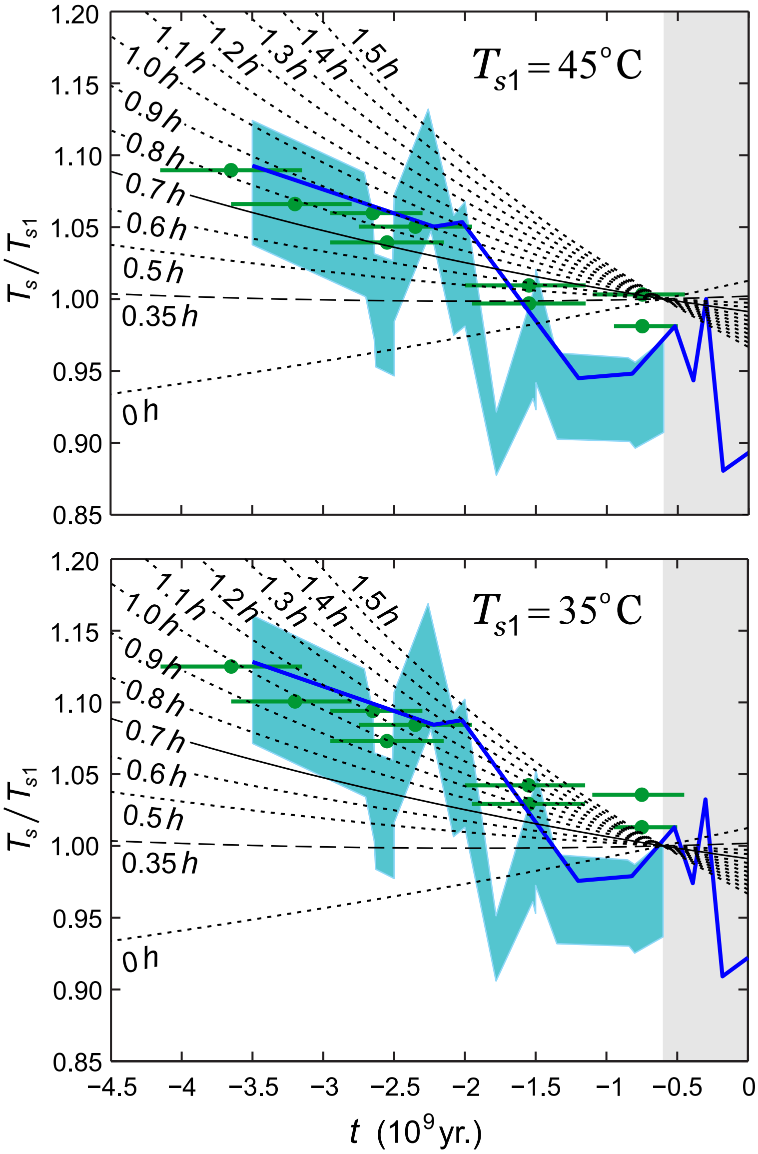

Figure 2: Temporal evolution of the Earth's surface temperature in the Precambrian period only. Top and bottom panels correspond to two different temperatures in the end of Precambrian T_{s1}, and all other designations are the same as in Fig. 1. Grey shaded area corresponds to the Cambrian epoch, excluded from the analysis.

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SPHERICALLY CLOSED DYNAMIC SPACE: A FRAMEWORK FOR JWST OBSERVATIONS AND LOCAL HUBBLE EXPANSION

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Abstract: Recent observations by the James Webb Space Telescope — massive, morphologically mature galaxies at $z > 10$ and compact angular sizes persisting to the highest observed redshifts — pose direct challenges to the standard ΛCDM cosmological model and motivate a reassessment of foundational assumptions. This paper presents the Dynamic Universe (DU) framework as a natural context for interpreting these observations. The DU rests on two postulates: that physical space is the three-dimensional surface of a four-dimensional sphere (a 3-sphere), and that the total energy of the universe is identically zero at all times. In DU, the rest energy of matter is obtained against the release of gravitational energy in a contraction–expansion cycle, providing an intrinsically high-energy early universe in which gravitational collapse and structure formation proceeded at rates far exceeding those of the present epoch. This accounts naturally for the massive early structures revealed by JWST without fine-tuned mechanisms. DU predicts a monotonic decrease of galaxy angular sizes with redshift, consistent with the full JWST dataset, and reproduces the supernova Hubble diagram without dark energy. DU also predicts local Hubble expansion of gravitationally bound systems — supported by Lunar Laser Ranging and three independent geological datasets converging on $H_0 \approx 70\,\text{(km/s)/Mpc}$ from Solar System data alone.

Keywords: cosmology, zero-energy principle, 3-sphere space, JWST, angular size, Type Ia supernovae, Lunar Laser Ranging, Dynamic Universe

PACS: 01.30.Cc, 95.30.Sf, 98.80.-k, 98.80.Bp, 98.80.Es, 98.80.Jk

1. Introduction

The standard cosmological model, ΛCDM, stands as one of the most empirically successful frameworks in the history of physics. Yet its success has come at a conceptual cost that is becoming increasingly difficult to ignore. Most dramatically, the model requires that all matter and energy in the universe appeared instantaneously from nothing in a Big Bang event — a postulate that lies beyond the reach of physical explanation and sits uneasily alongside the conservation laws that govern other domains of physics. Beyond this founding assumption, the model requires inflation, dark matter, and dark energy whose physical nature remains undetermined after decades of dedicated investigation. Dark energy alone, introduced to account for the observed curvature of the supernova Hubble diagram, contributes approximately 70 % of the total energy content of the universe while resisting every attempt at direct detection or theoretical derivation from first principles.

Recent observational developments have sharpened these tensions. The James Webb Space Telescope has revealed massive, morphologically mature galaxies at redshifts $z > 10$, implying that substantial gravitational structure formed within the first few hundred million years after the nominal Big Bang. Accommodating these observations within ΛCDM requires increasingly fine-tuned assumptions about early star formation efficiencies and seeding mechanisms Adams et al. [1], Melia [16], Gupta [9]. Simultaneously, the Hubble tension — the persistent discrepancy between H_0 inferred from early-universe CMB data and from local distance-ladder measurements — has resisted resolution within the standard framework and now stands at a significance level that challenges the model's internal consistency.

These difficulties share a common root: they arise not from the failure of any single measurement, but from the accumulating strain on the foundational assumptions of the model. This suggests that the appropriate response is not the addition of further parameters, but a reassessment of foundations.

The Dynamic Universe (DU) framework (see Suntola [22, 23]) takes precisely this step. It is built on two postulates, each of which has been independently recognized by leading physicists over more than a century, but which have never previously been united into a single dynamical framework. The first postulate is that physical space is the three-dimensional surface of a four-dimensional sphere — a 3-sphere of radius R_4 — a geometry considered by Schwarzschild [19] in 1900 and adopted by Einstein [5] in 1917 for its physical and philosophical coherence. The second postulate is that the total energy of the universe is identically zero at all times, with the kinetic energy of expansion exactly balanced by global gravitational energy. This zero-energy condition had been recognized by Tolman [24], Sciama [20], and Feynman [6], but treated in each case as a remarkable numerical coincidence rather than as a governing principle. In DU, it is promoted to the status of the primary postulate — the equation of motion of the universe itself.

In this picture, the rest energy of matter is not created from nothing but obtained against the release of gravitational energy in a contraction–expansion cycle, in the spirit of Aristotle's *entelecheia* — the actualization of potential. Mass obtains the meaning of the substance for the expression of energy; no external energy source is required, and no instant creation of matter, as energized mass, is invoked.

This single foundational choice has far-reaching consequences. The expansion dynamics of the universe, the velocity of light, the rest energy of matter, and all relativistic phenomena emerge from the global energy balance and its local redistribution — without a cosmological constant, without dark energy, and without inflation. The zero-energy approach also turns the description of relativity on its head. In the metric formalism of relativity theory, time and distance are functions of gravitation and motion, and the velocity of light and the rest mass are constants. In the energy-based DU framework, time and distance are universal coordinate quantities; the velocity of light is linked to the expansion velocity of the 3-sphere. Locally, the velocity of light and the rest mass are functions of the local energy state — the state of gravitation and motion — rather than of the geometry of spacetime.

A consequence that follows necessarily from the zero-energy dynamics: all gravitationally bound systems — from galaxy clusters to planetary orbits — expand in direct proportion to the expansion of space. Local expansion in DU is not an assumption but a prediction, supported by an observational chain spanning an extraordinary range of scales, from JWST angular sizes of early galaxies, through transponder measurements of planetary distances and Lunar Laser Ranging, to geological evidence preserved in ancient coral fossils. Three independent geological datasets derived entirely from Solar System data converge on $H_0 \approx 70\,(\mathrm{km/s})/\mathrm{Mpc}$ without any cosmological observation. The Earth–Moon recession measured by Lunar Laser Ranging, long noted as anomalously large relative to independent tidal estimates, decomposes naturally in the DU framework into a cosmological component and a tidal residual consistent with those independent constraints.

This paper presents the DU framework with emphasis on the choice of foundational assumptions and its observational consequences, the geometry of the universe as probed by JWST, the handling of cosmological distances and its effect on the interpretation of supernova data, and the evidence for local Hubble expansion in Solar System measurements. Section 2 examines the foundational choice between the kinematic-metric and zero-energy-dynamic approaches. Section 3 addresses the energetically rich early universe and JWST angular-size observations. Section 4 develops the DU distance framework and its application to the supernova Hubble diagram. Section 5 presents the local expansion evidence. Section 6 discusses what the DU framework adds and what remains open, and states the conclusions.

2. The choice of foundations: kinematics/metrics vs. zero-energy dynamics

2.1. Two cornerstones, long separated

The two principles on which the DU rests — spherically closed space and global zero-energy balance — each have a distinguished history, and their separation in the literature is itself instructive.

The idea that physical space might be finite and without boundary was argued on philosophical and physical grounds by Schwarzschild [19], who considered positively curved, closed (elliptic) geometry for space a full sixteen years before general relativity existed. Einstein [5] adopted precisely this geometry — a spatially closed 3-sphere — as the natural setting for his cosmological equations. He recognized its physical coherence immediately: a 3-sphere is the simplest finite geometry without a boundary or a preferred center, and it avoids the conceptual difficulties of an infinite universe.

However, the field equations demanded a dynamic universe, one that must either expand or contract, and to preserve the static cosmos he preferred, Einstein introduced the cosmological constant Λ as an explicit stabilizing term. The geometry was physically motivated; its dynamics were suppressed by hand. When Friedmann [7] in 1922 and Lemaître [13] in 1927 showed that the field equations without Λ admit expanding and contracting solutions, the 3-sphere geometry was retained, but the dynamical question was thereafter framed entirely within the metric formalism — as a property of certain solutions to a field equation, not as a consequence of an energy principle.

The zero-energy condition has an equally distinguished, and equally incomplete, history. Tolman [24] provided the first systematic treatment, demonstrating that a closed relativistic universe possesses net zero total energy. Sciama [20], working under Dirac, derived from Mach's principle the conclusion that the total energy of a particle at rest — inertial plus gravitational — is identically zero. Feynman [6], in his lectures on gravitation, identified the condition $GM^2/R = Mc^2$ as a "great mystery" and explicitly entertained the possibility that three-dimensional space is the surface of a four-dimensional sphere — without, however, constructing a cosmology from these observations. In each case, the zero-energy condition appeared as a striking numerical coincidence, noted and then set aside, rather than elevated to a governing principle.

The reason these two cornerstones remained separated is largely formal. The kinematic and metric language of general relativity, which became the standard framework of cosmology after 1915, is inherently local and geometric. It describes how energy-momentum curves spacetime and how test particles move in curved spacetime, but it does not naturally accommodate global energy balance as a dynamical constraint. Within this formalism, the zero-energy condition is not a statement one can straightforwardly impose on the field equations; it appears instead as an occasional observation about special solutions. The energy perspective — in which the rest energy of matter is kinetic energy borrowed from gravitation — is systematically obscured by the metric machinery.

2.2. The decisive step: zero-energy balance as equation of motion

The Dynamic Universe takes the step that the history of physics prepared but did not take. Adopting both cornerstones simultaneously — 3-sphere geometry and global zero-energy balance — and using the zero-energy condition itself as the equation of motion, the framework is expressed in its most compact form as

$$E_m + E_g = Mc_0^2 - \frac{GMM''}{R_4} = 0, \tag{1}$$

where M is the total mass of the universe, M'' is its mass equivalence at the 4-center obtained by integrating the Newtonian gravitational potential over the 3-sphere, R_4 is the 4-radius, and c_0 is the velocity of expansion in the fourth dimension, also identified as the velocity of light in space. This is not a derived property of a metric solution; it is the primary postulate.

Differentiating equation (1) with respect to universal coordinate time yields the expansion dynamics directly:

$$c_0 = \frac{dR_4}{dt} = \frac{2}{3}\frac{R_4}{T} = \left(\frac{2}{3}GM''\right)^{1/3} T^{-1/3}, \tag{2}$$

where T is the time elapsed from the turning point of the contraction–expansion cycle. The velocity of light decreases as the universe expands, and was correspondingly higher in the early universe — a consequence with direct observational implications for structure formation, as discussed in Section 3.

The contraction–expansion cycle provides the mechanism that ΛCDM cannot supply: the rest energy of matter is built up against the release of gravitational energy during contraction, and the process reverses after the turning point. No external energy source, no creation event, and no inflationary episode are required. Energy is conserved globally at every stage.

2.3. Structural consequences

The shift from a metric to an energy foundation reorganizes the entire theory structure. In the GR framework, the primary object is the metric tensor $g_{\mu\nu}$; matter curves spacetime, and spacetime directs the motion of matter. Energy conservation is a derived consequence — the vanishing of the covariant divergence of the energy-momentum tensor. The velocity of light c and the rest mass m are constants; relativistic effects are carried by dilated time, contracted length, and the geometry of spacetime.

In the DU framework, in harmony with common experience, time and distance are universal coordinate quantities — the same for all observers. The velocity of light and the rest mass are not constants but functions of the local energy state, determined by the depth of the gravitational potential wells in which an object resides and by its velocity within those wells. The rest energy of an object situated within a hierarchy of n nested gravitational frames is

$$E_{\mathrm{rest}(n)} = m_0 c_0^2 \prod_{i=1}^{n} \left[(1-\delta_i)\sqrt{1-\beta_i^2} \right], \tag{3}$$

where $\delta_i = GM_i/r_i c_0 c_{0\delta}$ is the gravitational factor and $\beta_i = v_i/c_i$ the velocity ratio in the i-th frame, Figure 1.

Physical process rates — including atomic clock frequencies — are proportional to the locally available rest energy. Gravitational redshift, time dilation, the Shapiro delay, and perihelion precession are not consequences of spacetime curvature but of the redistribution of energy in this nested system. The DU and GR predictions for all locally tested phenomena agree to the limits of current experimental sensitivity;

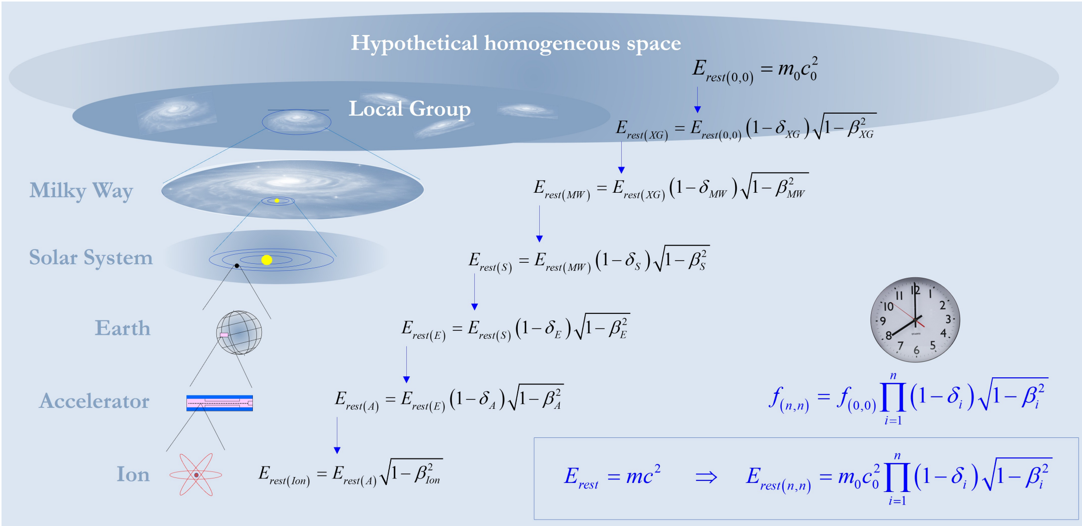

Figure 1: Relativity in zero-energy space. The system of nested energy frames links the rest energy of mass m in a local frame to the rest energy it would have at the state of rest in a hypothetical homogeneous space. The frame-dependent rest energy determines the rates of physical processes, like the frequencies of atomic clocks, according to the state of gravitation and motion in the local frame and the parent frames.

they differ only in the physical interpretation and, crucially, in the cosmological predictions that follow from each foundation.

The practical consequence of this structural difference appears most clearly at the cosmological level. In ΛCDM, the observed curvature of the Hubble diagram requires a dark energy component $\Omega_\Lambda \approx 0.7$ whose physical nature has remained elusive for more than 25 years. In the DU, that curvature follows from the zero-energy balance — it is the natural shape of the magnitude–redshift relation in an expanding 3-sphere, not a signal of an unknown energy component. Similarly, the expansion of gravitationally bound local systems, which in ΛCDM is absent by construction, emerges in the DU as an unavoidable consequence of the same global energy balance that governs the cosmological expansion.

3. The energetically rich early universe and JWST angular sizes

3.1. High energy density in the early universe

One of the most direct consequences of the zero-energy expansion dynamics derived in Section 2 concerns the physical conditions that prevailed in the early universe. As shown by equation (2), the velocity of light evolves as $c_0 \propto T^{-1/3}$, where T is the time elapsed from the turning point. At small T, c_0 was substantially larger than its present value. Since the rest energy of matter is $E_m = Mc_0^2$, and since all physical process rates — gravitational collapse, nuclear reactions, radiation emission,

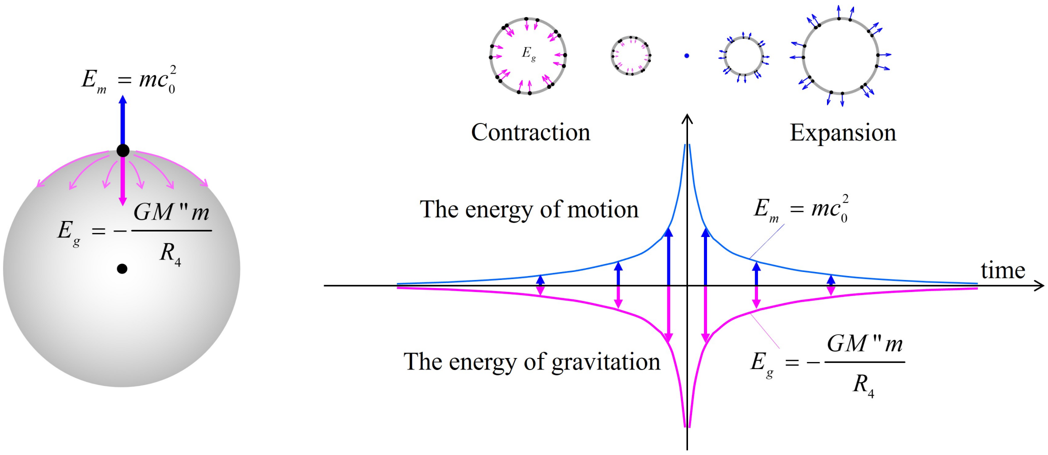

Figure 2: (a) Zero-energy balance in a spherically closed space. (b) Buildup of the rest energy of matter as kinetic energy against the release of gravitational energy in a contraction–expansion cycle of 3-sphere space. Any mass m at rest in space has momentum mc in the fourth dimension due to the expansion, and the total energy of mass with momentum p in space is $E = c\sqrt{(mc)^2 + p^2}$, equal to the Dirac formulation of relativistic total energy. Zero-energy balance ($E_m + E_g = 0$) is maintained throughout the contraction–expansion cycle. In the early expansion following the pass-through from contraction, due to high energy excitation, gravitational processes operated dramatically faster, enhancing atomic/stellar processes, which provided ample time for massive galaxy formation, as supported by JWST observations.

and atomic transitions — are proportional to the locally available rest energy, the early universe was not a slowly evolving dilute system but a dynamically high-energy state in which every physical process proceeded at rates far exceeding those of the present epoch, Figure 2.

This has an immediate and natural consequence for structure formation. Gravitational collapse, which under present conditions requires hundreds of millions of years to assemble a massive galaxy, would have proceeded on correspondingly shorter timescales in the early universe, in direct proportion to the higher available energy density. The energetically rich early universe is not an additional hypothesis grafted onto the DU framework; it follows inevitably from the same equation of motion that governs the present expansion.

This stands in sharp contrast to the situation in ΛCDM, where the formation timescales of massive structures are essentially fixed by the growth rate of density perturbations in a matter-dominated expansion. The discovery by the James Webb Space Telescope of massive, morphologically mature galaxies at $z > 10$ — structures whose existence requires formation timescales that strain the standard model — is therefore a natural expectation within the DU framework rather than a puzzle requiring resolution.

3.2. Angular size as a geometric probe

The geometry of 3-sphere space makes a precise and distinctive prediction for the observed angular sizes of galaxies as a function of redshift, directly testable with JWST data.

In the DU framework, galaxies are gravitationally bound systems that expand in proportion to the expansion of space. A galaxy with present-epoch diameter d_0 had diameter $d = d_0/(1+z)$ at the epoch of emission. The observed angular size at redshift z, accounting for the optical lensing effect of the 3-sphere geometry Křížek [11], is

$$\psi = \frac{d_0}{zR_4} \cdot \frac{\theta}{\sin\theta}, \qquad \theta = \ln(1+z), \tag{4}$$

where the lensing factor $\theta/\sin\theta$ becomes significant only near the antipodal point at $z \approx 22.1$. For objects well below the antipodal point, this reduces to essentially Euclidean scaling: $\psi \approx d_0/(zR_4)$, a monotonic decrease with redshift.

The ΛCDM prediction is qualitatively different. In the standard framework, galaxies are treated as objects of fixed physical size, and the angular diameter distance reaches a maximum at $z \approx 1.5$ before decreasing at higher redshifts, a direct consequence of the FLRW metric expansion history and the Etherington reciprocity relation. The ΛCDM model therefore predicts that galaxies should appear progressively *larger* in angular size as redshift increases beyond $z \approx 1.5$, with the angular size growing steeply at $z > 6$.

3.3. Observational confrontation

Figure 3 shows the angular size–redshift relation across more than four decades in redshift, combining data from the 2MASS Redshift Survey at low redshifts ($z < 0.1$, [10]), the ASTRODEEP–JWST CEERS catalogue at intermediate and high redshifts ($0.5 < z < 12$, [17]), JWST observations at $z \approx 10$, see [14], and the extreme-redshift galaxy candidate Capotauro at $z \approx 31$, see [8].

The observational data follow the monotonic DU trend across the full redshift range. The JWST data at $z > 6$ show no evidence of the angular-size inversion predicted by ΛCDM; high-redshift galaxies appear compact and continue to decrease in apparent size with increasing redshift, consistent with the DU prediction of Euclidean scaling for objects that expand with space. The Capotauro candidate at $z \approx 31$, if confirmed, extends this trend far beyond the ΛCDM angular-size minimum into a regime where the two frameworks diverge most strongly.

The width of the DU prediction band in Figure 3 reflects the intrinsic range of galaxy sizes in the population; no adjustable cosmological parameters are involved. The ΛCDM prediction, by contrast, requires the assumption that galaxy sizes are fixed in physical coordinates — an assumption that, together with the reciprocity assumption, the DU identifies as the source of the discrepancy, since in 3-sphere space the natural expectation is that gravitationally bound systems participate in the expansion.

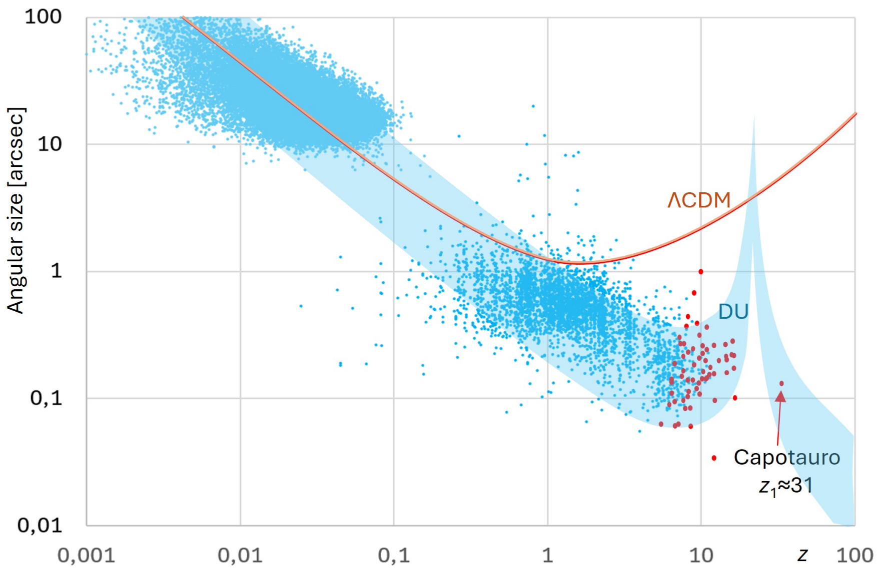

Figure 3: Angular size versus redshift for ΛCDM and the Dynamic Universe. Angular size is represented by the Kron-radius proxy r_{Kron} from the ASTRODEEP–JWST CEERS source catalogue [17], plotted versus photometric redshift. Low-z data ($z < 0.1$) are from the 2MASS Redshift Survey [10]. High-z data ($z > 0.5$) are from the ASTRODEEP catalogue. Red points at $z \approx 10$ are JWST observations from [14]. The point at $z \approx 31$ is the galaxy candidate Capotauro [8]. The red curve shows the ΛCDM prediction with its characteristic non-monotonic behaviour: angular sizes decrease to a minimum at $z \approx 1.5$ and increase at higher redshifts. The light blue band shows the DU prediction: a monotonic decrease following Euclidean scaling on the 3-sphere for objects that expand with space. The antipodal point occurs at $z \approx 22$, around which double images are predicted in opposite sky directions.

3.4. The antipodal double-image prediction

The 3-sphere geometry carries a further prediction that has no counterpart in ΛCDM and is in principle directly testable: an object near the antipodal point at $z \approx 22.1$ should produce a double image in opposite directions on the sky, since light from such an object can reach the observer by travelling either way around the 3-sphere. An object at redshift $z_2 > 22.1$ would produce a counter-image in the diametrically opposite sky direction at

$$z_1 = \mathrm{e}^{2\pi}/(z_2 + 1) - 1. \tag{5}$$

For the Capotauro candidate at $z_2 \approx 31$, equation (5) predicts a counter-image at $z_1 \approx 15.7$ in the opposite sky direction. This prediction is falsifiable with targeted deep-field observations and constitutes a signature unique to closed 3-sphere geometry. Its confirmation or refutation would provide a decisive geometric test of the DU framework at the largest accessible scales.

4. Cosmological distances and the supernova Hubble diagram

4.1. Distance definitions in the DU framework

The unambiguous geometry of 3-sphere space and the parameter-free expansion dynamics of equation (2) yield closed-form expressions for all cosmological distances, Figure 4. These contrast with the integral expressions of ΛCDM, which require numerical evaluation and depend on the density parameters Ω_m and Ω_Λ.

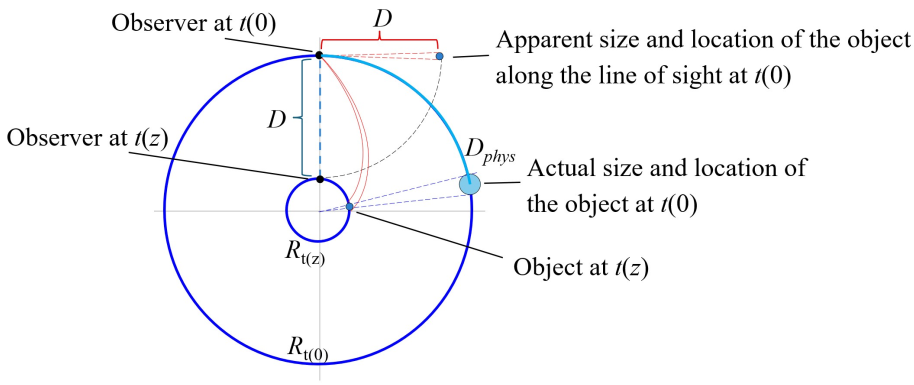

Figure 4: Distances in DU space. The optical distance is the tangential component of the light path spiral from the object at emission to the observer at observation. Because the velocity of light equals the expansion velocity at all times, the optical distance equals the increase of the 4-radius, $D = R_{t(0)} - R_{t(z)}$, during light transmission. Objects like galaxies expand in direct proportion to the expansion of space. The observer sees the object at the apparent location at distance D unexpanded.

In the DU framework, light travels along the surface of the expanding 3-sphere. Because the velocity of light equals the expansion velocity at every epoch, the optical distance equals the increase in the 4-radius during signal propagation:

$$D = R_{4(0)} - R_{4(z)} = \frac{z}{z+1} R_4, \tag{6}$$

where $R_4 = R_{4(0)}$ is the present 4-radius, related to the Hubble radius by $R_H = c/H_0$. This expression serves as both the angular size distance and the light-travel distance — quantities that are distinct in the ΛCDM framework but coincide in the DU. The luminosity distance, accounting for areal spreading and the energy dilution $(1 + z)$ carried by each cycle of radiation, is

$$D_L = \frac{z}{\sqrt{1+z}} R_4. \tag{7}$$

A full comparison of the distance definitions in the ΛCDM and DU frameworks is given in Table 1. The structural simplicity of the DU expressions reflects the absence of free parameters: once R_H is fixed, all distances are determined by the redshift alone.

Table 1: Distance definitions in the ΛCDM and DU frameworks. The ΛCDM expressions require numerical integration over Ω_m and Ω_Λ; the DU expressions are closed-form functions of z and R_4.

Quantity	ΛCDM	DU
Optical distance	$\dfrac{c}{H_0(1+z)}\displaystyle\int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_m z')-z'(2+z')\Omega_\Lambda}}$	$\dfrac{z}{z+1}R_4$
Luminosity distance	$\dfrac{c(1+z)}{H_0}\displaystyle\int_0^z \frac{dz'}{\sqrt{\cdots}}$	$\dfrac{z}{\sqrt{1+z}}R_4$
Angular size (expanding objects)	$\dfrac{d_0(1+z)}{c/H_0}\Big/\displaystyle\int_0^z \frac{dz'}{\sqrt{\cdots}}$	$\dfrac{d_0}{zR_4}\cdot\dfrac{\theta}{\sin\theta}$

4.2. The K-correction and its theoretical assumptions

Supernova catalogues such as Pantheon+ [2] do not report bolometric magnitudes. In the standard data reduction, observed magnitudes are subjected to a K-correction that removes two factors of $(1+z)$ from the received flux: one for the redshift-induced reduction of each photon's energy, $E_{\rm obs} = E_{\rm em}/(1+z)$, and one for the cosmological time dilation of the photon arrival rate. The combined effect is a flux reduction by $(1+z)^2$, equivalent to an increase in the distance modulus of $2.5\log(1+z)^2$.

This correction is internally consistent within ΛCDM. However, when DU predictions are compared against K-corrected catalogue data, the correction must be applied explicitly to the DU bolometric prediction, and its theoretical basis must be recognized as a ΛCDM-convention assumption rather than a theory-neutral operation. In the DU framework, the energy carried by a cycle of radiation is conserved during cosmological propagation: as the wavelength stretches with the expansion of space, the energy per cycle remains constant relative to the total energy of the universe, and it is the energy density that decreases, not the energy of the radiation itself. This distinction follows directly from the DU treatment of Planck's equation $E = hf = h/T$ and the intrinsic Planck constant $h_0 = h/c$.

The DU bolometric distance modulus is

$$\mu_{\rm DU,bol} = 5\log(R_H/10\,{\rm pc}) + 2.5\log\left[\frac{z^2}{1+z}\right]. \tag{8}$$

Adding the K-correction $2.5\log(1+z)^2$ yields the DU prediction in the ΛCDM-convention framework:

$$\mu_{\rm DU} = 5\log(R_H/10\,{\rm pc}) + 2.5\log\left[z^2(1+z)\right]. \tag{9}$$

Equation (9) contains no adjustable cosmological parameters. The corresponding ΛCDM expression requires specification of Ω_m and Ω_Λ determined by fitting to the data.

The conversion from bolometric to K-corrected magnitudes is not a theory-neutral calibration step; it embeds specific physical assumptions about the behaviour of photon energy and arrival rate during cosmological propagation. Applying this correction explicitly and transparently — rather than treating the corrected data as raw observables — is the appropriate response, and it is what equation (9) represents.

4.3. Observational confrontation: Pantheon+ and SN 2023adsy

Figure 5 shows the Pantheon+ Hubble diagram [2] together with the high-redshift JWST Type Ia supernova SN 2023adsy at $z \approx 2.9$, see [18], spanning the redshift range $0.001 < z < 2.9$ with over 1500 supernovae from multiple surveys.

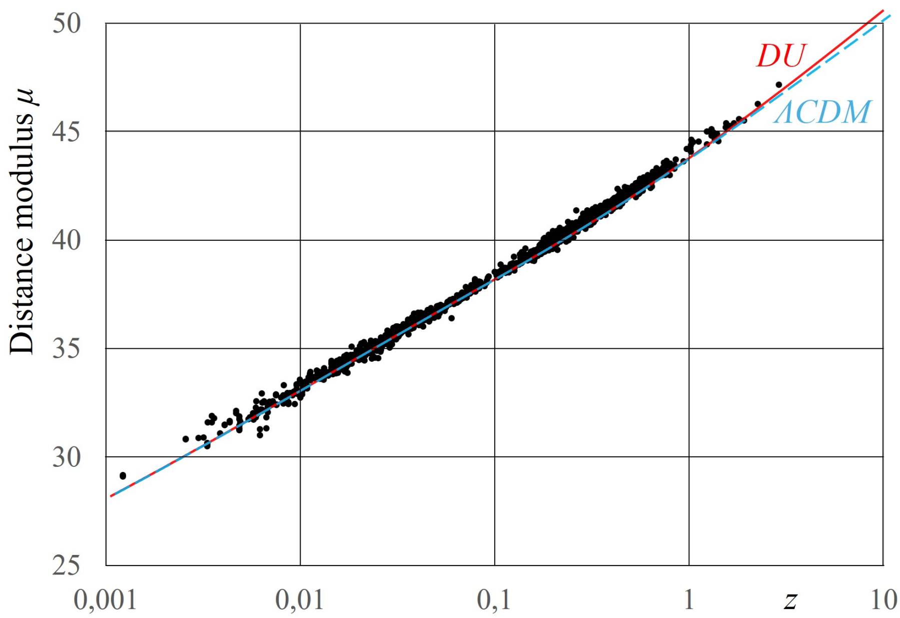

Figure 5: Type Ia supernova distance modulus versus redshift. Data points are from the Pantheon+ Hubble diagram [2]. The high-redshift JWST Type Ia supernova SN 2023adsy at $z \approx 2.9$ is added from [18]. The DU prediction (red curve) is the closed-form expression (9), derived from the geometry and expansion dynamics of spherically closed space and converted to the K-corrected framework. The ΛCDM fit (blue dashed curve) uses $\Omega_m = 0.334$ and $\Omega_\Lambda = 0.666$ from the Pantheon+ analysis. Both predictions are consistent with the data across the full redshift range; the DU prediction contains no adjustable cosmological parameters.

The DU prediction (9), plotted directly against the data with no fitting procedure, traces the supernova data from $z \approx 0.001$ to $z \approx 2.9$ at least as well as the two-parameter ΛCDM fit with $\Omega_m = 0.334$ and $\Omega_\Lambda = 0.666$.

The physical interpretation of this agreement is worth stating explicitly. The curvature of the Hubble diagram at $z > 0.5$ — the feature whose discovery earned the 2011 Nobel Prize and that in ΛCDM is attributed to accelerated expansion driven by dark energy — is reproduced in the DU by the geometry and energy dynamics of the expanding 3-sphere alone. What standard cosmology interprets as evidence for a cosmological constant Λ, the DU identifies as the natural shape of the magnitude–redshift relation in spherically closed space.

A visible divergence between the two predictions begins to develop at $z > 2$. The inclusion of SN 2023adsy at $z \approx 2.9$, well beyond the range used to calibrate the ΛCDM parameters, provides a first out-of-sample discriminating test. The growing separation of the curves suggests that a sample of supernovae at $z > 2$ — now becoming accessible with JWST — will provide a decisive observational test between the two frameworks in the near future.

5. Local Hubble expansion: Ranging measurements and geological evidence

5.1. Local expansion as a necessary prediction

In the DU framework, the expansion of space is not a background phenomenon that affects only cosmological distances while leaving local systems untouched. The rest energy of every mass in space is determined by the globally available energy, which evolves with R_4. As R_4 increases, the natural length scales of all gravitationally bound systems expand in direct proportion, following $r(t) = r_0(t/t_0)^{2/3}$. This is qualitatively different from ΛCDM, where gravitationally bound systems are decoupled from the cosmological expansion by construction. Local expansion in DU is not an assumption but a necessary prediction, subject to direct observational tests at Solar System scales.

5.2. What ranging experiments actually measure

Modern ranging experiments — whether Lunar Laser Ranging (LLR) or planetary transponder measurements — do not measure geometric distance directly. They measure round-trip light travel time using atomic clocks. The physical observable is the number of clock cycles $n = f \cdot T$ accumulated during the signal propagation time T. Converting this cycle count to a distance requires a model for the relationship between clock frequency, velocity of light, and coordinate framework — and it is precisely here that the DU and GR interpretations diverge.

In standard GR data reductions, the clock observable is carried through the IAU time-scale transformation chain (proper time → TT → TCG → TCB → TDB). As shown by Brumberg [3], expansion terms are systematically eliminated through this chain — an effect he termed the “Einstein effect.” In the GR reduction, the observed increase in the Earth–Moon distance is attributed entirely to tidal dissipation, with no cosmological component.

In the DU framework, no such transformation chain is needed. The clock frequency and the velocity of light are both determined by the same locally available rest energy, equation (3). Since both depend on the same energy state, the number of clock cycles is directly proportional to the physical distance at that epoch. The cosmological expansion appears directly in the ranging data as a real secular trend, not absorbed into a coordinate convention. Discriminating between the two interpretations requires measurements that are not based on atomic clocks — precisely the role played by the geological record.

5.3. Lunar Laser Ranging

The currently accepted secular increase of the Earth–Moon distance from LLR is approximately 3.82 cm/yr. In the standard GR interpretation, this entire recession is attributed to tidal dissipation. This attribution has been noted as problematic: independent estimates of tidal dissipation from satellite laser ranging, ocean tide models, and the historical eclipse record consistently yield rates substantially below 3.82 cm/yr.

In the DU framework, the observed recession decomposes naturally as

$$\dot{d}_{EM} = \dot{d}_{\text{cosm}} + \dot{d}_{\text{tidal}} \approx 2.8 + 1.0 = 3.8 \text{ cm/yr}, \tag{10}$$

where the cosmological component is $H_0 \times d_{EM} \approx 70\,(\text{km/s})/\text{Mpc} \times 384400\,\text{km} \approx 2.8\,\text{cm/yr}$, and the tidal residual of $\approx 1.0\,\text{cm/yr}$ is consistent with independent tidal estimates. The long-standing tension between the LLR-observed recession and independent tidal constraints dissolves naturally in the DU decomposition.

5.4. The annual eccentric-orbit test

As the Earth moves from perihelion to aphelion, the depth of the solar gravitational potential changes. In the DU framework, local distance scales are set by the zero-energy balance in the full hierarchy of nested energy frames. The annual variation of the solar potential and orbital velocity therefore produce an annual variation of the Earth–Moon geometric distance of approximately 12.6 cm between perihelion and aphelion. However, this variation is predicted to be exactly unobservable in LLR clock-cycle measurements: the fractional change $\Delta n/n = \Delta f/f + \Delta T/T$ vanishes exactly, because the increase in clock frequency at aphelion and the decrease in signal propagation time cancel to all relevant orders. The DU prediction is therefore consistent with the absence of any anomalous annual signal in LLR data, while maintaining that the geometric variation is physically real.

5.5. Geological evidence for local expansion

The most striking independent evidence for local expansion comes from three geological datasets analyzed by Sipilä [21], each probing the long-term evolution of Solar System distances through physical records entirely independent of atomic clocks.

Coral fossil data. Coral growth layers record the number of days per year going back hundreds of millions of years [26, 27, 25, 4]. The number of days per year is governed by tidal friction (which slows the Earth's rotation) and the expansion of the Earth–Sun distance (which lengthens the year). Under the no-expansion hypothesis, using the standard tidal friction rate of 2.5 ms per century [15], the predicted number of days per year falls systematically below the coral fossil observations over the past 900 million years. When orbital expansion at $H_0 = 70\,(\text{km/s})/\text{Mpc}$ is included, the prediction matches the coral record over the full span, Figure 6. Treating H_0 as the unknown and fitting to the coral data with the tidal friction rate fixed yields

$$H_0 = 70 \pm 2 \text{ (km/s)/Mpc}, \tag{11}$$

derived entirely from paleontological data, without any cosmological observation.

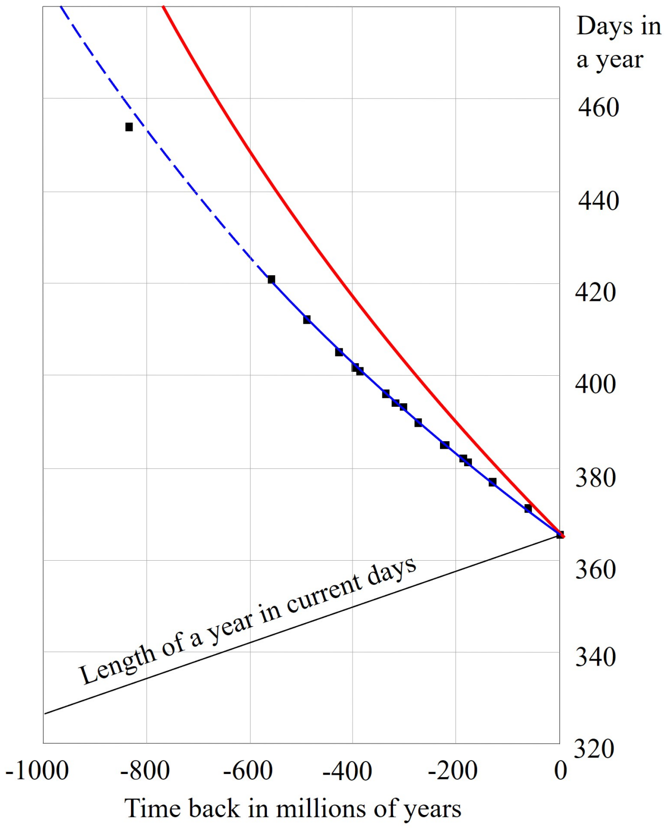

Figure 6: Days per year from coral fossil data over the past billion years. Black points: observational data spanning 100–850 million years [4, 25]. Red curve: prediction from tidal friction alone at 2.5 ms/century [15]. Blue curve: DU prediction incorporating both space expansion and tidal recession. Black curve: length of year in current days, showing year lengthening with expansion.

Tidalite data. Sandstone tidal rhythmites near Townsville, Australia, dated to approximately 635 million years ago, record the number of lunar months per year, see Williams [28]. Under the no-expansion hypothesis, the implied average Earth–Moon recession rate is only 2.1 cm/yr, substantially less than the modern LLR value of 3.82 cm/yr. Under the expansion hypothesis, applying Kepler's third law yields an average recession rate of 3.85 cm/yr — in close agreement with the modern LLR measurement — and a tidal component of ≈ 1.1 cm/yr, consistent with independent tidal estimates by Sipilä [21]. Calculations by Křížek and Somer [12] show that tidal recession is not enough to explain the observed lunar recession and requires local expansion of about 1.7 cm/yr based on their estimate of the tidal recession 2.1 cm/yr.

The Faint Young Sun paradox. Standard stellar evolution models indicate that the Sun's luminosity was approximately 25 % lower 3.85 billion years ago. Under the no-expansion hypothesis, this implies Mars was too cold for liquid water — yet geological evidence shows Mars had substantial surface water at that epoch. Under the expansion hypothesis, the Mars–Sun distance was proportionally smaller, placing Mars within the habitable zone despite the lower luminosity. No anomalous greenhouse atmosphere or additional heat source is required.

5.6. Convergence and significance

The three geological lines of evidence — coral fossils, tidalites, and the Faint Young Sun constraint — span timescales from 635 million to 3.85 billion years and involve entirely different physical recording mechanisms. Each is independently consistent with local expansion at $H_0 \approx 70\,(\text{km/s})/\text{Mpc}$, and each poses a difficulty for the no-expansion hypothesis unresolved within the standard framework. Their convergence on a single value of the Hubble constant, derived entirely from Solar System data, constitutes theory-independent local evidence for the expansion of gravitationally bound systems at the Hubble rate. Together with the LLR decomposition, they form a coherent observational chain spanning more than fifteen decades in timescale, all consistent with a single physical picture in which local and cosmological expansion are manifestations of the same underlying zero-energy dynamics.

6. Discussion and conclusions

6.1. The JWST observations in the DU perspective

The observations of the James Webb Space Telescope constitute the most direct current challenge to the standard cosmological model, and they are the natural focus for assessing what the DU framework adds. The challenge operates on two levels, and the DU addresses both from the same foundational principle.

At the level of structure formation, JWST has revealed massive, morphologically mature galaxies at $z > 10$ whose existence strains the standard timeline. In the DU, no fine-tuning is needed. The velocity of light — and with it the energy density and all physical process rates — evolves as $c_0 \propto T^{-1/3}$, so the early universe was intrinsically a high-energy state in which gravitational collapse proceeded at rates proportional to the available energy density. Massive structures form rapidly because the energy driving their formation was dramatically higher, not because any special mechanism is invoked. The energetically rich early universe is not an additional hypothesis; it is a direct consequence of the zero-energy expansion dynamics of equation (2). At the same time, the contraction–expansion cycle of the DU eliminates the most dramatic postulate of ΛCDM — the instant creation of matter and energy from nothing — replacing it with a physically transparent process in which rest energy is obtained against the release of gravitational energy, in the spirit of Aristotle's *entelecheia*, the actualization of potentiality.

At the level of angular sizes, JWST provides the most precise test of the two frameworks' geometric predictions over the widest redshift range yet available. DU predicts a monotonic decrease of galaxy angular sizes with redshift, reflecting the expansion of gravitationally bound systems with space and Euclidean scaling on the 3-sphere. ΛCDM predicts a non-monotonic inversion above $z \approx 1.5$. The JWST data follow the monotonic DU trend without exception across the full observed range. The Capotauro candidate at $z \approx 31$, if confirmed, places an observational point in the regime of maximum divergence between the two predictions, and the associated counter-image prediction at $z_1 \approx 15.7$ constitutes a falsifiable geometric signature unique to closed 3-sphere space.

6.2. The broader observational picture

The JWST results are the most vivid current evidence for the DU framework, but they are part of a broader observational picture spanning fifteen decades in redshift. The parameter-free DU prediction matches the Pantheon+ supernova Hubble diagram from $z \approx 0.001$ to $z \approx 2.9$ at least as well as the two-parameter ΛCDM fit, attributing the curvature of the diagram to the geometry and energy dynamics of the expanding 3-sphere rather than to dark energy. Local expansion — a necessary prediction of the same zero-energy dynamics — is supported by the LLR decomposition and by three independent geological datasets converging on $H_0 \approx 70\,(\mathrm{km/s})/\mathrm{Mpc}$ from Solar System data alone. Together, these results establish that the standard physical ontology of ΛCDM — dark energy, dark matter, inflation, and instant creation — is not required by the observational data as they currently stand. The DU achieves this from two postulates and a single shared empirical input, $R_H = c/H_0$, with no adjustable cosmological parameters.

6.3. Open questions and the road ahead

Quantitative treatments of the CMB acoustic spectrum, Big Bang nucleosynthesis, gravitational waveforms for compact binary mergers, and large-scale structure formation have not yet been completed within the DU framework. These are open tasks, not fundamental obstacles — ΛCDM itself required decades of development from its foundational equations to its current quantitative precision, a process still ongoing as evidenced by the Hubble tension and the S8 tension. The near-term observational agenda is clear: the growing JWST supernova sample at $z > 2$, where the DU and ΛCDM predictions visibly diverge, will provide a decisive test. Deep-field searches opposite to Capotauro will test the counter-image prediction. And a systematic analysis of JWST angular sizes at $z > 15$ will extend the angular-size confrontation into the regime where the two frameworks differ most strongly.

6.4. Conclusions

The Dynamic Universe framework, grounded in 3-sphere geometry and the global zero-energy balance as its primary postulate, offers a physically transparent account of what the James Webb Space Telescope observes. The high-energy state of the early universe follows necessarily from the same zero-energy expansion dynamics that governs the present epoch, providing ample energy density for the rapid formation of the massive and mature structures that JWST reveals at $z > 10$. The monotonic decrease of galaxy angular sizes across the full JWST redshift range — with no evidence of the angular-size inversion predicted by ΛCDM — is a direct consequence of the 3-sphere geometry and the expansion of gravitationally bound systems with space. These are not post-hoc explanations but predictions derived from two foundational postulates, without adjustable parameters.

The same postulates predict local Hubble expansion, supported by an observational chain from Lunar Laser Ranging through three independent geological records converging on $H_0 \approx 70$ (km/s)/Mpc from Solar System data alone. The same geometry and energy dynamics reproduce the supernova Hubble diagram without dark energy.

The Dynamic Universe does not yet answer every question that ΛCDM addresses quantitatively, but it answers the questions that ΛCDM answers most awkwardly — the origin of the energy of matter, the rapid formation of early structure, the angular sizes of high-redshift galaxies, and the local expansion of gravitationally bound systems — from a single coherent principle, without a patchwork of undetected components. The JWST observations, far from challenging the DU framework, are among its strongest current support.

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EARLY RECEPTION OF THE DYNAMIC UNIVERSE

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Abstract: Tuomo Suntola's *Dynamic Universe* (DU) is now 30 years old. DU is the result of Suntola's pursuit of unity and understanding in physics. DU is an alternative to relativistic physics in the same way that the system-oriented heliocentric model was an alternative to the observer-oriented geocentric model: while in the special theory of relativity a freely selected observer is in the state of rest, DU places the observer into an overall system with a clear geometry and laws, where the state of rest is the center of the universe. DU's causal-mechanical basic structure provides an understandable and unified explanation of central relativistic phenomena, and provides the basis for their mathematical descriptions. Unlike relativistic physics, DU is compatible with basic quantum mechanics and suggests an ontological interpretation for it. In this work, I review how DU has been received over the years. The overall pattern is that people whose theoretical and methodological commitments align with the pursuit of unity and understanding tend to perceive DU positively or neutrally. In contrast, those strongly committed to relativistic physics and contemporary instrumentalism —that settles for describing phenomena mathematically and disregards the goal of giving understandable causal explanations— tend to dismiss DU without investigating it.

Keywords: the dynamic universe, tuomo suntola, paradigm shift, dogmatism, stagnation, general theory of relativity, ΛCDM, cosmology

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1. Suntola's Classical Methodology

Tuomo Suntola's philosophical and scientific methodology aligns with the classical ideal of science, where a physical theory with few simple postulates provides understandable causal explanations and accurate mathematical descriptions of phenomena in its domain. It is not enough to produce mathematical descriptions that characterize the behavior of objects, but a physical theory must make their behavior understandable by explaining why they behave as they do, and provide a mathematical description of their behavior based on this explanation. Newtonian mechanics is a classical example of this kind of theory.

In the 20th century, classical physics turned out to be inadequate; therefore quantum mechanics was developed, and Newtonian mechanics was replaced or corrected by the special theory of relativity. These mathematics-driven theories do not conform to the classical ideal: quantum mechanics still lacks a consensual ontological interpretation that would make the focal phenomena understandable; the increasingly complex theory structure of relativistic physics fails to make nature understandable to a satisfactory degree. Furthermore, quantum mechanics and general relativity do not cohere.

Suntola decided to fix theoretical physics, so that its fixed version respects the classical ideal. And the Dynamic Universe (DU) is Suntola's remedy. Suntola argues that DU gives an understandable causal explanation for relativistic, celestial, and cosmological phenomena —as consequences of energy conservation— and describes them mathematically. Thus, DU totally replaces relativistic physics (RP), which comprises the special theory of relativity (SR), the general theory of relativity (GR), and GR-based relativistic cosmology (ΛCDM). DU is compatible with basic quantum mechanics (QM), and proposes an understandable causal mechanism for QM.

Suntola's means of fixing physics resemble those of Newtonian mechanics: (i) characterize the overall geometry of space where physical objects reside; (ii) explain their causal interactions by a few simple dynamic law hypotheses, applying a minimal number of auxiliaries; (iii) produce accurate predictions of their behaviour in terms of this causal-mechanical basic structure. Thus, although the content of the two theories is entirely different —their geometries and their law hypotheses are different, and while in Newtonian mechanics force is the primary quantity, in DU energy is the primary quantity— their methodological nature is very similar.

One could wish that DU or any other theory that conforms to the classical ideal would be welcomed with open arms as a remedy for contemporary theoretical physics. However, the most common reaction has been to reject DU without investigating it. The apparent reason behind this reaction is the widespread conviction that RP and its underlying methodology are correct. In turn, the reason this idea has become widespread is found in history. Namely, for the past 100+ years, RP and QM have produced mathematical descriptions of the behavior of physical objects, but have not provided an understandable causal mechanism that allows genuine understanding of their behavior across different scales of phenomena. As a result of their 100+ years dominance, the methodology of physics has gradually been transformed into *instrumentalism* that conforms to these theories and legitimizes them. Thus, instead of seriously doubting RP and QM due to their violation of the classical ideal, the classical ideal itself has been replaced by instrumentalism, just because mathematical descriptions have been all that RP and QM have been able to offer.

In the instrumentalist philosophy of science, the primary function of physical theories is to describe phenomena mathematically, whereas the task of understanding phenomena has been all but abandoned. That is, the concept of understanding has been revised to conform to instrumentalism, so that it now means grasping how a theory is applied in describing phenomena mathematically. In contrast, Suntola's aim has always been a theory that is understandable in itself and that enables genuine understanding of phenomena, and which in turn functions as the basis of describing the phenomena mathematically. Therefore, before entering the debate on the plausibility of DU, you should first and foremost ask what you want from physics. Is instrumentalism enough for you, or do you consider the classical ideal respectable?

The article is organized as follows. A short history of DU and Suntola's technological career is given in §2. The reception of DU is reviewed in §3. In §4, the review is analyzed. Most of the quotations are translated from Finnish by the author.

2. A Short History of the Dynamic Universe

Tuomo Suntola was interested in technology and manual skills from childhood. He built his first electrical device, a transistor radio, at the age of 12, when the first transistors became available in Finland. During his teenage years, he constructed radios and audio amplifiers, and, e.g., wooden miniature replicas of World War II fighter planes.

Tuomo began his studies in Electrical Engineering at Helsinki University of Technology in Otaniemi in 1963. He completed his Master of Science degree in 1967 and his Doctorate in Technology in 1971. He worked at the university's Electron Physics Laboratory from 1967 to 1971, then at the Semiconductor Laboratory of VTT Technical Research Centre of Finland from 1972 to 1973, where he developed the Humicap humidity sensor for Vaisala Oy. Vaisala developed this technology further and became a world leader in advanced humidity measurement. The Humicap sensor technology has been in continuous use and development ever since.

Currently, Tuomo lives in Kilo, Espoo. The architecture of his house was designed by his late wife Soilikki. Tuomo organized the building of his house between March and December 1973, and did its electrical work and most of its woodwork. The only major renovation to the house has been the replacement of the flat roof with a shed roof in the 1990s, as the original roof drains were prone to clogging.

In 1974, Tuomo was invited to establish and lead Instrumentarium's research unit, where he invented Atomic Layer Deposition (ALD) technology as a solution for manufacturing electroluminescent displays. ALD enables the growth of material one atomic layer at a time in a self-regulating process. Instrumentarium sold the EL/ALD project to Lohja Oy in 1978, and Tuomo moved with it. The project was completed successfully, and Lohja Oy began production of ALD-based displays in 1985. Tuomo continued working there until 1987, when Microchemistry Oy was founded as a subsidiary of Neste Oy to develop further applications for ALD. Tuomo became its Managing Director, first leading the development of ALD for photovoltaic devices and catalysts, then for machinery and technology for semiconductor manufacturing. Microchemistry was subsequently sold to ASM in 1999, which became the world leader in the technology.

Tuomo worked at Fortum from 1998 until retiring at age 60 in 2004. Thereafter, his connection to ALD continued as a Board member of Picosun Oy from 2004 to 2022 when it was acquired by Applied Materials, Inc. In 2004, he received the European SEMI Award for applying ALD to semiconductor component manufacturing, and in 2018, he was awarded the Millennium Technology Prize for ALD's central role in improving integrated circuit performance. Today, ALD is an enabling technology for modern semiconductor devices, and its range of applications is still expanding.[1]

The seeds of the Dynamic Universe were planted long before the theory itself took shape. Tuomo and his colleague Heikki Kanerva were both working at the Electron physics laboratory and VTT between 1967 and 1973. They engaged in deep discussions about the possibility of a comprehensible picture of reality. This discussion was intertwined with the project of understanding the theory of relativity and time. Time dilation —the idea that the metre and the second, or equivalently the local rate of the flow of time, vary from place to place— never sat well with Tuomo's thinking. He concluded that the quite incomprehensible relativistic spacetime geometry is perhaps not the only alternative for describing reality, and that perhaps the correct coordinate system had not yet been found. This initiated Tuomo's quest for a new perspective.

Kanerva found a book at an antiquarian bookshop, a newer edition of [32], containing original publications related to the birth of relativity theory. It included Einstein's 1917 cosmology article [8], which suggested a static 3-sphere geometry for the universe. Here, the familiar three-dimensional space is replaced by the 3-sphere, and its radius is the fourth, metric, dimension: a genuine spatial dimension, perpendicular to the three spatial directions. Kanerva became fascinated by the concept of an expanding 3-sphere —an idea that would later prove pivotal for DU. At that time, however, the connection of a 3-sphere to the theory of relativity and the problem of time did not open up. When Kanerva moved to Turku in 1978, he left the book and the problem for Tuomo.

From 1974 to 1990, Tuomo's career took him into the world of industrial innovation. The philosophical questions about relativity and a comprehensive picture of reality were put on hold, with one important exception. In the early 1980s, Tuomo tried to understand the worldview behind special relativity with the aid of vector diagrams that model the momentum mv (where m is the mass of an object and v is its velocity) as a complex quantity. This contemplation was directly related to the expression (x, y, z, ct), which represents the coordinates of an event in the 4-dimensional Minkowski spacetime. If one supposes that a mass object possesses momentum in spacetime, this momentum can be decomposed into two components: a spatial momentum directed along the (x, y, z) coordinates, which may be regarded as the real component of a complex number; and a momentum in the fourth dimension along ct, which may be regarded as the imaginary component. Thus, the theory of relativity defined the fourth dimension as a temporal dimension, and it was very difficult for Tuomo to consider that momentum is pointed in the direction of time.

The role of these contemplations in DU was to be crystallized by the media-

[1]See Puurunen [39] for the history of ALD until 2014.

tion of a new figure who entered the scene in the early 1990s. Jaakko Kajamaa, a Doctor of Science in Technology, was grappling with the foundations of physics. Kajamaa sought a physicist with whom he could discuss his unconventional ideas, which had been summarily dismissed by the physicists he had approached. No one was interested except for Tuomo. From 1991 to 1996, the two engaged in intensive discussions.

One of Kajamaa's many goals was to correct or to reformulate the relativistic expression for the total energy of a mass object in space:

$$E^2 = (cp)^2 + (mc^2)^2.$$

Here, cp denotes the object's kinetic energy or in Kajamaa's terms its energy of motion, the product of the velocity of ligth c and the object's momentum in space $p = mv$. mc^2 denotes the object's rest energy, or in Kajamaa's terms the energy of its mass. Kajamaa's [12, p. 140] alternative expression was:

$$E = pc + mc^2.$$

Kajamaa presented numeric examples of applying the formula, which Tuomo showed to be erroneous. Kajamaa did not provide a clear foundation or a physical meaning for this expression, which was the central interest of Tuomo. In February 1996, during the skiing holiday, Tuomo discovered the solution, by reflecting on his 1980s contemplation of momentum as a complex quantity. Tuomo wrote Kajamaa's equation in a complex form as

$$E = c(p + \mathrm{i}mc)$$

which resembles the Dirac equation. Here, $p = mv$ can be interpreted as the object's momentum in space, and the term $\mathrm{i}mc$ can be treated as the imaginary component of the complex momentum. The remaining problem was to figure out the ontological interpretation for the imaginary momentum $\mathrm{i}mc$. This required considering the fourth dimension as a metric dimension, where space, and so all objects in space, move at velocity c. (In fact, although in the theory of relativity the fourth dimension is called time, it is measured in meters: the unit of ct is meters — $[m/s] \times [s] = [m]$).

The consecutive question was: Where is space moving then? This is where the 3-sphere came in, an idea conveyed through Kanerva in 1967–1973. Space, as the surface of an expanding 3-sphere, does not move anywhere. Instead, c denotes the velocity at which the radius of the 3-sphere increases. This choice, so to speak, made Einstein's 1917 3-sphere dynamic. That is, Tuomo replaced the time-like fourth dimension of the relativity theory with a metric fourth dimension.

This insight raised a question: why do space and all mass objects in space — on the surface of the 3-sphere— have the velocity c in the fourth dimension? The 3-sphere geometry and the idea that all mass lies on its surface opened up further components of what was to become DU's basic structure. Since all mass in the universe moves in the metric fourth dimension, the universe has a total energy of motion E_m. And since all mass resides within a distance from the center of the 3-sphere, the universe has a total energy of gravitation E_g, which is the main form of potential energy.

Tuomo immediately checked the quantities of E_m and E_g. As c was known, checking the magnitude of $E_m = Mc^2$ required only the calculation of M. The current estimate of the Hubble radius, interpreted as the radius of the 3-sphere, R_4, gave the volume of the universe. The volume times the current estimate of the critical mass density of the universe gave M. The total gravitational energy could be calculated as $E_g = \frac{GM^2}{R_4}$, where G is the gravitational constant. The calculation showed that E_m and E_g are essentially equal. This indicated that the universe could be considered as a spherical pendulum moving in the fourth dimension, a process of contraction and expansion. The current energy of motion Mc^2 was obtained from the preceding contraction stage of the universe, so there was no need for a mysterious Big Bang or quantum fluctuations as its cause.

These ideas are expressed by DU's energy balance equation [69, eq. 1.2.1:1], which characterizes the zero-energy principle within the 3-sphere. The equation states that the sum of E_m and E_g is zero:

$$E_m + E_g = 0.$$

When written out, the equation appears as:

$$c_0^2 M - \frac{GMM''}{R_4} = 0.$$

M is the total mass of the universe, situated on the surface of the 3-sphere. R_4 is the radius of the universe, i.e., the radius of the 3-sphere. c_0 is the velocity of the expansion of space, i.e., the velocity of the increase of R_4. M'' is the mass equivalence, i.e., the total mass of the universe considered to be situated at the center of the 3-sphere. G is the gravitational constant.

Tuomo decided to consider a single contraction-expansion cycle. The contraction starts from the initial state of rest very far in the past, where R_4 is, figuratively speaking, infinite.[2] In the initial state, E_m is 0, which is the minimal energy of motion, and E_g is 0, which is the maximal energy of gravitation. The contraction was caused by E_g, and E_m is obtained as kinetic energy against the release of E_g. As the contraction advances from the state of maximal expansion to the singularity, E_g varies in the interval $[0, -\infty]$, and E_m varies in the interval $[0, \infty]$. As the contraction advances, E_g increases in the negative direction toward $-\infty$ at the singularity, which is the minimal gravitational energy, and E_m increases toward ∞ at the singularity, which is the maximal energy of motion. In turn, from the singularity onward, toward

[2]The talk of infinite magnitudes is figurative. Tuomo states in [69, p. 78]: "It is natural to think about the possibility of closing also the fourth dimension which would turn the expansion of space back to a new contraction and expansion cycle."

the forthcoming maximal stage of expansion, E_m decreases from ∞ toward 0, whereas E_g increases from $-\infty$ toward 0.

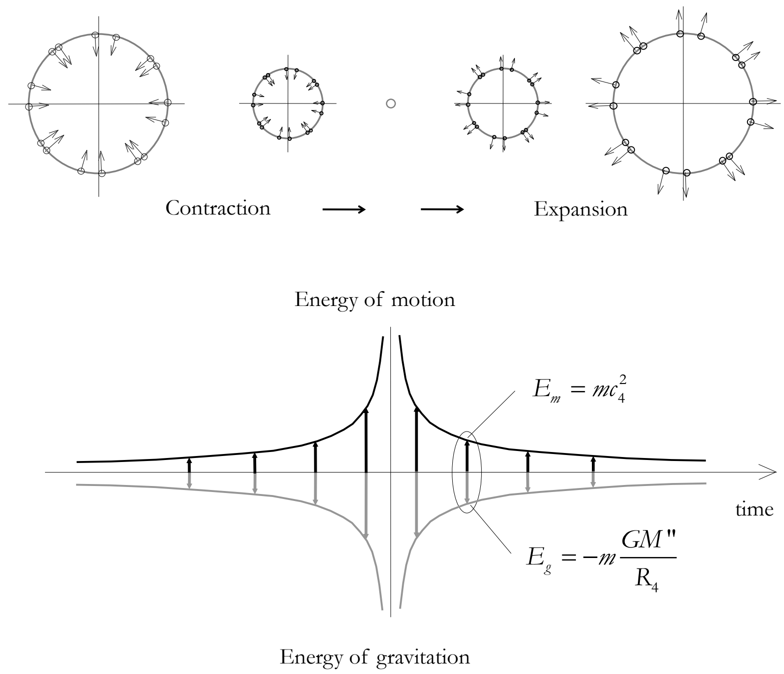

Figure 1: Energy buildup and release in spherical space [69, Sec. 1.2.1].

The energy of motion in the fourth dimension is identified as the rest energy of matter. This idea differs essentially from relativistic physics, where rest energy mc^2 is an intrinsic, irreducible property of an object. Thus, in DU, mc gains an unambiguous physical meaning: it is the rest momentum, i.e., the momentum of mass m in the fourth dimension. The famous equation $E = mc^2$ for the rest energy of m can be written in complex form as $E = c_0p = c_0mc$, where c_0 is the velocity of the expansion of space, and c is the velocity of m in the local fourth dimension. Yet, in hypothetical homogeneous space, $E = c_0p = c_0mc_0$.

Another great difference is that while in relativistic physics c is constant, in DU c is variable. Specifically, in DU c_0 is variable, and c_0 is also the maximum velocity of light in space. In DU, the local velocity of light c in space is variable also in another sense: it is a function of the local gravitational potential.[3]

[3]One may ask: How can c vary when it is always measured to be constant? Here c appears constant because our measuring instruments — atomic clocks — are subject to the same laws as the quantities they measure. That is, when we measure c with an atomic clock, the result is the same at all altitudes, because the clock's frequency changes in direct proportion to the change of c: light moves faster the higher you are, but the clock also ticks faster, so you always get the same result. Tuomo has often characterized this as what the late Finnish professor Raimo Lehti called *the conspiracy of the laws of nature.*

When M, G, and R_4 are inserted into the energy balance equation, the current c_0 and thus also, roughly, the current c, is obtained: $c_0 = \sqrt{\frac{GM''}{R_4}}$. The equation indicates that c_0 is variable. For the increase of Mc_0^2 in the contraction phase is possible only through the increase of c_0 —this is the only available alternative when M is considered a constant, which is the case in DU. Conversely, the decrease of Mc_0^2 in the current expansion phase is possible only through the decrease of c_0.

The next morning, after the discovery of the energy balance equation, Tuomo picked up the phone and called his colleague Eeva-Liisa Lakomaa at Microchemistry: "The speed of light is not constant," he told her. Soon after in 1996, Tuomo collected the basic ideas in what became the first DU book. In 1998, Tuomo was invited to present his emerging theory at The Finnish Society for Natural Philosophy (Luonnonfilosofian seura), a platform for bridging ideas in physics and philosophy. The venue was the historic Estates House (Säätytalo) in Helsinki. For the first time, the Dynamic Universe was publicly articulated, and the response was immediate. In the audience that day were three individuals who were captivated by the vision Tuomo presented, and who would become parts of the DU story: Heikki Sipilä, Ari Lehto, and Tarja Kallio-Tamminen.[4]

Later, they gathered with Tuomo to discuss the implications of DU, the need for a community to explore these ideas, and the philosophical and scientific challenges ahead. The conversation marked the birth of what would first become the Universe Club. Its first meetings were held at VTT Micronova where Ari Lehto worked, and later regularly at Tuomo's house in Kilo. Later, the group also had pleasant summer gatherings at Ari Lehto's cottage, and once met at Marskin maja in Loppi. In addition to the four founding members, the visitors at the meetings included, inter alia, Jouko Seppänen, Paul Talvio, and Pentti Passiniemi. In 2008, the Physics Foundations Society was registered. Among the regular members in the Physics Foundations Society meetings since the beginning have been Mervi Hyvönen-Dabek and Jan Dabek.

Together, Tuomo, Ari, Heikki, and Tarja represented a remarkable convergence of deep technical expertise and philosophical curiosity. Their technologies —Tuomo's humidity sensor from 1973, Heikki's gamma and X-ray detectors, and Ari's pressure sensors— had been applied widely and found their way even into spacecraft and Mars rovers. Now, they turned their collective intellect toward the foundations of physics. Tuomo's focus has been on the development of DU, Tarja's on the connection of DU with quantum mechanics, and Heikki's on the DU explanation of phenomena within the Solar System, specifically on the expansion of the Solar System, and on fundamentals such as Mach's principle. Ari has focused on the development of his own theory, Period Doubling [24, 25, 26, 27, 28, 29, 30], which coheres with DU.

[4]Tuomo had met Ari while working at Instrumentarium in the 1970's. Tuomo had met Heikki at a leadership camp in the summer of 1960, but they had not interacted in the meanwhile, except for participating in the same lectures of Einar Stenij on mechanics in Helsinki University of Technology in Otaniemi.

I met Tuomo through the Finnish Society for Natural Philosophy. In 2011, I started as the secretary of the society, while Tuomo acted as its Treasurer, and before long we started communication about physics-related issues. Well before I met Tuomo, I had understood that comprehensible metaphysics cannot be done in the context of relativistic simultaneity, and so I sought ways out. Tuomo offered a comprehensive alternative: absolute simultaneity as the starting point. I found this more straightforward than the neo-Lorentzian path of adding a privileged reference frame to the special theory of relativity, as Franco Selleri, William Lane Craig, and Michael Tooley have suggested. Since then, I have participated in the organization of several events and the preparation of works concerning DU. On one hand, for me, philosophy of physics remains instrumental for making absolute simultaneity and eternal universe scientifically respectable metaphysical concepts. On the other hand, I see great prospects in applying the classical method of science in metaphysics, and in this sense the criteria for a good metaphysical theory and a good physical theory are closely aligned (Styrman [56]).

The latest chapter in the development of DU is the project *Appearance and Reality in Physics and Beyond* at the University of Helsinki, Faculty of Humanities, in the Department of Philosophy, History, and Art Studies. The project has been scheduled to last from June 1, 2023, to the end of 2027. Tuomo has funded the project. In addition to DU, it investigates the Bohmian interpretation of quantum mechanics and the philosophy of physics in general. The principal investigator of the project is Paavo Pylkkänen, and I am a senior researcher. Saara Wuokko prepared her PhD in the project until the end of 2025. The central collaborators of the project are: the Physics Foundations Society; Marja-Liisa Kakkuri-Knuuttila, whose central interest lies in the links between DU and Aristotle, and who has studied the philosophy of science underlying Bohm's causal explanation model of quantum mechanics; Petri Lievonen, who has studied the mathematics of DU intensively; Michal Křížek, who arranges the Cosmology on Small Scales conference series in Prague and has visited us twice in Helsinki; and Paavo's Bohm-Hiley circle in Britain, which for a long time gathered around Basil Hiley, who passed away in 2025.

Tuomo has documented the development of DU in a sequence of updated monographs, which were titled *The Dynamic Universe* (1996–1999), *The Dynamic Universe: A New Perspective on Space and Relativity* (2000–2003), *Theoretical Bases of the Dynamic Universe* (2004), and *The Dynamic Universe: Toward a Unified Picture of Physical Reality* (2009–2018). The fifth edition of this sequence is currently under preparation. Each stage deepened the framework, revealing how the same principles could unify phenomena across scales. The 1996 book concerned the main principles only. Therefter, DU was gradually expanded to cover general physics, relativistic phenomena, cosmology and celestial mechnics, and DU was linked to quantum mechanics.[5]

[5]Covering relativistic phenomena required complementing DU by the system of nested energy frames, which were needed for connecting the local to the global. Expansion of DU into the quantum realm required considering mass waves as the ultimate constituents of mass objects. In DU mass

Tuomo documented the intellectual journey behind DU in three editions of *The Short History of Science — or the long path to the union of metaphysics and empiricism* (2012–2018) [70]. A Finnish language edition, *Tieteen lyhyt historia — vai pitkä tie luonnonfilosofian ja empirismin kohtaamiseen*, was published in 2018 [71].

Peer-reviewed papers on DU have been published by Suntola, his collaborators, and some commentators and critics in: *Apeiron* [57, 58] (2000), *Episteme* [59] (2002), *Galilean Electrodynamics* [60] (2003), the book *The Nature of Light: What is a Photon?* [61] and arXiv [62] (2005), AIP conference proceedings [63] (2006), *International Journal of Astrophysics and Space Science* [64, 65] and *Tieteessä tapahtuu* [66, 67] (2014), a PhD thesis [50] and proceedings of the *Scientific Models and a Comprehensive Picture of Reality* workshop in *La Nuova Critica* [47, 68, 51] (2016), *Physics Essays* [52] (2018), proceedings of the *Unification in Physics and Philosophy* workshop [72, 48, 16, 53] in *Journal of Physics: Conference Series* (2020), *Physics Essays* [73, 20] (2021–2022), proceedings of *Cosmology on Small Scales* conferences [74, 49, 54] (2022 and 2024), and in proceedings of *Physics and Reality* conference in *Journal of Physics: Conference Series* [31, 6, 75, 55, 17, 13](2025), in *Yliopisto-lehti* [36, 37, 42, 18, 80, 15] and in *Kanava* [19] (2025–2026), and in *Frontiers in Astronomy and Space Sciences* [77].

Tuomo and his colleagues have given presentations about the Dynamic Universe at numerous conferences, workshops, and other events in Finland and abroad. One may seek them out from the websites of the Physics Foundations Society[6] and the Finnish Society for Natural Philosophy.[7]

DU continues to be developed, discussed, and debated. Journal articles about DU, as well as proceedings of workshops and conferences arranged by the *Appearance and Reality in Physics and Beyond* research group, are under preparation. DU's journey from a student's quiet reflections in the 1960s, through discussions with Kanerva and a chance find in an antiquarian bookshop, through intense discussions with Kajamaa, to a fully articulated theory and decades-long dissemination and collaboration, is a testament to the power of persistent inquiry. The next section shows that a very high level of persistence is needed in defending DU, to match the level of repulsion and opposition it has faced and faces.

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waves come in two forms: standing waves and moving waves. Tuomo found the concept of a mass wave and the conception of a particle as a Compton resonator by re-interpreting the Planck equation and photon. His essential discovery was that the Planck equation should not be interpreted to describe radiation, but the emission of radiation. That is, the energy emitted in one period by the transition of a single elementary charge. Furthermore, Tuomo linked the Planck equation to Maxwell's equations. This derivation strengthened Tuomo's idea that the Planck constant contains an extra c, which he had inferred earlier from the relation of atomic clock frequency and c.

[6] `https://physicsfoundations.org/`

[7] `https://www.luonnonfilosofia.fi/`

3. The Reception of DU

Dismissive comments about DU are reviewed in §3.1. Miscellaneous comments are reviewed in §3.2, the debate in Yliopisto-lehti in 2025–2026 is reviewed in §3.3, and comments from the Bohm-Hiley circle are received in §3.4.

3.1. Neglect and Dismissal

In the following, I review outright negative and dismissive comments about DU, in chronological order. The overall decision-making mechanism behind these comments appears plainly as the acknowledgement that DU deviates from relativistic physics, and the conclusion that this deviation entails that DU is incorrect. In other words, DU is rejected because it goes against tradition. The quoted texts are translated from Finnish.

1. **1996** *No comments.* When Tuomo had assembled the basic ideas into the first DU book, he delivered it to, inter alia, several cosmologists and physicists, with the expectation that it would raise immediate interest. Nobody seemed to be interested. However, it turned out that Ari Lehto came across the first DU book in this process, which led him to participate in the 1998 seminar at the Estates House.

2. **1996** *Grant application.* Tuomo applied for a grant for developing DU from the Academy of Finland in 1996. From the expert statement:

   > The goal of the work is naive and based on an overly simplistic conception of a theory describing natural phenomena . . . Irrelevant . . . No contacts with other researchers, nor a broader knowledge of mathematical physics . . . No significant results can be expected . . . The grant shall not be awarded.

   History after 1996 has shown that 'No significant results can be expected' was a false prediction, with the proviso that one respects the classical ideal of science.

3. **Late 1990's** *No comments.* Tuomo, Mervi Hyvönen-Dabek, and Erik Spring went to Tuorla Observatory in the Southern Finland, to visit a renowned astrophysicist.[8] Tuomo presented DU to him, but he did not get interested. Mervi recollects: "I got the impression that he listened to Tuomo's presentation on DU without really commenting at all. I had a feeling that the trip was of no use, but at least we tried."

4. **2000** *We do not want to talk.* The Finnish Society for Natural Philosophy organized the seminar *Are there alternatives to the premises of the theory of relativity?* on May 11th 2000. Prior to the event, the panelists were asked to compare DU explanations of specific phenomena to their relativistic explanations: (1) Frequency,

[8]Mervi Hyvönen-Dabek is a long-time member of the Physics Foundations Society. Her colleague Erik Spring was a professor of Applied Physics at the University of Helsinki, for whom Tuomo completed a licentiate degree in medical electronics as a minor subject.

wavelength, and physical length changes of moving resonators; (2) Clock and electromagnetic radiation gravitational shift (comparison of clocks at different altitudes and frequency change of radio signal propagating in gravitational field); (3) Light travel time delay near mass center (Mariner 6 and 7 experiments and the difference between path lengthening and speed reduction contributions); (4) The effect of ellipticity of Earth's orbit on Earth-Moon distance measurements (based on round-trip light time or hypothetical interferometric measurement); (5) Free fall into a black hole (effective mass change of an object approaching the event horizon).

At the event, one of the panelists informed the chair that they were unwilling to discuss these questions. This was both surprising and disappointing, for the seminar was arranged to get comments from GR specialists. At the seminar dinner, one of the panelists asked Suntola about Mercury's perihelion precession. Tuomo had not contemplated the case earlier, but he promised to do this. After a few weeks, Tuomo sent him the DU analysis about the case. The panelist's comments reveal something about the sociology of physics:

> I familiarized myself with the text, but I do not master this kind of heavy-duty mechanics. If the result is that DU reproduces GR (at least as far as GR is testable), then it cannot be considered incorrect, and its merit is to offer an alternative presentation of GR. In that case it is of course publishable — congratulations! Possible predictions in which DU deviates from GR are of course interesting. I doubt, however, for reasons of tradition, that DU would gain supporters.

First, the panelist considers DU incorrectly as a model of GR. Second, the panelist makes the conditional statement: if DU reproduces GR at least as far as GR is testable, then DU is correct. The antecedent of the conditional is false, because DU does not reproduce GR and this is not even the goal of DU. Certainly, DU aims to reproduce the *correct* predictions of GR, but not the incorrect ones.[9] The panelist assumes that DU will not gain supporters even if it is correct, due to traditional reasons. In light of the present study, although DU has gained some supporters, the panelist's prediction turned out to be largely correct.

5. **2003** *It must be a coinsidence.* Tuomo participated in the colloquium *Human Approaches to the Universe: An Interdisciplinary Perspective* at the University of Helsinki, 26–27 September 2003, with the presentation *Universal Order in Absolute Time.* After the panel discussion, Tuomo had a brief conversation with a Finnish cosmologist about the notion of the zero-energy principle, namely, that the total rest

[9] Suntola [69, §1.2.6] argues that DU produces a stable orbit for Mercury, whereas three textbook GR equations that explain Mercury's perihelion shift entail that Mercury departs its orbit. For a single period, the increase is small and omitted as a second order effect by Weber [81, pp. 64–67], Berry [1, p. 83] and Foster and Nightingale [10, ch. 4.5]. However, in their solutions the expansion of Mercury's main axis entails its escape in less than 400,000 years.

energy of all mass in space is equal to its total gravitational energy. The cosmologist commented as follows: “It must be a coincidence . . . not worth further investigation.”

It is remarkable that a Finnish cosmologist rejects an idea that even celebrated physicists have embraced. Tuomo [77] reminds that DU rests on 3-sphere geometry and the zero-energy principle —the postulate that the positive energy of motion is exactly in balance with the negative gravitational energy— and points out that these have been independently recognized by renowned physicists, but prior to DU have not been united into a single dynamic theory. Schwarzschild [45] argued for a positively curved, closed geometry of space, and Einstein [8] adopted the 3-sphere geometry. The zero-energy principle has been contemplated by Tolman [78] (who showed that in a closed relativistic universe, the sum of all positive energy of mass and all negative gravitational energy is zero), Sciama [46] (who concluded that the sum of total inertial and total gravitational energy of a particle at rest is zero), and Feynman [9, pp. 9–10, 164] (who recognized that the condition $GM^2/R = Mc^2$ constitutes a ‘great mystery’ and also entertained the possibility that three-dimensional space is the surface of a four-dimensional sphere).

6. **2005** *Cancelled newspaper article.* An article about DU was scheduled to appear in Helsingin Sanomat, the leading Finnish newspaper. Two days before the article was to appear in a Sunday issue, a photographer came to Suntola’s house to photograph him. However, the article never appeared. No reason was given then or after —I have asked Helsingin Sanomat at least twice.

7. **2012** *Suntola is dangerous because he knows mathematics.* At a Finnish Society for Natural Philosophy meeting, while working in the Society board, I informed a physics professor that DU is coupled with mathematics that yields accurate predictions of several phenomena. He replied: “Suntola is dangerous because he knows mathematics so well.” His conviction appears to stem from an unshaken belief in RP. Consider a hypothetical reconstruction of his line of thought: (1) RP is correct; (2) therefore, its alternatives are incorrect; (3) therefore, it is malicious to develop such incorrect alternatives; (4) and therefore, Suntola, who masters mathematics quite well, is especially dangerous, as he might succeed in misleading people with a mathematically rigorous but incorrect theory.

8. **2013** *Not Physics — Wrong.* Tuomo gave a presentation titled *Short History of Science* on January 22 2013. The event was arranged by the Finnish Society for Natural Philosophy at the House of Science and Letters, back then known as room 505, now known as the Cedercreutz hall. I noticed that a physics professor was unesy while listening to Tuomo’s presentation, and I encouraged him to speak up. Finally he commented: “DU is not physics.” He was asked “Why is it not physics?” To this, the professor replied: “Because it is wrong.” In turn, he was asked “Why is it wrong?” The professor replied: “Because it is not physics.” This resulted in a comic reaction.

9. **2013** *DU is not workable because it has not been accepted.* At *The Finnish Society for Natural Philosophy 25 Years, K.V. Laurikainen Honorary Symposium* November 11–12 2013, I informed a Finnish physics professor that DU has been largely disregarded by the physics community. He replied by presenting a thought experiment: (1) science is open to new workable ideas; (2) if DU were workable, it would already have been accepted; (3) DU has not been accepted; (4) Therefore, it is not workable. It seems that the professor overestimated the openness of science and the rate of its self-correcting nature.

10. **c. 2015** *The sixth decimal argument.* I went to invite a cosmology professor to comment on Suntola's forthcoming presentation at The Finnish Society for Natural Philosophy. He insisted that before he comments on DU, it must explain the polarization of the cosmic background radiation spectrum. Thus, he did not appreciate DU's success in explaining —classically virtuously— several central phenomena, with a much more efficient ratio of phenomena per person-years than relativistic physics. Instead, he only noted that DU has not been worked out in one specific respect, perhaps in his own special area, which happens to be highly hypothetical due to observational and theoretical challenges.

    Consider the logical form of such a *sixth decimal argument*: a proponent of a paradigm rejects a relatively mature rival —one that explains several central phenomena in the paradigm's domain— on the grounds that it fails to explain a relatively insignificant and poorly understood phenomenon that the paradigm itself explains poorly —for instance, by applying several auxiliaries and offering a mere mathematical description rather than a causal explanation. In this setting the paradigm always wins, for an army of mathematicians can generate mathematical descriptions of whatever phenomena far faster than a small group developing causal explanations based on the rival. Consequently, the classically virtuous rival never receives credit for its successes, while from the viewpoint of scientific progress it would be rational to allocate resources to its further development.

11. **2017** *Courage-themed Science Forum.* While acting as the chair for the Finnish Society for Natural Philosophy, Tuomo submitted a session proposal "Courageous thinking — the foundation of understanding reality" for the 2019 Science Forum, whose theme was *Courage*. The Science Forum (Tieteen päivät) is a biennial science festival taking place in Helsinki, Finland, open to all visitors. The suggested talks were (1) Ari Lehto: Science corrects itself — or does it? (2) Tuomo Suntola: From describing observations to describing phenomena — perceiving the whole is the key to a unified theory and an understandable worldview. (3) Heikki Sipilä: The zero-energy principle. (4) Avril Styrman: Economy as the engine of paradigm change.

    The program committee decided that the session "does not fit into the overall program and is too challenging for the audience." Later, Tuomo was requested to appear in the chemists' 100th anniversary celebration in the Science Forum, and at the same time he was informed that the program committee "did not approve the . . . session proposal due to disagreements regarding the Dynamic Universe theory." Tuomo replied: "Within the framework of the Science Forum's Courage theme, it would feel somewhat frustrating to me to talk about a courageous choice made over 40 years ago while simultaneously remaining silent about a corresponding choice today."

12. **2018**. I invited a physics lecturer to comment on Suntola's presentation at the Finnish Society for Natural Philosophy. He could not make it, and I asked if he could recommend some other cosmologist or a GR specialist.

    Lecturer: I once was in an event where Tuomo Suntola was explaining his own cosmological ideas, and I would not wish the same waste of time on any cosmologist or expert in general relativity.

    Me: A few other physicists have also dismissed DU, but no one has said what is wrong with it. DU has different premises than standard physics, but surely this is not a flaw if it yields accurate predictions and unifies gravity and QM? Could you tell me or even come and give a presentation on what is wrong with DU? This would be a really big thing for the Society and for many people.

    Lecturer: Dozens of ideas in cosmology and particle physics are published every day. Which ones to engage with must be evaluated based on the interest and credibility of the ideas. Suntola's ideas do not rise above the threshold that I would spend time on them.

13. **2000–2019** *Editorial rejections*. Tuomo submitted DU articles to Physical Review D in 2000, 2007, and 2019. The editorian rejection letters conform to the reject-based-on-deviation-from-the-standard policy.

    2000 *New cosmology model shows relativity in universal time and distant observations in Euclidean geometry*: "Physical Review D does not publish theoretical speculations if they do not have rather substantial motivation or if they are based upon ad hoc assumptions."

    2007 *Zero-energy space cancels the need for dark energy*: "Papers that lie outside the mainstream of current research must justify their publication by including a clear and convincing discussion of the motivation for the new speculation, with reasons for introducing any new concepts. This discussion should be at a level of detail and precision comparable to that of the accepted theory, and should be at a level of discourse appropriate to the current state of research in the field. If the new formulation results in contradictions with the accepted theory, then there must be both a discussion of what experiments could be done to show that the conventional theory needs improvement, and an analysis showing that the new theory is consistent with existing experiments. Upon reading your manuscript, I conclude that your paper fails to satisfy all of these requirements."

2019 *Zero-energy universe*: "After examining your manuscript, I find that it is too speculative and too far from the level of current research in the field to be suitable for Physical Review D."

The editorial rejections that Ari Lehto received are quite similar.

2013 *Is there a periodic system of elementary particles?* in *Nature Physics*: "...we have no doubt that analysis of elementary particles using period doubling will be of interest to other specialists. But, I regret that we are unable to conclude that the manuscript provides the sort of significant conceptual advance in understanding or technology to be likely to excite the immediate interest of the broad readership of Nature Physics. We therefore feel that it would find a more appropriate audience in a more specialist journal, rather than Nature Physics."

2019 *Planck scale origin of the electron properties* in *Physical Review Letters*: "We regret to inform you that we have concluded that it is not suitable for publication in Physical Review Letters."

2019–2020 *On the wave-particle duality and the electron properties*. In *Foundations of Physics*: "The author of this manuscript fails to make clear how his/her work relates to current discussions in the foundations of physics. This is not a judgement about the quality of the contents of the submission, but, regrettably, does place the submission outside the scope of Foundations of Physics. Perhaps a theoretical physics journal would be more suitable?" In *International Journal of Theoretical Physics*: "The paper did not undergo technical review and is not being declined for any technical error. We wish you every success in finding an alternative place of publication." In *arXiv*: "This repository is only for substantive self-contained scientific research results that would be considered refereeable for publication in a conventional journal. Our moderators have determined that your submission is not of plausible interest for arXiv."

2020 *Period Doubling Phenomenon as the Origin of the Electron Properties*. In *Chinese Journal of Physics*: "The work reported in the manuscript may provide useful knowledge or information for some experts in the research field. However, it does not appeal to the broader interest of our readers." In *arXiv*: the same as above.

3.2. Miscellaneous Views

Individuals viewed DU positively or neutrally when their philosophical views aligned with it, or when they saw it as a useful tool, or when they considered expressions of alternatives useful due to the current state of physics.

> Johannes Hopiavuori, originally 'Hammaren', was a researcher at the University of Jyväskylä. He once participated in a Universe Club meeting, and invited Tuomo to give a presentation at the University of Jyväskylä. The presentation titled *The Dynamic Universe* took place on January 29 1999.

The assistent of Paul Kustaanheimo (1935—1997), who had operated as a professor of mathematics and astronomy in Finland, contacted Tuomo, and informed him that Kustaanheimo had worked on something similar to DU. This is not surprising in light of Kustaanheimo's [23, pp. 240–1] remark: "But above all we need a new theoretical vision, one that again strikes deeper into the essence of the material world than previous ones, and which moreover should be mathematically simpler than previous ones. ... A truly successful theory must be extremely simple, *trivial*."

Dr. Johan Silén from the Finnish Meteorological Institute (FMI) invited Tuomo to give a presentation about DU. The request materialized at a GPS seminar at FMI on November 17 2002, which was arranged by Seppo Haarala and Antti Lange. Tuomo delivered the speech *On the theoretical foundations of the GPS system: Relativistic phenomena in clock readings and signal travel times* (GPS-järjestelmän teoreettisista perusteista: Relativistiset ilmiöt kellojen näyttämissä ja signaalien kulkuajoissa). Silén recollects: "It is in the nature of science to be interested in alternatives. Clearly, physics is in a dead end and needs a new approach. ... What was interesting then was the corrected computational verification of the atomic clock experiment."

In Physical Interpretations of Relativity Theory (PIRT) conference in London in 2004, prof. G. H. Keswani understood DU very well and was very enthusiastic about it, but passed away soon after the meeting.

Kalevi Kalliomäki, the developer of Finland's time standard at VTT MIKES, was impressed by DU, and invited Tuomo to give a DU presentation at MIKES in Otaniemi. Tuomo Delivered the speech *The Dynamic Universe — a holistic perspective to space* on February 7, 2011. Kalevi and his colleagues considered it natural that clocks at different altitudes tick at different frequencies. Kalevi recollects on April 24, 2026:

> Tuomo is certainly a clever guy; he can just whip up all kinds of mathematics on the spot. I had to make relativistic corrections with the Russians when we could still talk to them. I preferred to use the corrections given by Suntola's DU theory rather than those from the relativity theory, because they were simpler. We transported clocks between Helsinki and Moscow. Moscow is much higher than Helsinki. I specifically preferred to use the formulas found in Suntola's books rather than Einstein's stuff, because it was easier.

Tuomo met the Finnish philosopher Jaakko Hintikka in Haikko Manor to speak at a theme evening of the Finnish Society for Natural Philosophy, titled *The Encounter of Philosophy and Empirical Science* on May 12, 2015 [11]. Hintikka received the Leibnizian-Aristotelian ideas about the actualization of potentiality and the relation of the local and the global very well. A week later, they met again in Haikko. Both meetings lasted for several hours. Unfortunately, Hintikka passed away soon after, on August 12, 2015.

Several laymen have expressed their consent for DU throughout the years. One active Society member, Heikki Mäntylä, even produced two self-publications about DU [34, 33].

3.3. Debate in Yliopisto-lehti 2025–2026

The debate in the University of Helsinki's popular science magazine *Yliopisto-lehti* began by Juha Merimaa's [36] aricle about DU. Below, I quote in a chronological order the essential content of the exchange that followed. The article sparked a critical reaction from the cosmologist Syksy Räsänen. Maria Pemberton [37], the editor of the Yliopisto-lehti, replied:

> Yliopisto-lehti came under fire from a physicist. According to the feedback, we publish pseudoscience uncritically and give column space based on large donations. . . . Citizens need to know not only about research results but also about how science is done and how it develops. Both open-minded ideas and criticism are necessary. It may be that the Dynamic Universe theory ends up in physics's waste basket, but it may also have a great deal to offer to philosophy. . . . People may continue to argue about the truth, but in science the matter is clear — truth changes. There is a currently valid truth, which new research refines and shapes. The explanation of the universe is no more complete than any research result discussed on the pages of this magazine. In your hands is the current knowledge, which must be challenged.

A dismissive commentary by Syksy Räsänen [42].

> Suntola's ideas about the theory of relativity are not physics. They contain elementary errors, no journal in the field has accepted them for publication, and no researcher in the field takes them seriously. Suntola's thoughts on the theory of relativity have as little significance for physics as nonsense about the flatness of the Earth does for geography. Why does Yliopisto-lehti give them space? Perhaps the editorial board denies that there is truth independent of opinions. Theories change and views are refuted, but we can still discover things that cannot be reasonably doubted. Mainland Åland is an island. HIV causes AIDS. Mercury moves with great precision as the theory of relativity predicts. Or perhaps the magazine does not understand that something can be beyond reasonable doubt even if it is not within the realm of common sense. ... It is a dangerous direction that Yliopisto-lehti refuses to correct its course based on expert feedback and defends pseudoscience.

A defensive commentary by Tarja Kallio-Tamminen [18].

> Physics and philosophy aim at understanding the phenomena in reality. Yet the foundation of the world view has been shrouded in obscurity for a hundred years. No interpretation has been found for quantum mechanics, nor does it fit together with the theory of relativity. The situation does not seem to bother physicists much: it is sufficient that a mathematical description is found for observable phenomena. The philosopher also seeks to understand the reality behind theories by examining the applied assumptions. She/He considers a theory better the greater set of phenomena it explains with the smallest possible number of postulates. In scientific revolutions, metaphysical postulates are renewed. Räsänen assumes the theory of relativity to be true. Within normal science, there is no reason to doubt its foundations. Since DU does not fit into the prevailing paradigm and does not even need a separate theory of relativity, it is left unexamined. Such fundamentalism should not be disguised in defense of science. In science, one does not possess the truth, but only strives toward it. From the perspective of developing science and our conception of reality, it would be more responsible to seek errors in DU.

A dismissive commentary by Kimmo Tuominen [80].

> Natural science requires systematic handling of assumptions and their consequences, as well as clear argumentation for predictions derived from a model. Predictions are compared with observations so that the analysis is reproducible by other researchers. It appears that the proponents of DU do not understand how natural science operates and how physical theories develop. The writings also create a false picture of how those presenting their results from outside the mainstream must fight against a dogmatic hedgehog defense. The experts' indifference to DU is framed as Einsteinian stubbornness, and the justifications resort to a kind of appeal to bitterness, *argumentum ad odium*.
>
> Perhaps the background to the writings of DU's proponents is that the theory of relativity and quantum mechanics, which form the basis of the modern worldview, are perceived as challenging, and the unsupported but "clear and understandable picture of reality" offered by DU is more appealing. The natural scientific method does not seek to prove 'common sense' but to find regularities that make accurate prediction of natural phenomena possible. Modern physics predicts phenomena that we cannot grasp intuitively but which turn out to be experimentally true. They have enabled technological development that has changed society and our everyday life in an irreversible way. Scientists are open to new ideas and the development of models. These are measured by scientific rationality. The Dynamic Universe does not offer a model worth serious consideration in this respect.

A defensive commentary by Kallio-Tamminen and Kakkuri-Knuuttila [15].

This writing [of Tuominen] that exudes the instrumentalist attitude generally adopted by physicists dismissed the need for a clear picture of reality by reducing natural research to "finding such regularities that make accurate prediction of natural phenomena possible". It should not be a disadvantage if the causes of these precisely predicted and experimentally verified phenomena could also be explained in an understandable way. When a comprehensive theory that offers explanations is left unexamined within the physics community, a philosopher of science can justifiably interpret the situation as dogmatic hedgehog defense.

Tuomo Suntola's work and publications show that he acts in accordance with the ideals of science and knows the foundation and history of physical theories well. ... The atomic layer deposition [developed by Suntola] was not easily accepted by the scientific community either. Doubtful messages about the impossibility of the method were still coming when the test production of thin films had already begun. Fortunately, chemistry researchers at the University of Helsinki understood the value of the innovation early on. More than 50 doctoral dissertations have been completed on the subject, and the pace is not slowing down. New and strange ideas are not the enemies of science but its fuel. Many scientists have initially had to defend their theory alone. Suntola's youthful innovation has already had a global impact. Can we afford to leave unexplored where DU might lead?

3.4. Comments from the Bohm-Hiley Circle

The *Appearance and Reality in Physics and Beyond* group from the University of Helsinki, together with Peter Van Reeth from UCL, organized the workshop *Quantum Potential Meets The Dynamic Universe* on May 16–17, 2025 at UCL, London, and the workshop *Understanding Quantum and Relativistic Phenomena* on November 7–8, 2025 at the University of Helsinki. Two commentaries on DU related to these events are presented below.

3.4.1. The Dynamic Universe and the Bohmian Interpretation

Below, I investigate the coherence of DU and the de Broglie-Bohm interpretation of quantum mechanics (dBB). This endeavor is oriented by the remarks of Chris Dewdney, who is well known for his 1979 calculation of particle trajectories (Philippidis, Dewdney, and Hiley [38]), based on Bohm's [2, 3] work. In his speech, Dewdney [7] suggests that the "dynamic universe theory solves a problem for the Bohm theory and perhaps the Bohm theory can solve a problem for the dynamic universe as well."

The underlying assumptions and goals of DU and dBB overlap. They are both nonlocal realist interpretations. They are nonlocal, as they commit to faster-than-light influences, and therefore they practically commit to a privileged reference frame and absolute simultaneity. And unlike antirealist interpretations, they hold that physical properties exist independently of measurement and that a quantum system has a definite state prior to measurement, so that particles have definite paths and intrinsic properties.[10]

Let us first consider the contribution of DU to dBB. The major problem with the non-locality assumption is that it violates special and general relativity. Therefore, if DU were accepted as a viable alternative to relativistic physics, the coherence of dBB with the rest of physics would increase significantly in this sense. Dewdney suggests this:

> Bohm of course was well aware of this and he said that a simple way out of this is just to say that there is a . . . preferred reference frame which tells us how things actually happen, right, and other observers in different frames are just going to give us their view of that thing which actually happened in the preferred frame. . . . What's interesting I think from the point of view of this meeting is that you know how are we going to get our preferred frame and the answer is sitting right there because Suntola's Dynamic Universe theory has an undetectable absolute cosmic rest frame and in that theory the Lorentz invariance doesn't arise from the symmetry postulates but from this constraint that every object shares a fixed cosmic energy budget between internal activity and transformational motion. Chris Dewdney [7]

Let us then consider the contribution of dBB to DU. In his comment to Suntola [76], Dewdney pointed out that DU lacks an equation for particle paths, and he suggested that the Schrödinger equation and its Bohmian interpretation could provide these:

> *Dewdney*: So you know when you show it going through a slit, how do we know that it diffracts? But you don't have a wave equation, so we don't know that. But what you could do is just take your geometry and then put the Schrödinger equation on your geometry, and so you would then have the Bohm trajectories in your picture as well, because you would just take the Schrödinger equation with its Bohm interpretation and sort of put that on your geometry. So that would be one sort of completion of your theory, because I mean the thing is what I don't understand is when you draw your trajectory from the upper slit down to one of the interference maxima, how do you know that that's the path that your particle would take?
>
> *Suntola*: It should be understood more like a conceptual level which I have achieved here, so that the interference pattern is something I think that's called Fraunhofer interference, so I haven't made any calculations.
>
> *Dewdney*: So are you thinking of a classical wave?
>
> *Suntola*: It's very classical, so that's what was in my mind when drawing the trajectory —that it follows the classical interference. So there was nothing new.

[10] Bohm and Hiley [5, pp. 106–8] give the dBB account of what happens in the measurement.

So, can DU help dBB and vice versa? This question can be approached via their stances on *quantum non-locality*, the thesis that measurement of one particle affects the other particle(s) at a superluminal velocity.

Bohm accepted quantum non-locality as a fact and interpreted it as inherent in standard QM.[11] Bohm [2, 3] and Bohm and Hiley [5, pp. 28–31] derive the mathematical form of the quantum potential and the particles' guidance condition from the polar form of the Schrödinger equation. The dBB complements the formalism of quantum mechanics with the implicit hypothesis that particles have actual, well-defined positions at all times —the 'hidden variables' of the title of Bohm's 1952 articles. The quantum potential is interpreted as a real physical field that guides the motion of particles: in an N-particle system, the particles reside within a 3N-dimensional quantum potential field that explains the assumed non-locality. In dBB as presented by Dewdney, in a 2-particle system, what changes immediately due to a remote measurement of particle A is the trajectory of particle B, whereas B's intrinsic properties —angular momentum, mass, and charge— remain unaffected. Later, in collaboration with Basil Hiley during the 1970s and 1980s, Bohm extended dBB by proposing that *active information* is the mechanism that conveys nonlocal influences between particles via the quantum potential field.

In DU, nonlocality appears as instantaneous gravitational and Coulomb interactions; Suntola has not considered strong and nuclear interactions. Suntola has not considered whether quantum nonlocality is a genuine phenomenon, nor has he attempted to explain it in terms of DU. Were quantum nonlocality accepted as a genuine phenomenon and one were to explain it comprehensively in terms of DU, this would require derivation of the DU-equivalent of the Schrödinger equation from DU, including the derivation of particle trajectories. Thus, dBB gives a precedent for calculating the particle paths.

3.4.2. Pros and Cons

Calum Robson commented on DU on November 17 2025.

> Having thought about what was discussed, I am inclined to the same conclusion as at the conference—the instincts behind DU are correct, but the approach via a Euclidean space is not.
>
> 1. The desire for a dynamic, rather than a merely kinematic, description of General Relativity is one I share. I have wondered myself if the energy in each frame was the key to understanding this, and engaging with the DU approach has made me more convinced of this. If we are measuring relative time by equivalent clocks, if there is more energy, e.g., in a star, then maybe there is a way of understanding time dilation by the extra energy making the clocks tick faster—this is something I'm thinking about, and I find the DU idea of energy frames very interesting in this regard. I also agree with DU that the energy

[11] See Bohm [4] and Kakkuri-Knuuttila [14] for the development of Bohm's understanding.

balance in GR is an intriguing mystery which needs to be explored. I was not previously aware of this point, which I think is important.

That said, why am I not convinced by the use of Euclidean space to explore these insights? First of all the justification for using Euclidean space in DU does not (as I see it) hold up. The idea is to have, 'the same universe for all observers, not one which looks the same'. But in DU different observers (at different points on the sphere) will describe the universe differently. There is still an underlying shared universe of course—but that is also the case in the Minkowski/GR case, where there is an underlying shared reality which is invariant under hyperbolic rotations. There is no difference between the spherical symmetry of DU and the hyperbolic symmetry of GR in this respect. Both involve different observers having different descriptions of a shared universe.

There is no advantage, therefore, to giving up the Minkowski structure, and there are very good reasons for keeping it. Minkowski structure must arise whenever there is a finite rate of casual propagation, since any such finite rate will split spacetime around a point into 'spacelike' and 'timelike' parts, depending upon whether the points in question are causally reachable from that point. This structure is preserved by the hyperbolic rotations in the Lorentz group. This also implies that time dilation and length contraction in SR are purely kinematic—they are the symmetry requirements of a universe in which this causal structure is preserved (Of course it would still be good to have an explanation of **why** there is a finite speed of causal propagation, which would indirectly provide an explaination for these kinematics. DU, of course, attempts to provide such an explanation).

Such a structure should also be present in the DU, since DU contains a finite speed of light. Around each point, there will be, 'light cones' defined by the speed of light at each point. This will induce a hyperbolic structure on DU, with light cones which ,'tip' just as in GR, due to the varying speed of light. It would be very interesting to examine this induced light cone structure and compare it explicitly with GR. However on the whole I don't see the advantage to starting with a Euclidean space rather than just investigating the energy balance question within GR itself. Of course, I could be wrong here and I'm willing to be persuaded if I am wrong.

2. I also have five technical questions:

a) if you get tipping of reference frames near mass centers, would this not distort the spherical symmetry if the mass distrubution is not initially symmetric? Is it possible to use python to model the evolution of a DU model from some initial mass distribution?

b) On a more philosophical point, does the absolute space exist, 'before' the sphere has reached a particular time? Is what is expanding and contracting the absolute space, or it is matter within the absolute space?

c) I note that the radius of atoms goes like $1/\sqrt{1-v^2/c^2}$. Does this not imply that the radius of atoms goes to infinity as the speed goes to c? We don't seem to observe this in colliders if so.

d) The Uncertainty Principle is central to Quantum Mechanics — we cannot simultaneously measure x and p within an arbitrary accuracy. This does not seem to be a feature of DU, which makes it look a bit like a hidden variable theory. How does it avoid the Bell inequalities, and how does the underlying deterministic dipole model of quantum reality give rise to the quantum mechanical stochastic picture?

e) The cosmological graphs in the recent DU papers are fascinating since they imply better fits with DU than the standard cosmological model. But such graphs need to be backed up with the calculations used to derive them, I think. It was mentioned at the conference that such calculations were in the DU book, but I have been unable to find them. Can you let me know exactly where in the Physics Foundations Society literature I can find the derivations so I can examine them more closely?

I'm aware these questions may require a fuller answer than can be given in an email exchange. Overall, I think the DU approach is very interesting, both in itself and in the challenge it poses to deeply established principles in physics. Whilst I ultimately think that these principles, like Relativity, remain true, it is very instructive to think about the justifications for things which usually are unchallenged and not thought about.

Tuomo replied on November 20 2025.

For me, the choice of the Euclidean coordinate system is to keep the description closely linked to the physical, observable reality. The fundamental finding in DU was that any mass m in space has momentum $p_4 = mc$ in the fourth dimension due to the expansion of the 3-sphere space. When summed up to a momentum p in space as an orthogonal component, we have the total momentum $p_{tot} = \sqrt{(mc)^2 + p^2}$ and the total energy of motion, $E_{tot} = c_0\sqrt{(mc)^2 + p^2}$, which is identical with the Dirac formulation of the relativistic total energy. It is illustrative to express momentum and energy as complex quantities with the effects of motion **with** space as the imaginary component and the effects **in** space as the real components. The complex plane presentation also allows expressing the balancing gravitational energy components in the same coordinate system, which is very important for illustrating the symmetry and balance of the energies of gravitation and motion. Combining the effects of gravitation and motion, we end up with the system of nested energy frames, which illustrates relativity as a direct consequence of the conservation of the overall energy balance, and illustrates the linkage between local and the rest of space. In DU, the effect of SR time dilation is conveyed by the effect of motion on the rest mass, and the effect of GR gravitational time dilation by the effect of local gravitational potential on the local (coordinate) velocity of light.

In the DU complex plane presentation, Minkowski's 'spacelike' means closest to the real axis and 'timelike' the imaginary axis. In DU, time is considered a scalar. It is applied equally for motion and dynamics (motion-related quantities) in space and the motion and dynamics of space.

DU is not a novel description of GR, but an alternative to GR. DU does not need postulates like the relativity principle, the equivalence principle, time dilation, length contraction, or the constancy of the velocity of light. ...

2.a,b) It is useful to consider the initial state or the reference state as the state of rest with all mass uniformly distributed = hypothetical homogeneous space (with momentum only in the 4th dimension). I assume that the homogeneous space exists in the contraction phase before the turning point to expansion. The turning point or pass-through point may trigger the structure formation.

2.c) Yes, the radius of atoms goes like $1/\sqrt{1-v^2/c^2}$. In the DU framework, Bohr radius is directly proportional to the Compton wavelength and the emission wavelength, which we know increases like $1/\sqrt{1-v^2/c^2}$ with velocity. Max velocity obtained for atoms in a collider is an interesting question (Is it a relevant question? ...

2.d) Uncertainty Principle needs thorough pondering. Introduction of the intrinsic Planck constant $h_0 = h/c$ may offer new insights into the interpretation of the uncertainty principle. This is a very interesting question.

2.e) The angular size of a standard rod is discussed in Chapter 6.2. in the DU book. Importantly, galaxies in DU are expanding objects, which is essential for the Euclidean appearance of the angular size. The lensing effect of the 3-sphere is not included in the DU book. Optical distance, redshift, and Hubble law, see Chapter 6.1.2. Magnitudes and supernova observations, Chapter 6.3.2, Bolometric magnitudes in multi-bandpass detection 6.3.3, K-corrections 6.3.4. You may find that the interpretation of observations and the conversion to DU framework is quite tricky. The problem goes back to Tolman's (1930) derivation of the observed luminosity of distant objects. This is very interesting; we may discuss this in more detail later on.

4. Conclusions

The present review indicates that people whose theoretical and methodological commitments align with the classical ideal of science tend to perceive DU positively or neutrally, whereas those strongly committed to relativistic physics and the underlying instrumentalist methodology tend to dismiss DU without investigating it.

The examples in §3.1 and §3.3 reveal the most common rejection mechanism: (1) assume that standard physics and its instrumentalist methodology are the only correct alternatives; (2) detect DU's deviation from the standard; (3) conclude that therefore DU is false and not worth further investigation. The only variance in the examples is in how the rejection is formulated and justified: some explicitly cite deviation from standard physics or instrumentalist methodology; others give no justification but may express negative expectations or sentiment, or mistake DU as a model of general relativity. In no case is DU argued to violate objective criteria. The national leaders of this movement are Syksy Räsänen and Kari Enqvist. They expressed similar sentiments already in 2014:

> Well-known examples of denialism include denying climate change, evolution, the connection between HIV and AIDS, and the effectiveness of vaccines. ... Among denialists there is also a smaller contingent: the relativity theory denialists. ... In fact, the validity of the special theory of relativity is beyond reasonable doubt, and the purpose of particle accelerators is not to test it. It is more accurate to say that their operation is based on the fact that the theory of relativity is correct. ... The validity of the theory of relativity is inseparably tied to all our physical understanding. The relativity theory denialist unknowingly denies the foundations of physics. Räsänen and Enqvist [40, pp. 55–6]

Let their view be called *relativistic instrumentalism.* I attempt to construct it: (1) the predictions of SR are in many cases correct; (2) therefore SR must be correct, that is, its validity is beyond reasonable doubt; (3) since SR is correct, there is no need to discuss its alternatives. It appears that relativistic instrumentalism is behind the systematic practice of rejecting everything that violates SR without investigation. Juha Himanka suggests that this tenet has been a part of SR since its inception:

> In Lorentz's opinion, Einstein extended his theory into philosophy and presented his personally favored theory as a general truth without justification. In Lorentz's opinion, Einstein's theory was just one of many possible models. Lorentz himself considered that scientific evidence should not be directly extended to general, philosophical claims about reality. Himanka [41, pp. 47–8]

Then again, if alternatives to SR and to RP are not a priori rejected, we arrive at the highly relevant question about criteria by which we may objectively decide between rival theories. Let us, for the sake of the example, accept *neutral* instrumentalism, where accurate predictions are the sole criterion, whereas consistency with RP is not required. In this case, all equally accurate theories for the same phenomena should be considered equally correct as RP. Currently, DU gives accurate predictions for several central phenomena in the domain of RP. To get a correct perspective, one should acknowledge that the best mathematicians have developed RP in the past 100+ years for perhaps thousands of person-years. Therefore, we should not require that a rival like DU, which has been developed for less than 20 person-years, should immediately provide mathematical descriptions for *all* phenomena that RP currently does. It is more intelligible to look at the ratio of mathematically described phenomena based on a theory and person-years spent on creating those descriptions. If DU's ratio is significantly greater, which I believe it is, it would be intelligible to fund DU —for it is rational to assume that DU would eventually yield more 'correctness' faster.

Thus, even accuracy alone, when qualified by the applied person-years, supports the fitness of DU as a research program. However, accuracy alone gives a very limited picture of a theory. For the reason why DU provides a good ratio of person-years and phenomena is that it is genuinely unified: its relatively simple and deep causal mechanism, together with a minimal number of auxiliaries, yield understandable explanations and novel predictions for heterogeneous phenomena. It has been known for a long time that deep causal explanations yield novel predictions, and thus help us apply mathematics as a tool more efficiently than without them (Salmon [43, p. 259], Schurz [44, p. 74], Styrman [56, §3.2]). For reference, one may also look at how virtually all other natural sciences, except theoretical physics, actually work.

Räsänen and Enqvist [40, p. 55] agree that the relativity theory's "abstractness and detachment from the immediate sphere of experience ... cause deep disbelief." But at the same time they argue that the relativity theory is correct. That is, they argue that the relativity theory is correct, despite the fact that it does not make nature understandable. This underlines that understanding nature has no weight in the instrumentalist mindset, and suggests that the possibility of a genuine unified alternative to the relativity theory is not even in the spectrum.

Kimmo Tuominen started by noting that it "appears that the proponents of DU do not understand how natural science operates and how physical theories develop" and concluded that instrumentalism is the correct method: "The natural scientific method does not seek to prove 'common sense' but to find regularities that make accurate prediction of natural phenomena possible." He also says that the writings of the proponents of DU "create a false picture of how those presenting their results from outside the mainstream must fight against a dogmatic hedgehog defense."

Now, if you indeed suppose that instrumentalism is the only correct method, then you reject the initial rationality behind DU. Let us recall the historical development again: (1) the classical ideal was gradually replaced by instrumentalism in physics because the 20th century theories did not meet this standard; (2) DU respects the classical ideal, but now DU is rejected because instrumentalism is the house philosophy; (3) thus, DU is so to speak rejected based on *a philosophy of failure*.

There are good reasons to consider relativistic instrumentalism as a philosophy of failure. First, *the paradox of instrumentalism* shows that its initial rationale to legitimize the lack of causal explanations and the understanding they would provide is misdirected. For it is rational to believe that a theory which conforms to the classical ideal will, in the long run, produce accurate mathematical predictions more efficiently than a theory that violates the classical ideal. Therefore, an exclusive commitment to an instrumentalist theory cannot be justified based on the primary instrumentalist criterion. Second, the disregard of anomalies and introduction of explanations that apply auxiliaries appears to be the gold standard of relativistic instrumentalism (Styrman [55]). Third, Suntola [77] has detected cases of data modification, which are, however, most probably not acknowledged. Fourth, the pragmatic virtues that DU's simpler formalism provides are in contrast to the highly complex mathematics of GR (Křížek and Somer [21]).

In this light, common sense indicates that novel insights that aim to fix the problems of RP should be investigated. The greatest 20th-century philosophers of science —Karl Popper, Thomas Kuhn, Imre Lakatos, and Paul Feyerabend— have taught us that to fully understand a theory's weaknesses, it must be juxtaposed with an alternative theory. In the current debate, Pemberton [37] reminded in the same vein that "[b]oth open-minded ideas and criticism are necessary," and Kallio-Tamminen and Kakkuri-Knuuttila [15] reminded that "[n]ew and strange ideas are not the enemies of science but its fuel."

But the present study supports the notion that the proponents of such novel ideas are more likely to face dismissal or moral outrage and be branded denialists; even Yliopisto-lehti got its share of outrage due to allowing the discussion of such views. In general, the present study reminds us that it is extremely hard to change physics, even when there are good reasons to do so. So do not live under the false hope that someone might take you seriously without a significant effort. If this is extremely hard even for individuals like Suntola, it appears almost impossible for people with fewer credentials.

Perhaps a paradigm shift becomes practically possible only when relativistic physics is so troubled by anomalies, auxiliaries, and data modification that they are apparent to virtually everyone. At that stage, a sufficiently mature rival is ready and waiting. Whether the new paradigm will be DU or some other theory we cannot know. But I am certain that the philosophy of failure and the narrative where we do not need to understand nature are gradually coming to an end. I have high hopes for this new era of understanding, like in the 18th and 19th centuries. Were DU or another theory to the same effect become the paradigm, this change would feel most natural. For virtually all other natural sciences —like chemistry, biology, geology, and meteorology— except for theoretical physics are founded on causal mechanisms already. This change would only set physics in the same causal basket with the others.

The dismissive attitudes that have been discussed are in dire contrast with the atmosphere and the discussion culture in the Helsinki-UCL meetings, as well as in the Czech Academy meetings in Prague arranged by Michal Křížek. In these meetings the primary goal was not to judge and dismiss alternative thinking, but to try to understand it and to investigate whether there are any prospects in it. I included two comments related to these meetings as proofs that one can talk about DU without reproach or moral outrage. Chris Dewdney suggested that DU and the de Broglie-Bohm theory might help one another, and Calum Robson saw some pros and cons in DU. Perhaps, people who have been exposed to minority positions are more prone to neutral feedback than people stricly in the standard theories' sphere of influence?

It is important to see that a mature discussion of fundamental theories requires not just an open mind, but also somewhat extensive knowledge of the foundations of physics, as well as the interest in exploring the ideas of other people. All these requirements were met in the Helsinki-UCL and Prague meetings, whereas in various earlier meetings where Tuomo presented DU, although the participants did not reject alternatives, they did not have the interest or the skills to make intelligible comments.

Now let us explore how the atmosphere could be made more mature and open to genuine dialogue, so that important novel ideas that go against tradition would be assessed more objectively and systematically by people with proper skills. I suggest an interplay of three measures: development of transparent criteria via public scrutiny, systematic integration of philosophy in science education, and greater openness in peer review up to an optimal level.

First, I suggest an institutional requirement for transparent criteria for each science. That is, journals and public science institutions should openly articulate their evaluation criteria, respect them in practice, and subject them to public scrutiny. This suggestion aligns with Maxwell's aim-oriented empiricism [35], which encourages positive feedback between improving physical theories and improving the aims and methods of physics. Transparent criteria could help to tackle both excessive proliferation of theories[12] and dogmatism in peer review —aka cognitive bias, old boyism, cronyism, or particularism (Travis and Collins [79]).

I have suggested unity as the basis of deciding when a theory deserves to be properly evaluated: the ratio of the number of explained phenomena to the weight of the applied ontological commitments (Styrman [56]). In short, an emerging theory should be serously evaluated when its relatively simple basic structure explains the central phenomena that the paradigm explains and also resolves its main anomalies, while applying considerably fewer auxiliaries. This conforms to Kuhn's [22, p. 153] remark that "[p]robably the single most prevalent claim advanced by the proponents of a new paradigm is that they can solve the problems that have led the old one to a crisis." This criterion would ensure that DU's inconsistency with relativistic physics would not entail its rejection, and it would also tackle excessive proliferation of theories by setting a precedent for alternatives. However, also much less justified alternative ideas may turn out helpful, and so the question of just how strict the basic requirement for rationality should be is complex.

Second, I suggest that philosophy of science be strongly integrated into all education, from kindergarten to universities. If scientists generally understood the cor-

[12]Renowned physicists receive a steady flow of theoretical proposals, and it would be a full time job to get properly acquainted with them. I have found no comprehensive study of this feat, but I have heard this personally from several physicists, and Merimaa [36] writes: "Physicists receive a large number of contacts from individuals outside the scientific community who, at least in their own opinion, have valid solutions to physics problems." One well known source is Percy Bridgman's archive of alternative proposals he received between 1925–1961. See `https://snaccooperative.org/vocab_administrator/resources/8238256`

nerstones of the philosophy of science, as well as the history and the greatest defects of contemporary theories, they would inevitably be more open to alternatives and more willing to engage in dialogue. One may contrast the current typical first-year physics student, being groomed for absolute conformism, with the future first-year student who understands, inter alia, that observations are often theory-laden, that they are not being taught theories that are beyond reasonable doubt but provisional theories that should be replaced if they face irredeemable anomalies, that at least some alternative theories should be seen primarily as means of improving physics rather than as creations of science denialists, and that there are other immensely important criteria in addition to the accuracy of predictions.

Third, I suggest increasingly open peer review in journals and public science institutions. Namely, I suggest an active search for the optimal degree of openness, somewhere between the current blind review and total openness, which should be currently seen merely as the extreme end of the spectrum. Here, the decisions and identities of people who make decisions concerning the acceptance or rejection of scientific articles, funding of science, and public science-related jobs are known.

An overall system comprises certain criteria, a certain philosophy of science, and a certain degree of openness in peer review. The current institutional system in theoretical physics comprises more or less instrumentalist criteria and philosophy, and blind review. We have seen that the current system is incapable of tackling stagnation, and so the path of revolutions appears to be the only possibility within the current system. Therefore, to enable a smoother path, it appears rational to increase the openness of peer review in tandem with explicating and openly scrutinizing the criteria, and strengthening the role of philosophy in science education.

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CALCULATION OF THE GALAXY ROTATION CURVE

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Abstract: This paper deals with the gravitational model suitable for the rotation curve calculation as well as the mass calculation of the SAb-class non-barred spiral galaxies with flat disc and spherical bulk. The model consists of a spherical bulk, flat disc, and spherical halo. A very thin disc is considered as a flat 2D mass continuum with exponentially distributed surface mass density σ. Gravitational acceleration of this disc cannot be calculated analytically (hyper-elliptic integral). Therefore, the disc is substituted by a centrally symmetrical system of point masses whose gravitational influence is calculated individually, and summed as vectors. The whole gravitational acceleration $g(r) = g_{\mathrm{bulk}}(r) + g_{\mathrm{halo}}(r) + g_{\mathrm{disc}}(r)$ is compared with the centripetal acceleration. From the equation $g(r) = v^2(r)/r$ the orbital velocity $v(r)$ can be to determined, and displayed as the rotation curve. In this way, the rotation curve of the M31 galaxy was calculated. Best fitting of the calculated curve to the measured curves occurs at the calculated total galaxy mass $4.7 \times 10^{11} M_{\odot}$. This implies that this very small galaxy mass consists exclusively of baryonic matter. The presence of dark matter is absolutely not necessary for the explanation of high orbital velocities.

Keywords: galaxy, bulk, disc, halo, rotation curve, dark matter.

PACS: 95.35+d; 95.30.Sf; 98.35.Hj; 98.80-k

1. Introduction

The first more precise measurement of the M31 galaxy rotation curve was published in 1970, see [1]. Rotation curves of many other galaxies have since been measured, e.g. [2], [3], [4], [5], [6]. It is well known that in all cases, the measured orbital velocities are much higher than expected from the Keplerian curve $v(r) \approx r^{1/2}$. This discrepancy is considered as evidence of the dark matter existence within the galaxy.

The total mass of the M31 has been calculated by many authors, see [7] to [14]. The results are spread over a wide range, approx. from $7 \times 10^{11} M_{\odot}$ up to $23 \times 10^{11} M_{\odot}$. These values differ in ratio 3 : 1, which suggests that the results are inconsistent, see Table 1.1. Based on numerical simulations, most studies conclude that the source of dark matter is primarily the vast spherical halo surrounding the galaxy. The mass of the halo itself has been computed by various authors, see [3], [10], [15] to [19]. The results are also spread over a wide range, approx. from $7 \times 10^{11} M_\odot$ up to $22 \times 10^{11} M_\odot$. The ratio 3 : 1 indicates inconsistency again. Furthermore, it is seen that the total galaxy masses (i.e. including the halo) lie within the same range as the halo masses themselves. That is somewhat illogical.

Author	Radius [kpc]	Radius [R_{25}]	Total mass [$10^{11} M_\odot$]
This work	70	3.00	4.70
This work	∞	∞	4.78
Ibata et al. (2004), [7]	125	5.36	7.5 ± 1.3
Evans et al. (2003), [8]	100	4.29	7–10
Tollerud et al., (2012), [9]	139	5.97	8 ± 4
Sofue (2015), [10, Table 3]	200	8.58	13.9 ± 2.6
Watkins et al. (2010), [11]	300	12.88	14 ± 4
van der Marel et al. (2008), [12]	385	16.52	17.2 ± 2.5
Lee et al. (2008), [13]	100	4.29	19.0 ± 1.3
Fardal et al. (2013), [14]	200	8.58	19 ± 5
Sofue (2015), [10, Table 3]	385	16.52	19.9 ± 3.9
			Halo mass [$10^{11} M_\odot$]
This work	70	3.00	0.3
Geehan et al. (2006), [15]	200	8.58	7.1
Seigar et al. (2008), [16]	200	8.58	7.3
Chemin et al., (2009),[17]	160	6.87	10.0
Tamm et al. (2012), [18]	200	8.58	11.3–12.7
Sofue (2015), [10, Table 3]	200	8.58	12.3 ± 2.6
Corbelli et al. (2010), [3]	200	8.58	13 ± 3
Veljanovski et al. (2014), [19]	200	8.58	12–16
Sofue (2015), [10, Table 3]	385	8.58	18.3 ± 3.9

Table 1.1: Overview of the calculated total and the halo masses for M31 galaxy. The radii are provided in two different units: kpc and $R_{25} = D_{25}/2$.

In the following chapters, we show that high orbital velocities in SAb-class galaxies can be easily explained without the presence of dark matter. Simultaneously, we prove through the precise analytical calculation in closed algebraic form (not by numerical simulation) that the gravitational influence of the spherical halo on the rotation curve is almost negligible.

2. Calculation strategy of galaxy rotation curve

Galaxy is substituted by a *static gravitational model* consisting of a spherical bulk, flat 2D-disc, and spherical halo, see Figure 5.1. Mass parameters and dimension parameters of a bulk, disc and halo are easily adjustable in the model.

- The bulk is taken as homogeneous sphere with constant volumetric mass density ρ_b. Gravitational acceleration *inside* and *outside* bulk is analytically calculated in *closed algebraic form.*
- The halo is taken as non-homogeneous sphere with exponentially distributed volumetric mass density ρ_h. Gravitational acceleration *inside* and *outside* halo is analytically calculated in *closed algebraic form.*
- Very thin disc is taken as the flat 2D mass continuum with exponentially distributed surface mass density σ. Gravitational acceleration of disc cannot be calculated analytically (hyper-elliptic integral).
- Therefore, the disc is substituted by centrally symmetrical system of point masses, gravitation influence of which is calculated individually, and summed as vectors in the *closed algebraic form.* The mass of individual points is calculated in the way to approximate the original exponential distribution of the surface density σ.
- Thus, the gravitational accelerations of bulk, halo and all the individual point masses of the disc are calculated *analytically in closed algebraic form* (not numerically) for any distance r to the galaxy center. All accelerations are summed up, $g(r) = g_\mathrm{bulk}(r) + g_\mathrm{halo}(r) + g_\mathrm{disc}(r)$.
- This calculated centripetal gravitational acceleration $g(r)$ is compared with centrifugal acceleration $v^2(r)/r$. From equation $g(r) = v^2(r)/r$ the orbital velocity $v(r)$ can be to determine, and to display as the rotation curve $v(r) = [g(r) \cdot r]^{1/2}$.

3. Gravitation of the disc cannot be solved analytically

To understand better the gravitational influence of the mass disc, we will first analyze a much simpler case, i.e. gravitational influence of the mass circular filament, see Figure 3.1. The filament, with mass M and radius R, is infinitely thin. The task will be solved in polar coordinates r, φ. So, the angular mass density $\gamma = \mathrm{d}M/\mathrm{d}\varphi = M/(2\pi)$ can be defined via its circumference. This yields

$$\mathrm{d}M = \gamma \mathrm{d}\varphi = \frac{M}{2\pi}\mathrm{d}\varphi. \tag{3.1}$$

Differential mass $\mathrm{d}M$ acts on the testing point mass m by differential gravitational force

$$\mathrm{d}F = \frac{Gm\mathrm{d}M}{c^2} = \frac{GmM}{2\pi} \cdot \frac{1}{c^2}\mathrm{d}\varphi, \tag{3.2}$$

where G stands for the gravitational constant. The longitudinal (horizontal) component $\mathrm{d}F_\mathrm{g}$ is

$$\mathrm{d}F_\mathrm{g} = \mathrm{d}F \cdot \cos\alpha = \mathrm{d}F\frac{a}{c} = \frac{GmM}{2\pi} \cdot \frac{a}{c^3}\mathrm{d}\varphi. \tag{3.3}$$

The lengths c, a must be expressed by the polar coordinates r, φ. This follows

$$\mathrm{d}F_\mathrm{g}(r) = \frac{GmM}{2\pi} \cdot \frac{r - R\cos\varphi}{(R^2 + r^2 - 2Rr\cos\varphi)^{3/2}}\mathrm{d}\varphi. \tag{3.4}$$

The whole gravitation force in position r will be obtained by integration:

$$F_\mathrm{g}(r) = \frac{GmM}{2\pi} \int_0^{2\pi} \frac{r - R\cos\varphi}{(R^2 + r^2 - 2Rr\cos\varphi)^{3/2}}\mathrm{d}\varphi. \tag{3.5}$$

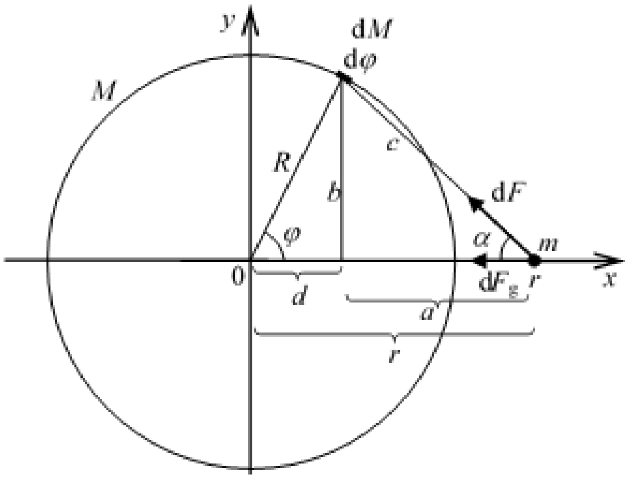

Figure 3.1: Gravitational influence of the mass circular filament.

To determine what type of integral it is, we use substitution $\cos\varphi = x$. The result of this substitution is the integral

$$F_\mathrm{g}(r) = \frac{GmM}{2\pi} \int \frac{Rx - r}{\sqrt{(R^2 + r^2 - 2Rrx)^3(1 - x^2)}}\mathrm{d}x. \tag{3.6}$$

It is known that the *elliptic* integrals must have the polynomial of either 3rd or 4th degree under the square root. In integral (3.6), the 5th degree polynomial occurs. This means that both integrals (3.6) and (3.5) are of *hyper-elliptic* type. Elliptic, and even more so hyper-elliptic integrals, cannot be solved analytically in closed algebraic form.

Surprisingly, integral (3.5) can be solved in the only special case $r = R$, i.e., if the testing point mass m lies directly on the circle. The result is $F_\mathrm{g}(R) = \infty$. It is a case of gravitational singularity (similarly as in the proximity of a point mass).

4. Substitution of the circle filament by the system of point masses

It is evident that the circular filament cannot be determined analytically. Therefore, we will substitute the continual filament by the system of discrete point masses lying on circle, see Figure 4.1. Point masses, equidistantly distributed on circle, are marked by numbers $q = 1, 2, 3, \dots, Q$.

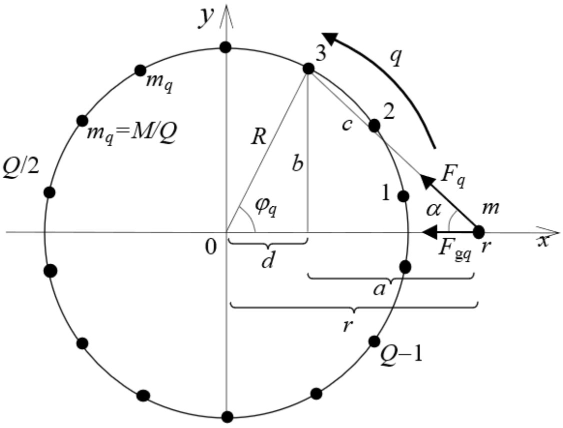

Figure 4.1: Substitution of the circle filament by the system of point masses.

The task is solved in the polar coordinates r, φ. The angle φ_q of the beam, on which the qth point lies, will be

$$\varphi_q = \frac{2\pi}{Q}(q - \tfrac{1}{2}), \qquad q = 1, 2, 3, \dots, Q. \tag{4.1}$$

The whole mass of all Q points must be the same as mass M of the original filament. So the mass m_q of one point will be $m_q = M/Q$. The point mass m_q acts on the testing point mass m by the force

$$F_q = \frac{Gmm_q}{c^2} = \frac{GmM}{Q} \cdot \frac{1}{c^2}. \tag{4.2}$$

The longitudinal (horizontal) component F_{gq} is

$$F_{gq} = F_q \cos\alpha = F_q \frac{a}{c} = \frac{GmM}{Q} \cdot \frac{a}{c^3}. \tag{4.3}$$

The lengths c, a must be expressed in the polar coordinates r, φ_q. This yields

$$F_{gq}(r) = \frac{GmM}{Q} \cdot \frac{r - R\cos\varphi_q}{(R^2 + r^2 - 2Rr\cos\varphi_q)^{3/2}}. \tag{4.4}$$

Expression (4.1) must be substituted into (4.4) instead of angle φ_q. The whole gravitation force F_g is got by the sum of all partial forces F_{gq}. The whole gravitational acceleration $g(r) = F_{\text{g}}(r)/m$ will be

$$g(r) = \frac{GM}{Q} \sum_{q=1}^{Q} \frac{r - R\cos(2\pi(q - \frac{1}{2})/Q)}{(R^2 + r^2 - 2Rr\cos(2\pi(q - \frac{1}{2})/Q))^{3/2}}. \tag{4.5}$$

Figure 4.2 shows the gravitational acceleration $g(r)$ in the normalized shape $GM = 1$, $r_{\text{norm}} = r/R$, with gradually increasing number Q of point masses. The whole mass of particular circles is constant. With increasing Q the germ of singularity accentuates. The singularity was predicted by equation (3.5) for case $r = R$.

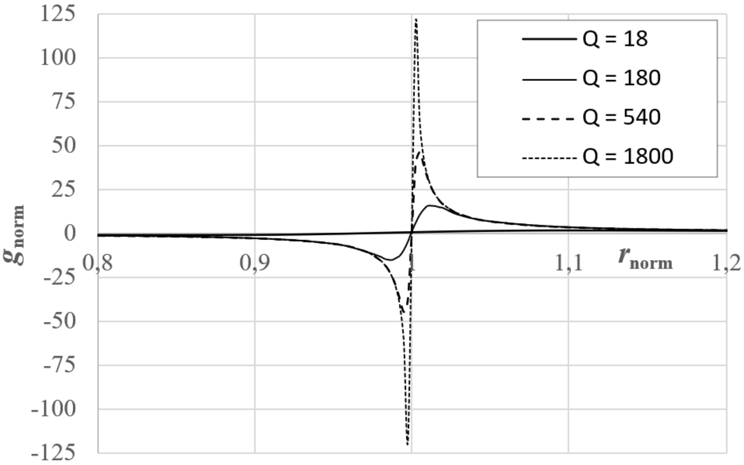

Figure 4.2: Gravitation acceleration of circles with Q points. Detail view in the neighborhood of the circle circumference (i.e. $r \to R$).

Figure 4.3 shows that from the point of view of gravitation influence, the circle differs radically from the empty shell, homogeneous full sphere, and point mass. Inside circle the acceleration is not zero (in difference from the shell). Inside the circle the acceleration is negative (point mass is pulled away from center), outside the acceleration is positive and *much higher* than in other cases. This implies that the galaxy disk cannot behave in a Keplerian manner.

5. Static gravitational model of galaxy

To calculate the orbital velocity of stars in galaxy the static gravitation model according to Figure 5.1. is used.

Basic *dimension parameter* of the model comes out, after deep consideration, from the galaxy standard isophotal diameter D_{25}, which is defined as the disc diameter at the 25 mag/arcsec^2 isophote of the apparent surface brightness measured at the blue-band. The basic dimension parameter of the model is a corresponding disc radius, i.e. $R_{25} = D_{25}/2$. All the other dimensions in the model are adjustable by dimensionless coefficients k_{xy} as multiples of radius R_{25}. It is suitable to calculate orbital velocities $v(r)$ up to distances at least $2R_{25}$ to $3R_{25}$, beyond the boundary of a visible disc.

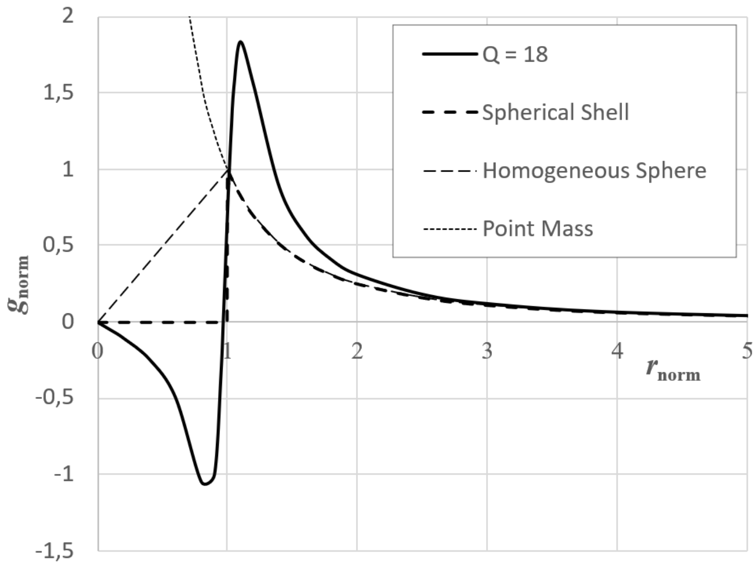

Figure 4.3: Gravitation acceleration of the circle with $Q = 18$. Comparison with the spherical shell, full homogeneous sphere, and point mass.

As the basic *mass parameter* of the model the pure whole stellar galaxy mass M_{st25} was chosen. This mass is found out on diameter D_{25} and is determined on the base of photometric analysis. All the other partial masses are then adjustable in model by dimensionless coefficients k_{xy} as multiples of pure stellar mass M_{st25}.

The model consists of four building elements with following properties:

- The spherical bulk at mass M_{b}, with constant volumetric density ρ_{b}.
- The vast spherical halo at radius of several R_{25}, with exponential volumetric density $\rho_{\mathrm{h}}(r)$.
- The disc is substituted by centrally symmetric system of point masses. The masses are located on P equidistant circles, numbered $p = 1, 2, 3, \ldots, P$ and on Q beams, numbered $q = 1, 2, 3, \ldots, Q$. The point masses are distributed to approximate the exponential surface density $\sigma_{\mathrm{d}}(r)$.
- The testing point (i.e. star) of mass m lies always on x-axis to avoid the collision with some fixed disc point m_{pq} (gravitational singularity). The testing point position, i.e. the distance r from the center, is arbitrarily continually selectable from 0 to ∞.

Model parameters

R_{25} – nominal disc radius; $R_{25} = D_{25}/2$.
M_{st25} – whole stellar mass measured photometrically in region D_{25}.
R_{infra} – disc radius in infrared band; $R_{\mathrm{infra}} \cong (1 \text{ to } 1.3) \cdot R_{25}$.
n – multiple of R_{25} radius; nR_{25} is the radius of analyzed region, usually $n = 3$ to 5.

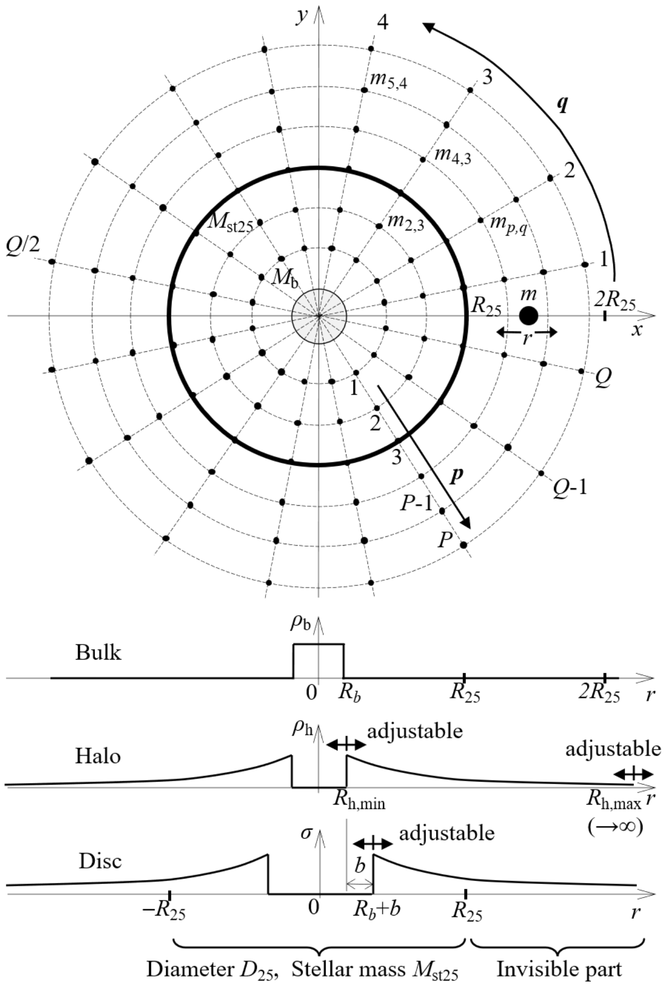

Figure 5.1: Static gravitational model of the galaxy.

M_b – pure stellar mass of central spherical bulk; $M_\mathrm{b} = k_\mathrm{Mb} M_\mathrm{st25}$.
k_Mb – mass coefficient of bulk; $k_\mathrm{Mb} \cong 0.1$ to 0.3.
R_b – radius of central spherical bulk; $R_\mathrm{b} = k_\mathrm{Rb} R_{25}$.
k_Rb – dimensional coefficient of bulk; $k_\mathrm{Rb} \cong 0.06$ to 0.2.
M_h – halo mass; $M_\mathrm{h} = k_\mathrm{Mh} M_\mathrm{st25}$.
k_Mh – mass coefficient of halo; $k_\mathrm{Mh} \cong 0.05$ to 1.5, uncertain value, data vary in order.
$R_\mathrm{h,min}$ – radius of halo inner boundary; $R_\mathrm{h,min} = k_\mathrm{Rh,min} R_{25}$.
$k_\mathrm{Rh,min}$ – dimensional coefficient of halo inner boundary; $k_\mathrm{Rh,min} \cong k_\mathrm{Rb} \cong 0.1$ to 0.2.
$R_\mathrm{h,max}$ – radius of halo outer boundary; $R_\mathrm{h,max} = k_\mathrm{Rh,max} R_{25}$, possibly $R_\mathrm{h,max} \to \infty$.
$k_\mathrm{Rh,max}$ – dimensional coefficient of halo outer boundary; $k_\mathrm{Rh,max} \cong 3$ to ∞.
M_d25 – pure stellar mass of the disc itself; $M_\mathrm{d25} = (1 - k_\mathrm{Mb}) M_\mathrm{st25}$, $M_\mathrm{st25} = M_\mathrm{b} + M_\mathrm{d25}$.
b – width of eventual gap between bulk and disc; $b = k_\mathrm{b} R_{25}$.
k_b – dimensional coefficient of gap between bulk and disc; $k_\mathrm{b} \cong -0.05$ to 0.15.
M_G25 – whole galaxy baryon mass in region D_{25}; $M_\mathrm{G25} = k_\mathrm{MG25} M_\mathrm{st25}$.

k_{MG25}– coefficient of whole baryon mass in region D_{25}; typically $k_{\mathrm{MG}} \cong 2$ to 2.5.
ρ_{b}– volumetric mass density of central bulk; it is approximately constant.
ρ_{h} – volumetric mass density of halo; $\rho_{\mathrm{h}} = \rho_{\mathrm{h,max}} \exp(-r/\lambda_{\mathrm{h}})$.
λ_{h} – longitudinal constant of halo exponential; $\lambda_{\mathrm{h}} = k_{\lambda\mathrm{h}} R_{25}$.
$k_{\lambda\mathrm{h}}$ – coefficient of halo longitudinal constant; $k_{\lambda\mathrm{h}} \cong 1$ to 6.
σ – surface mass density of disc; $\sigma = \sigma_{\mathrm{d,max}} \exp(-r/\lambda_{\mathrm{d}})$.
λ_{d} – longitudinal constant of disc exponential; $\lambda_{\mathrm{d}} = k_{\lambda\mathrm{d}} R_{25}$.
$k_{\lambda\mathrm{d}}$ – coefficient of longitudinal constant of disc exponential; $k_{\lambda\mathrm{d}} \cong 0.5$.
P – number of circles in disc.
p – index of pth circle in disc; $p = 1, 2, 3, \ldots, P$.
Q – number of beams in disc; even number.
q – index of qth beam in disc; $q = 1, 2, 3, \ldots, Q$.

Constant parameters entering to calculations: M_{st25}, R_{25}, k_{MG25}, k_{Mb}, k_{Rb}, k_{Mh}, $k_{\mathrm{Rh,min}}$, $k_{\mathrm{Rh,max}}$, k_{b}, $k_{\lambda\mathrm{h}}$, $k_{\lambda\mathrm{d}}$, P, Q.

Variable parameter entering to calculations: Distance r from the center of a testing star. It is changed by steps from 0 to $3R_{25}$.

Calculation results: The gravitation acceleration $g_{\mathrm{b}}(r)$ of bulk, $g_{\mathrm{d}}(r)$ of disc, $g_{\mathrm{h}}(r)$ of halo, whole acceleration $g(r)$, orbital velocity $v(r)$ are calculated for each chosen r.

Calculation of orbital velocity

The whole gravitation acceleration $g(r)$ in distance r will be determined as the sum of bulk acceleration $g_{\mathrm{b}}(r)$, halo acceleration $g_{\mathrm{h}}(r)$ and the sum of longitudinal components $g_{pq}(r)$ in the x-axis direction from all point masses of disc. The transverse components in y-axis direction, from upper and lower disc halves, are abolished to each other. So

$$g(r) = g_{\mathrm{b}}(r) + g_{\mathrm{h}}(r) + \sum_{p=1}^{P} \sum_{q=1}^{Q} g_{pq}(r) \qquad \text{for } r \in [0, \infty). \tag{5.1}$$

The orbital star velocity $v(r)$ can be clearly calculated by the comparison of centripetal gravitation acceleration $g(r)$ determined by equation (5.1) with centrifugal acceleration:

$$g(r) = \frac{v^2(r)}{r}. \tag{5.2}$$

From equation (5.2) we will get the star orbital velocity

$$v(r) = \sqrt{r g(r)}, \tag{5.3}$$

where the acceleration $g(r)$ is calculated according to equation (5.1). Equation (5.3) forms the demanded galaxy rotation curve in the graphic way. On the right-hand side of equation (5.1) it is necessary to calculate gravitational accelerations of galactic bulk, halo, and disc. Their calculations are introduced in following chapters.

6. Gravitational acceleration of galactic bulk

A galactic bulk behaves approximately as a homogeneous sphere with the constant volume density ρ_b in its whole volume. When calculating the gravitational acceleration it is necessary to proceed differently *outside* and *inside* the bulk.

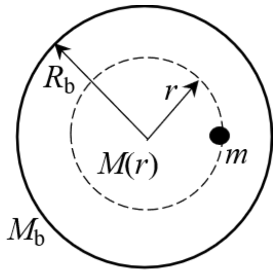

Figure 6.1: Point mass m lying inside the bulk sphere M_b.

The acceleration calculation *inside* the spherical bulk is based on the known fact that in the concentric spherical cavity of the empty sphere the gravitation field is zero, see R. Feynman [21], Volume II, Chapters 5.7 and 5.8. It is clear from this fact that inside the full sphere, in the distance r from center, the mass lying above this radius is not gravitationally applied. The gravitation field will be generated only by the inner imaginary sphere at the radius r and mass $M(r)$. If the whole sphere at radius R_b and mass M_b is *homogeneous*, its volume density is *constant* and has the value

$$\rho_\mathrm{b} = \frac{\mathrm{d}M}{\mathrm{d}V} = \frac{M}{V} = \frac{3M_\mathrm{b}}{4\pi R_\mathrm{b}^3}. \tag{6.1}$$

This leads to equation (6.2a). Acceleration outside the bulk in given by (6.2b):

$$g_\mathrm{b}(r) = \begin{cases} GM_\mathrm{b}\frac{r}{R_\mathrm{b}^3} & \text{for} \quad 0 < r < R_\mathrm{b}, \\ GM_\mathrm{b}\frac{1}{r^2} & \text{for} \quad r \geq R_\mathrm{b}. \end{cases} \tag{6.2a, b}$$

In model, the stellar mass of bulk is given by equation $M_\mathrm{b} = k_\mathrm{Mb} M_\mathrm{st25}$, where M_st25 is the basic mass parameter of model, and k_Mb is adjustable coefficient. Its usual value is being $k_\mathrm{Mb} \cong 0.1$ to 0.3. Gravitational acceleration $g_\mathrm{b}(r)$ of the bulk will be substituted into equation (5.1).

7. Gravitational acceleration of galactic halo

Halo has the shape of sphere which volumetric density $\rho_\mathrm{h}(r)$ decreases exponentially with radius r. The spherical region belonging to bulk is necessary to be removed out of the center because the bulk is solved separately. So whole halo mass is found in interval $R_\mathrm{h,min}$ to $R_\mathrm{h,max}$. Both radii in the model are free adjustable: $R_\mathrm{h,min} \geq R_\mathrm{b}$, $R_\mathrm{h,max} \in (R_{25}, \infty)$.

Volumetric density must be expressed

$$\rho_\mathrm{h}(r) = \frac{\mathrm{d}M_\mathrm{h}}{\mathrm{d}V} = \frac{\mathrm{d}M_\mathrm{h}}{4\pi r^2 \mathrm{d}r} = \rho_\mathrm{h,max} \mathrm{e}^{-(r - R_\mathrm{h,min})/\lambda_\mathrm{h}}, \tag{7.1}$$

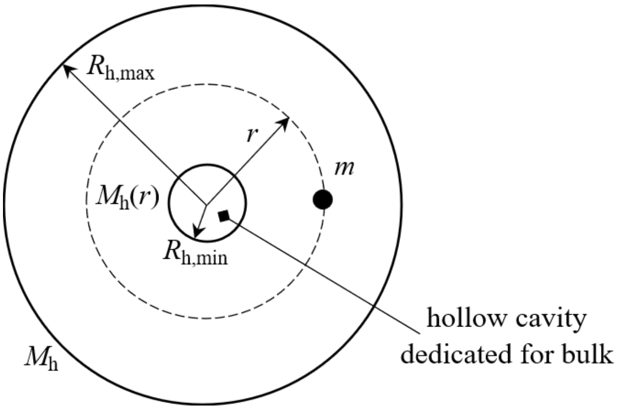

Figure 7.1: Point mass m lying inside the halo sphere $M_{\rm b}$.

because the beginning of the exponential is shifted by value $R_{\rm h,min}$ to the right. Quantity $\lambda_{\rm h}$ is arbitrarily adjustable longitudinal constant determining the exponential slope.

The following calculations are rather extensive. Therefore, only the results will be presented. Details are available in [22]. First, value $\rho_{\rm h,max}$ must be determined using the chosen halo mass $M_{\rm h}$, and the chosen longitudinal constant $\lambda_{\rm h}$:

$$\rho_{\rm h,max} = \frac{M_{\rm h}}{4\pi\lambda_{\rm h}\Big[(R_{\rm h,min}+\lambda_{\rm h})^2+\lambda_{\rm h}^2-\Big((R_{\rm h,max}+\lambda_{\rm h})^2+\lambda_{\rm h}^2\Big)\mathrm{e}^{(R_{\rm h,min}-R_{\rm h,max})/\lambda_{\rm h}}\Big]} \tag{7.2}$$

With help of it, we can calculate mass $M_{\rm h}(r)$ of the inner sphere at radius r, and subsequently the gravitational acceleration $g_{\rm h}(r)$:

$$g_{\rm h}(r) = \frac{4\pi G\rho_{\rm h,max}\lambda_{\rm h}}{r^2}\Big[(R_{\rm h,min}+\lambda_{\rm h})^2+\lambda_{\rm h}^2-((r+\lambda_{\rm h})^2+\lambda_{\rm h}^2)\mathrm{e}^{(R_{\rm h,min}-r)/\lambda_{\rm h}}\Big]. \tag{7.3}$$

In the model, the whole mass of galactic halo $M_{\rm h} = k_{\rm Mh}M_{\rm st25}$, and the longitudinal constant $\lambda_{\rm h} = k_{\lambda\rm h}R_{25}$, are determined with help of the adjustable coefficients $k_{\rm Mh}$, $k_{\lambda\rm h}$. The usual value of $k_{\rm Mh}$ is being 0.01 to 1. It is the totally uncertain value, the estimations vary widely, especially at authors working with dark matter, see Table 1.1. We will show further that mass of halo does not matter at all, because its gravitational acceleration can be completely neglected when compared with the disc (!). The halo mass $M_{\rm h}$ in the model is understood as the additional mass added optionally to stellar mass $M_{\rm st25}$. Gravitational acceleration $g_{\rm h}(r)$ of the halo will be substituted into equation (5.1).

8. Gravitational acceleration of galactic disc

Pure stellar mass of the disc itself at radius R_{25} is marked $M_{\rm d25}$. In region R_{25} the whole pure stellar mass $M_{\rm st25}$ is divided between the bulk and disc only, see Figure 5.1. The halo mass $M_{\rm h}$ is put to the model additionally. So we have $M_{\rm st25} = M_{\rm b} + M_{\rm d25}$.

As the mass $M_{\mathrm{b}} = k_{\mathrm{Mb}} M_{\mathrm{st25}}$ of the bulk has already been determined by the chosen coefficient k_{Mb}, then the pure stellar mass of disc must have the complementary value $M_{\mathrm{d25}} = M_{\mathrm{st25}} - M_{\mathrm{b}} = (1 - k_{\mathrm{Mb}}) M_{\mathrm{st25}}$.

Figure 8.1. shows the galactic bulk with one qth beam of a galactic disc on which the stars numbered $p = 1, 2, 3, \ldots, P$ are located (see also Figure 5.1). The radius of galactic bulk is marked R_{b}. The possible chosen gap between the bulk and disc has the width b. Radius of analyzed region should be at least 2-times to 3-times higher than radius R_{25}, the figure shows its actual size $2R_{25}$, generally nR_{25} (number n need not be integer). The stars on the beam are located equidistantly with the spacing d. The stars lying on pth circle of radius r_p (also marginal stars) correspond to the symmetrical annulus at width d (i.e. $r_p \pm d/2$). Thanks to exponential behavior of the surface mass density $\sigma(r)$, each annulus must have different mass. Then the mass of pth annulus must be divided equally among Q stars lying in this annulus. So to calculate the mass m_p of one star lying in pth annulus is the following task. All annuli have the same width

$$d = \frac{nR_{25} - R_{\mathrm{b}} - b}{P}. \tag{8.1}$$

The radius on which the pth star lies

$$r_p = R_{\mathrm{b}} + b + \left(p - \tfrac{1}{2}\right) d. \tag{8.2}$$

Areal mass density of the disc has an exponential behavior

$$\sigma(r) = \frac{\mathrm{d}M_{\mathrm{d25}}}{\mathrm{d}A} = \frac{\mathrm{d}M_{\mathrm{d25}}}{2\pi r\,\mathrm{d}r} = \sigma_{\max} \mathrm{e}^{-(r - R_{\mathrm{b}} - b)/\lambda_{\mathrm{d}}}, \tag{8.3}$$

because the beginning of the exponential is shifted to the right by the value $(R_{\mathrm{b}} + b)$.

The following calculations are very extensive. Therefore, only the results will be presented. Details are available in [22]. First, the value $\sigma_{\max}$ must be determined using the disc mass M_{d25}, and the chosen longitudinal constant λ_{d}:

$$\sigma_{\max} = \frac{M_{\mathrm{d25}}}{2\pi\lambda_{\mathrm{d}}\left[R_{\mathrm{b}} + b + \lambda_{\mathrm{d}} - (R_{25} + \lambda_{\mathrm{d}})\mathrm{e}^{(R_{\mathrm{b}} + b - R_{25})/\lambda_{\mathrm{d}}}\right]}. \tag{8.4}$$

Second, the mass m_p of one star lying in pth annulus, i.e. lying on pth circle, can be calculated as follows

$$m_p = \frac{2\pi}{Q}\sigma_{\max}\lambda_{\mathrm{d}}\mathrm{e}^{-pd/\lambda_{\mathrm{d}}}\left[(R_{\mathrm{b}} + b + (p-1)d + \lambda_{\mathrm{d}})\mathrm{e}^{d/\lambda_{\mathrm{d}}} - (R_{\mathrm{b}} + b + pd + \lambda_{\mathrm{d}})\right]. \tag{8.5}$$

Third, the gravitational acceleration caused by the star with index pq is

$$g_{pq}(r) = Gm_p \times \tag{8.6}$$

$$\frac{r - (R_{\mathrm{b}} + b + (p - \frac{1}{2})d)\cos(2\pi(q - \frac{1}{2})/Q)}{(r^2 + (R_{\mathrm{b}} + b + (p - \frac{1}{2})d)^2 - 2r(R_{\mathrm{b}} + b + (p - \frac{1}{2})d)\cos(2\pi(q - \frac{1}{2})/Q))^{3/2}}$$

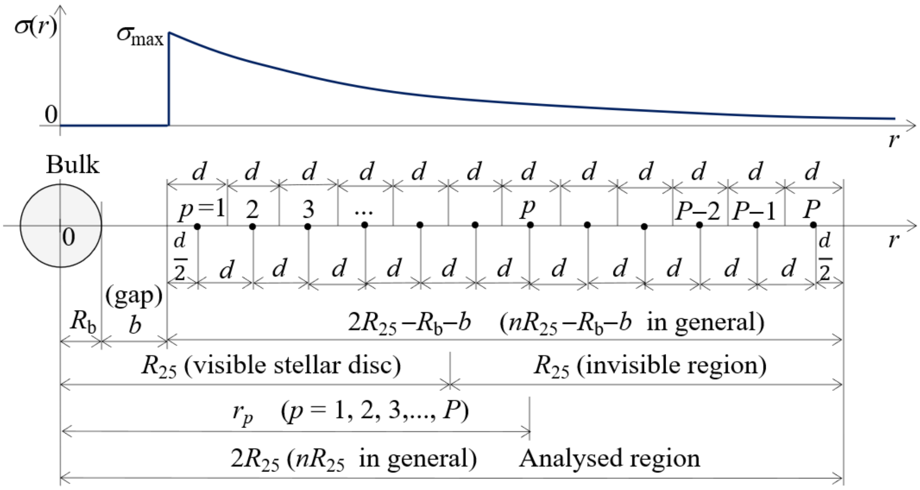

Figure 8.1: Galactic bulk drawn together with one beam of the galactic disc.

In Excel, the calculation of the gravitational acceleration $g_{pq}(r)$ consists of following steps:

To calculate $\sigma_{\max}$ according to (8.4), and numerical result to substitute into (8.5). To calculate m_p according to (8.5), and numerical result to substitute into (8.6). The whole gravitational acceleration $g_d(r)$ of the disc will be gotten by the sum of all the partial accelerations $g_{pq}(r)$:

$$g_{\mathrm{d}}(r) = \sum_{p=1}^{P}\sum_{q=1}^{Q} g_{pq}(r) = 2\sum_{p=1}^{P}\sum_{q=1}^{Q/2} g_{pq}(r). \tag{8.7}$$

Gravitational acceleration $g_{\mathrm{d}}(r)$ of the disc will be substituted into equation (5.1).

9. Calculation of M31 galaxy rotation curve

As the example of calculation, the galaxy M31 (NGC 224) in Andromeda constellation was chosen. This galaxy was chosen from following reasons:

- M31 is the nearest of large galaxies, so its distance (2.52×10^6 ly) is determined sufficiently accurately. Therefore, its absolute dimensions ($D_{25} = 152\,000$ ly) are also sufficiently accurately determined.
- This galaxy is a typical representative the spiral galaxies of SAb-class with a flat disc and central spherical bulk. Introduced model is constructed just for this galaxy type.
- Regarding its nearness, the galaxy has probably the most accurate determined whole pure stellar mass M_{st25} in region D_{25}. Calculation methodology yields to historical development, data from various authors differ ($0.9 \times 10^{11} M_\odot$ to $1.5 \times 10^{11} M_\odot$), in model we used average value $M_{\mathrm{st25}} \cong 1.2 \times 10^{11} M_\odot$.

The calculation was realized in Excel software. Before the calculation, all the model quantities were transferred into basic SI units.

Model parameters for M31 galaxy

Basic model parameters:
$R_{25} = D_{25}/2 = 76\,000$ ly $= 7.19 \times 10^{20}$ m (credible value).
$M_{\mathrm{st25}} = 1.2 \times 10^{11} M_{\odot} = 2.39 \times 10^{41}$ kg (uncertain average value).
Auxiliary parameter, i.e. disc radius measured in infrared band, has the value: $R_{\mathrm{infra}} = 1.25\, R_{25}$.

The model coefficients were selected in the way to fit the calculated rotation curve as near as possible to the measured one:
$k_{\mathrm{Mb}} = 0.16$ – bulk mass coefficient.
$k_{\mathrm{Rb}} = 0.1$ – bulk dimension coefficient.
$k_{\mathrm{Mh}} = 0.1$ – halo mass coefficient (chosen intuitively).
$k_{\mathrm{Rh,min}} = 0.1$ – halo inner boundary (intentionally chosen $k_{\mathrm{Rh,min}} = k_{Rb}$).
$k_{\mathrm{Rh,max}} = 3$ – halo outer boundary ($3R_{25}$).
$k_{\mathrm{b}} = 0$ – dimension coeff. of the gap between the bulk and disc (without gap).
$k_{\mathrm{MG25}} = 2.5$ – whole baryon galaxy mass in the region R_{25} ($M_{\mathrm{G25}} = 2.5 M_{\mathrm{st25}}$).
$k_{\lambda\mathrm{h}} = 1$ – longitudinal constant in exponential volumetric density of the halo.
$k_{\lambda\mathrm{d}} = 0.5$ – longitudinal constant in exponential surface density of the disc.
$n = 3$ – radius of analyzed region is $3R_{25}$.

The disc was modeled by $72 \times 36 = 2592$ point masses, arranged in $P = 72$ circles, $Q = 36$ beams. Number P was chosen greater than Q, regarding a very wide analyzed region at radius $3R_{25}$. The size of numbers P, Q does not nearly influence the quality of the calculation.

Results of calculation

In Figure 9.1 the dashed line marks the measured rotation curve of M31 galaxy gained as average values from many various information sources, see [10]. The rotation curves was calculated for three different values of the mass coefficient $k_{\mathrm{MG25}} = 1; 2.5; 3.5$.

The best fitting was achieved at coefficient $k_{\mathrm{MG25}} = 2.5$. So, numerically it is:

$$M_{\mathrm{G25}} = k_{\mathrm{MG25}} \cdot M_{\mathrm{st25}} = 2.5\, M_{\mathrm{st25}} \cong 5.97 \times 10^{41} \text{ kg } \cong 3 \times 10^{11} M_{\odot}.$$

Let us notice that the mass coefficient k_{MG25} of the whole mass in the visible region D_{25} is unexpectedly very low (!). The whole mass is 2.5-times greater than the stellar mass. This implies the ratio (*interstellar mass*)/(*whole mass*) $= (2.5 - 1)/2.5 = 0.6 = 60\,\%$. The commonly reported value is approx. 15 %. But in [23], L. Kohoutek admits value 25 %, or even more. Besides, the value 60 % corresponds very well to equation $M = \rho V$, where ρ is the typical interstellar disk density 2 (H-atoms)/cm^3, and V is the disc volume in the visible region D_{25}. In this sense, the demand for dark matter within the galaxy seems totally useless (!).

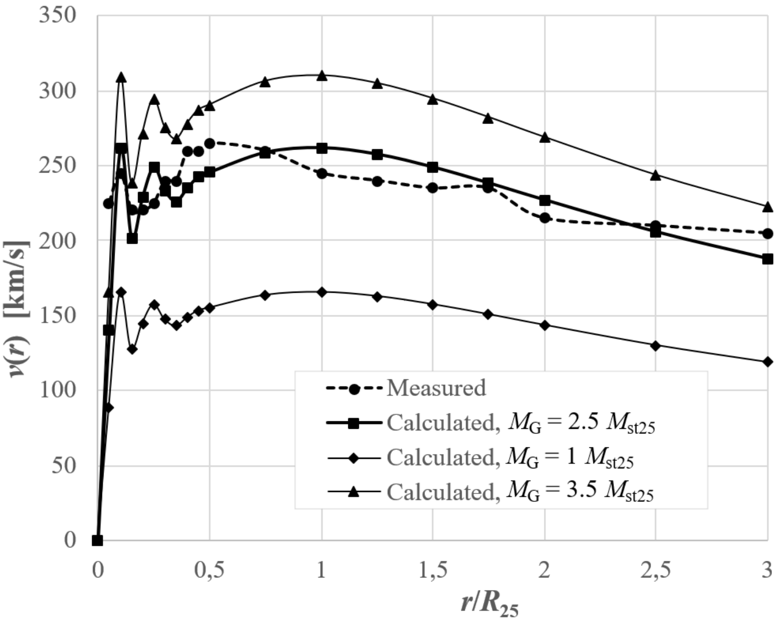

Figure 9.1: Galaxy M31. Rotation curves calculated for $k_{\mathrm{MG25}} = 1; 2.5; 3.5$.

It is possible to assume that the value 5.97×10^{41} kg provides in principle the most accurate determination of the total mass in region D_{25}. The reason is following:

Stellar mass M_{st25} acts only as an initial guess during the fitting process. If the photometrically determined stellar mass M_{st25} has in the reality different value, then the coefficient k_{MG25} would change too, but in such a way the product $k_{\mathrm{MG25}} \cdot M_{\mathrm{st25}}$ to be permanently equal to value 5.97×10^{41} kg (if the fitting should occur).

We may think that this method leads generally to the most accurate determination of the galaxy masses because:

- The method is based on validity of the original Newton gravitation law.
- The result does not depend on the accuracy of the M_{st25} determination (the "rough" result of photometric analysis is sufficient).
- Measuring the rotation curves by Doppler effect is very accurate in principle.

Figure 9.2 shows calculated gravitation accelerations of individual galaxy components. We see that in the region at radius $0.1R_{25}$ the acceleration of bulk dominates. Outside the bulk this acceleration decreases rapidly according to $1/r^2$, in accordance with Newton gravitation law. Within the radius of $0.2R_{25}$ the negative disc acceleration corresponds to theoretical results in Chapter 4. In region $r > 0.2R_{25}$ the disc acceleration is positive, and it is *essentially dominant* against both the bulk and halo accelerations. The halo acceleration seems to be zero. It is given by the fact that its value is in three orders smaller than the disc or bulk acceleration. So, the halo acceleration is shown separately in Figure 9.3. In the region at the radius $0.1R_{25}$ the

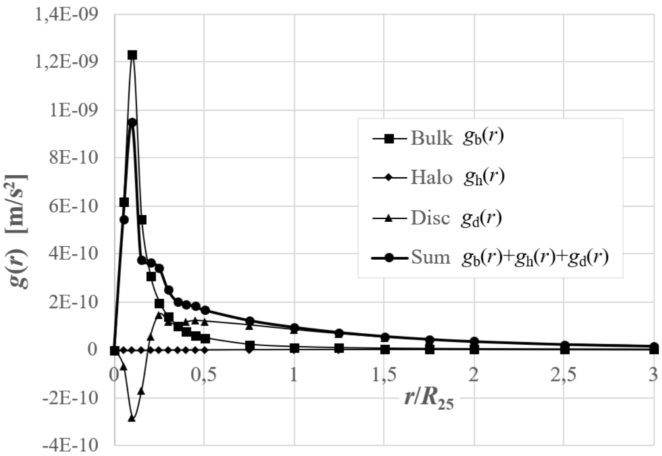

Figure 9.2: Galaxy M31. Gravitational accelerations of the bulk, halo, disc, and summarized acceleration.

acceleration is truly zero, see radius $R_{\mathrm{h,min}}$ in Figure 7.1. For demonstration purposes the mass $M_{\mathrm{h}} = k_{\mathrm{MG25}} \cdot k_{\mathrm{Mh}} \cdot M_{\mathrm{st25}} = 2.5 \cdot 0.1 M_{\mathrm{st25}}$ was chosen in the model. Let us note that only at the nonsensical state $M_{\mathrm{h}} \cong 250 M_{\mathrm{st25}}$, all three accelerations of the halo, bulk, and disc would be comparable. The halo influence on the rotation curve is *negligible*. The reason is following: The vast outer halo regions have in principle no gravitational influence on the disk, see Chapters 6 and 7.

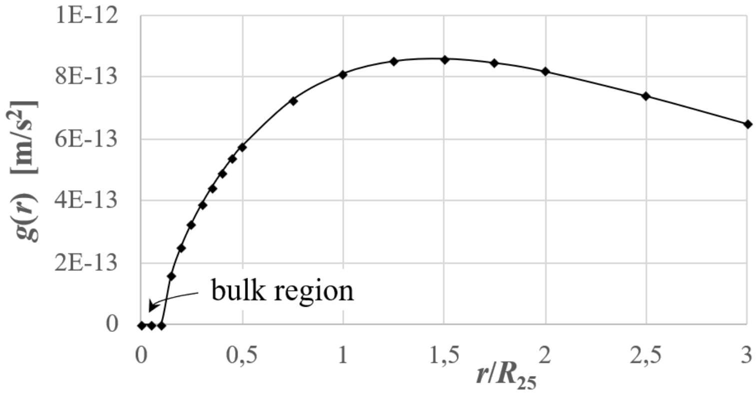

Figure 9.3: Galaxy M31. Gravitation acceleration of halo. Calculated for $M_{\mathrm{h}} = 2.5 \cdot 0.1 M_{\mathrm{st25}}$.

In some galaxies the special phenomenon occurs, i.e. the sharp decrease of the orbital velocity on the boundary between the bulk and disc. This is not easy to measure it because of high mass density, and it has not been explained successfully so far. It was also measured in galaxy M31, see Figures 9.4 and 9.5. The phenomenon can be very easily simulated and explained. In Figure 9.6, the curves differ only in one parameter k_{b}. This parameter adjusts the width of the gap b between the bulk surface and inner disc edge, see Figures 5.1 and 8.1. The explanation is simple:

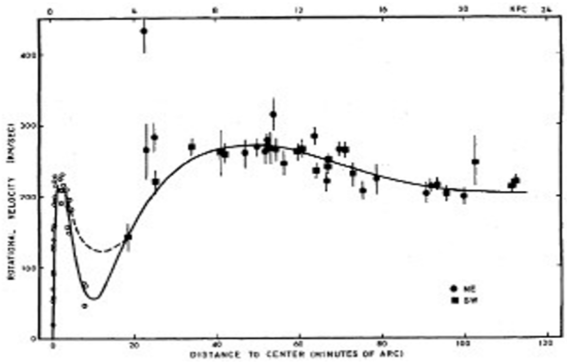

Figure 9.4: Galaxy M31. The velocity decrease measured in the region between the bulk and disc. Taken from [1].

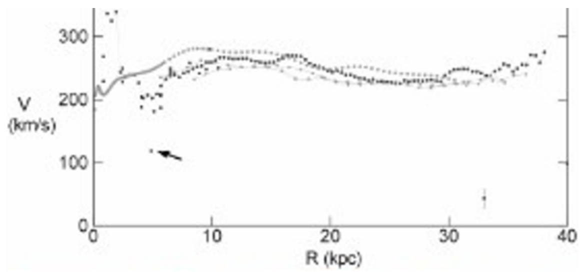

Figure 9.5: Galaxy M31. The decrease marked with an arrow was measured by Loinard et al. [4] and taken from [10].

With the growing gap b, the inner disc edge shifts to the right, so even the negative minimum of the disc acceleration shifts also to the right, see Figure 9.2. The subsequent local decrease of the *whole summed acceleration* necessarily leads to the local decrease of the orbital velocity in accordance with equation (5.3).

Mass of outer parts of M31

So far, the mass of the galaxy has been calculated only in region $r \in (0, R_{25})$. We shall be interested in the disc mass of the infra-region $r \in (R_{25}, R_{\mathrm{infra}})$, i.e. $r \in (R_{25}, 1.25\, R_{25})$, of the region $r \in (R_{\mathrm{infra}}, 3R_{25})$, and of residual region $r \in (3R_{25}, \infty)$. Listed regions concern *only the disc* (not halo), and have the shape of annulus. We will calculate the annuli mass $M_{\mathrm{min,max}}$ in the general boundaries r_{min}, r_{max}:

$$M_{\mathrm{min,max}} = \tag{9.1}$$

$$k_{\mathrm{MG25}} M_{\mathrm{d25}} \cdot \frac{(r_{\mathrm{min}} + \lambda_{\mathrm{d}})\mathrm{e}^{-r_{\mathrm{min}}/\lambda_{\mathrm{d}}} - (r_{\mathrm{max}} + \lambda_{\mathrm{d}})\mathrm{e}^{-r_{\mathrm{max}}/\lambda_{\mathrm{d}}}}{R_{\mathrm{b}} + b + \lambda_{\mathrm{d}} - (R_{25} + \lambda_{\mathrm{d}})\mathrm{e}^{(R_{\mathrm{b}}+b-R_{25})/\lambda_{\mathrm{d}}}} \cdot \mathrm{e}^{(R_{\mathrm{b}}+b)/\lambda_{\mathrm{d}}}.$$

Details are available in [22]. Masses of individual regions in galaxy M31 are listed in Table 9.1. Items 1, 2, 7 are the result of the fitting process. Items 3, 4, 5, 6 are calculated according to equation (9.1). Masses are introduced in two various units.

Table 9.1: Calculated masses of individual regions in galaxy M31.

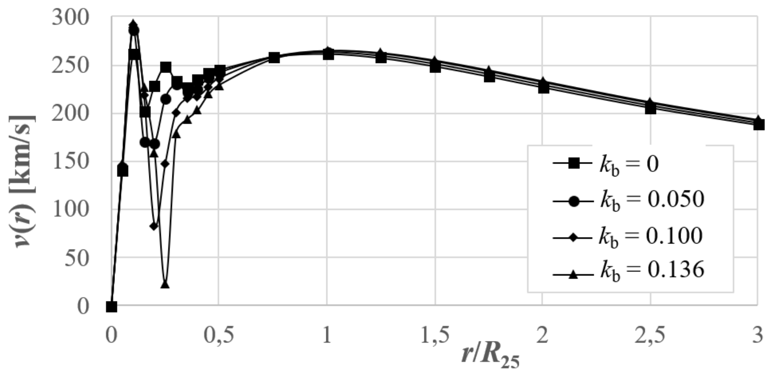

Figure 9.6: Galaxy M31. Rotation curves calculated for $k_\mathrm{b} = 0; 0.050; 0.100; 0.136$

no	Region	Radius r_min	Radius r_max	Mass $[M_\mathrm{st25}]$	Mass $[M_\odot]$
1	Bulk	$0\,R_{25}$	$0.1\,R_{25}$	0.400	0.48×10^{11}
2	Visible region of disc	$0.1\,R_{25}$	$1\,R_{25}$	2.100	2.52×10^{11}
3	Residual infra-region of disc	$1\,R_{25}$	$1.25\,R_{25}$	0.433	0.52×10^{11}
4	Disc region of interest	$1.25\,R_{25}$	$3\,R_{25}$	0.983	1.18×10^{11}
5	Residual region of disc	$3\,R_{25}$	∞	0.063	0.08×10^{11}
6	Whole disc	$0.1\,R_{25}$	∞	3.580	4.30×10^{11}
7	Bulk + visible region of disc	$0\,R_{25}$	$1\,R_{25}$	2.500	3.00×10^{11}
8	Whole galaxy (bulk + disc)	$0\,R_{25}$	∞	3.980	4.78×10^{11}

10. Conclusion

The paper deals with the gravitational model suitable for the calculation of a rotation curve of the SAb-class non-barred spiral galaxies with the flat disc and central spherical bulk. Galaxy M31 was analyzed in detail by this model. The paper leads to several unusual, but very important findings: The most important disc properties are:

1. Negative centrifugal gravitational acceleration inside its empty central region.

2. Extraordinary great centripetal gravitational acceleration within the disc region.

3. The acceleration decreases very slowly with the distance, much more slowly than according to the law $1/r^2$.

The gravitational model is based on equations (5.1), (5.2) and (5.3). Their correctness cannot be discussed. The evidence of this validity is the good agreement of the calculated and measured rotation curves.

From achieved results it is possible to judge that Newton gravitation law is valid with high accuracy even in great intergalactic distances. The so-called Modified Newtonian Dynamics (MOND) appears to be totally baseless.

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Conference *Cosmology on Small Scales 2026*
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TRACING HUBBLE FLOW AND PECULIAR MOTIONS AROUND THE LOCAL GROUP OF GALAXIES

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Abstract: Data on radial velocities and distances of 76 isolated galaxies with distances $D_c < 5.0$ Mpc relative to the barycenter of the Local Group are presented. Also, a dozen more isolated dwarf galaxies within this volume are noted, which currently lack accurate velocity or distance measurements. For 40 objects with $D_c < 3.5$ Mpc, under the assumption of a spherically symmetric Hubble flow around a point mass, estimates of the local Hubble parameter, $H_0 = (72 \pm 5)$ km s^{-1}Mpc^{-1}, and the total group mass

$$M(< 3.5 \text{ Mpc}) = (2.72 \pm 0.58) \times 10^{12} M_\odot$$

are obtained. It is noted that the dispersion of peculiar velocities in the vicinity of the Local Group increases from 22 km s^{-1} within $D_c < 1.5$ Mpc to 48 km s^{-1} within $D_c < 3.5$ Mpc and 65 km s^{-1} for $D_c < 5$ Mpc, and this is not caused by distance measurement errors. The dispersion of peculiar velocities of nearby isolated galaxies depends little on their degree of isolation, gas resources, or star formation activity. The barycenter of the Local Group has its own peculiar velocity of (21 ± 9) km s^{-1} relative to the surrounding isolated galaxies towards a supergalactic longitude of $L \sim (271 \pm 34)$ deg.

Keywords: cosmology, local group of galaxies, Hubble parameter

PACS: 98.80-k

PROBLEMS OF COSMOLOGY ON SMALL SCALES OF THE UNIVERSE

Abstract: Six challenges for the standard cosmological model lambda-CDM are listed, which arise when comparing its conclusions with observational data on scales of ~ 1 Mpc. The parameters of luminous and dwarf galaxies in the local sphere with a radius of 12 Mpc are presented. The average densities of stellar mass, neutral gas and dark matter are determined depending on the distance in the Local volume. A future of the distribution of angular momentum of nearby galaxies are noted.

A comparison of mass estimates for systems of galaxies based on the motions of its internal (virialized) members and neighboring galaxies are given. The reasons for the low value of the dark matter density, $\Omega_m = 0.08$, in the local Universe are discussed.

ALTERNATIVE COSMOLOGICAL THEORIES

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EARTH EXPANSION – CAUSED BY COSMOLOGICAL FACTORS?

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Abstract: Based on highly accurate measurements of the Earth's radius at various times, it is assumed that it is approximately constant in numerical terms. However, little attention has been paid to the scale drift observed in these measurements. The drift rate found is of the same order of magnitude as the rate of cosmological expansion, which may have important implications for geophysics and cosmology.

Keywords: Drift rate, Earth expansion, Hubble constant

PACS: 98.80-k

1. Introduction

The velocity of expansion of the universe is characterized by the parameter discovered by Hubble and Lemaître (the Hubble constant) and refers to a distance of one megaparsec (Mpc). It is approximately $70 \pm 4\,(\mathrm{km/s})/\mathrm{Mpc}$. One Mpc corresponds to approximately 3.26×10^6 light-years or 3.09×10^{24} cm. By comparison, the Earth's radius is 6.4×10^8 cm, which is approximately 4.8×10^{15} times smaller. If we convert the increase in velocity per Mpc to the Earth's radius R_{Earth}, we get ~ 0.05 cm/yr, which corresponds to $H_0 = 73$ (km/s)/Mpc. However, according to current cosmological assumptions, this is not permissible, as cosmic expansion is assumed only for cosmologically relevant distances. It is therefore assumed that the Earth's radius is not cosmologically relevant.

Occasionally, instead of the Hubble constant H_0, the resulting cosmological expansion rate (here: α) is used. This is derived from the Hubble constant to

$$\alpha = \Delta r \times (r \times t)^{-1}, \tag{1}$$

where t is a time period, r is a distance, and Δr is a distance traveled during the time period t. For a distance of, say, 1 Mpc, the following can then be written for $H_0 = 73$ (km/s)/Mpc

$$\alpha = 73 \text{ km} \times (3.1 \times 10^{19} \text{ km} \times 1\,\mathrm{s})^{-1} = 2.35 \times 10^{-18}\ \mathrm{s}^{-1}.$$

For a Hubble constant of 73 (km/s)/Mpc, the cosmological expansion rate is 2.35×10^{-18} s^{-1}. About 40 years ago, S. W. Carey came up with the idea that the expansion of the Earth could be explained cosmologically [1]. The reason for this is not known. Perhaps the findings of L. Egyed played a part, as he assumed that the Earth's radius was expanding at a rate of approximately 0.5 mm per year, which corresponds to the rate of cosmological expansion. It now appears that X. Wu et al. [2] have refuted the notion that the Earth is expanding. Although the study presents strong arguments in favor of Earth expansion, the measurement results indicated an expansion of the Earth's radius of 0.1 ± 0.2 mm/year. Wu's results thus confirm that Earth expansion – that is, a significant relative velocity between the center of mass and the Earth's surface – can be ruled out. We concur with this conclusion.

2. Arguments

In addition to the assumptions put forward by S. W. Carey and L. Egyed, there are many other arguments in favor of the expansion of the Earth; here, we shall confine ourselves to a few cosmological arguments. A cosmological argument in favor of the expansion of the Earth can be put forward if, for example, the results of a study show that the expansion of the Earth's radius is close to 2.35×10^{-18} s^{-1}, i.e. close to the Hubble constant. Here are a few examples:

1

The value for the expansion of the Earth's radius, found by L. Egyed to be approximately 0.5–0.66 mm/year, yields an expansion rate of $\alpha \approx 2.49 \times 10^{-18}$ s^{-1} when equation (1) is applied, and can be interpreted as having a cosmological origin. The result agrees with the cosmological expansion rate to 18 decimal places.

2

In 1990, H. Ruder, M. Schneider and M. Soffel carried out measurements of the Earth's body using the Wettzell geodetic fundamental station [3]. Among other things, this yielded a result for the polar distance between the Earth's poles. This distance is apparently increasing by 1 mm/year. The polar radius of the Earth is increasing by 0.5 mm/year, which corresponds to an expansion rate α as given by relation (1),

$$\alpha = 0.05 \text{ cm } \times (6370 \times 10^5 \text{ cm } \times 31.56 \times 10^6 \text{ s})^{-1} = 2.49 \times 10^{-18} \text{ s}^{-1}.$$

Although this value corresponds to the rate of cosmic expansion (the Hubble constant), post-glacial uplift was assumed. The resulting deceleration of the Earth's rotation is given as approximately 8×10^{-11} per year. This corresponds to 2.53×10^{-18} s^{-1}.

3

Records of eclipses and fossils findings have shown that the length of the day is subject to a secular delay, in addition to sporadic and periodic changes. As early as

1959, P. Ahnert stated that the secular rotational delay amounts to 29.95 seconds per Julian century. Meanwhile, in 2008, N. A. Bär [4] expressed this difference as follows:

$$\Delta t = 29.2208 \times T^2, \tag{2}$$

where the time interval Δt is given in seconds and T is the number of Julian centuries. Over the past 1000 years, the Earth's rotation has lagged behind SI time by ≈ 2922 seconds. Taking into account the length of the Julian century of 31.56×10^8 seconds, equation (2) yields the delay rate

$$\alpha \approx 29.22 \times (1/31.56 \times 10^8)^2 = 2.93 \times 10^{-18}\ \mathrm{s}^{-1}.$$

The Earth's rotation is slowing down at roughly the same rate as the expansion of the universe. Note that formula (2) refers not to the length of the day but to any given period of time.

4

The marine crust is approximately 200×10^6 years old, whilst the continental crust is approximately 4 to 4.2×10^9 years old, which is $\sim 30\,\%$ of the so-called age of the universe. As H_0 has remained approximately constant over the past 4.1×10^9 years, distances in the universe have expanded by $\sim 30\,\%$ during this period. If the Earth had expanded along with the universe, then 4.1×10^9 years ago – disregarding other factors – it would have had a radius of 70 % of its current radius, i.e. approximately 4459 km. The radius would have increased by ≈ 1911 km. Applying equation (1) gives

$$\Delta r = 6370\ \mathrm{km} \times 4.1 \times 10^9 \times 31.56 \times 10^6\ \mathrm{s} \times 2.35 \times 10^{-18}\ \mathrm{s}^{-1} = 1937\ \mathrm{km}.$$

According to (1), the resulting value for radial expansion differs by only 26 km. The agreement between the two growth values is remarkable, and it appears that the Earth's radius is expanding along with the universe. However, this is not possible within the framework of standard cosmology, a fact that appears to be confirmed by the measurement results of Wu et al. [2].

The problem of a very high density at that time ceases to apply once the next point is accepted. Consequently, the Earth's radius remained numerically constant, and with it the density.

5

Wu's results, as reported in [2], are not being questioned here. However, it seems to be generally overlooked that this study yielded a further result: a origin shift or scale shift was observed. The scale used for measurement drifts by approximately 0.05 cm over the course of a year, relative to the Earth's radius. This means that, whilst the Earth's radius remains numerically constant, its value changes by this amount due to scale drift. It should be noted that equation (1) yields a drift rate α for the Earth's radius (6370×10^5 cm) over the course of a year (31.56×10^6 s), which is equal to the cosmological expansion rate! It should be noted here that even for cosmic distances $z > 2$, the recession velocity cannot be a relative velocity of the observed objects because $v > c$. Therefore, scale drift must also be present in these cases.

3. Conclusions

The Earth is expanding without changing its numerical radius or density. This may seem strange, but the same must be assumed for the radius of distant galaxies, which are evidently expanding at the same rate of expansion without any apparent cause, as the author explains in [5]. In this case too, cosmological expansion is generally ruled out. Reference [5] also lists further examples in which the same expansion rate or scale drift is observed. This includes, for example, the Moon's orbit. Tidal friction is likely to be negligible in this case because the rate of expansion or drift corresponds exactly to other drift rates. The fact that this expansion, observed in many places, is not a numerical expansion but rather scale drift can also be inferred from the fact that, for very distant galaxies, the recession velocity exceeds the speed of light. It is therefore not a matter of relative velocity but of scale drift, as is the case with the Earth's radius. For intermediate distances, such as the distance to the Moon or the radius of a galaxy, scale drift can also be assumed to account for their expansion. Since the distance to the Moon is approximately 60 times greater than the Earth's radius, the annual drift value is calculated as 60×0.05 cm/yr = 3.0 cm/yr. To this drift value must be added a small value for relative velocity due to tidal friction, and the LLR measurement of 3.82 ± 0.07 cm/yr provides a margin of error: With a relative velocity of ~ 0.8 cm/yr, the Earth's Roche limit was not a problem for the Moons existence, even in the early days. This becomes problematic at a relative velocity of 3.82 ± 0.07 cm/yr.

Galaxy radii appear to have been smaller with increasing distance, suggesting they have expanded [6]. According to conventional wisdom, they cannot be undergoing cosmological expansion, yet according to V. Müller [5] they are. The drift of peripheral parts away from the center resolves these issues. It appears that the drift always increases with the distance from the respective gravitational center at the rate of cosmological expansion α, provided there are no forces other than gravitational attraction at play.

As the distance increased, the spatial and temporal units become smaller and approach zero at 13.8×10^9 light-years: The days and UT seconds were shorter, the galaxies were smaller [6], the distance to galaxies was shorter, the cosmological distance scale z encompassed shorter time scales, our lunar orbit was smaller, as were other lunar orbits [7], [8], etc. All these examples relate to spaces and objects whose internal structure is dominated by gravity. According to T. van Flandern [9], these objects behave "dynamically". One can assign them their own scale, which differs from the SI scale by the cosmological expansion rate ($\sim 2.35 \times 10^{-18}$ s^{-1}). Cosmic expansion and the Big Bang do not occur on this scale. Objects dominated by other fundamental forces are not included: Continental plates, small asteroids < 200 km, everyday objects, pendulums, etc. are prevented from expanding or drifting by stronger internal forces (electromagnetism). Mars is undoubtedly one of the objects dominated by gravity. In our view, it should therefore rotate with a deceleration of approximately 2.35×10^{-18} per second, just like these objects and the Earth. Given the current state of technology, demonstrating a joint delay in rotation should pose no problem, which amounts to proving that the Earth is expanding along with the universe.

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