Journal of Physics:
Conference Series

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To cite this article: Michal Křížek 2025 *J. Phys.: Conf. Ser.* **2948** 012007

Journal of Physics: Conference Series **2948** (2025) 012007 doi:10.1088/1742-6596/2948/1/012007

Infrared measurements of the JWST suggest that our dynamic universe is spatially closed

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Abstract. Global geometry and shape of the physical universe may be revealed by observing objects at large cosmological redshift z, since for small z the universe seems almost flat. Recent infrared measurements of the James Webb Space Telescope (JWST) indicate that there exist very luminous galaxies at distances $z \geq 13$ that should not exist according to the standard ΛCDM cosmological model for the flat universe with curvature index $k = 0$. We introduce a spacetime-lens principle that could explain why these very distant galaxies shine so much. We show that the observed large flux luminosities may be mere optical effects due to the positive curvature index $k = 1$ of an expanding 3-sphere modeling our physical universe in time. For Euclidean or hyperbolic geometries such large flux luminosities seem implausible. This suggests that the right model of a homogeneous and isotropic physical universe for each fixed time instant is a 3-sphere. The standard cosmological model is based on the normalized Friedmann equation $\Omega_{\mathrm{M}} + \Omega_\Lambda + \Omega_k = 1$, where $\Omega_{\mathrm{M}} + \Omega_\Lambda \doteq 1$ by measurements. We also show that this does not imply that $\Omega_k = 0$ and $k = 0$ as it is often claimed.

1 Introduction

At present, there are never-ending discussions about the global geometry of our physical universe, if it is spherical, flat, or hyperbolic, see [3]. The JWST recently found very distant and luminous galaxies whose masses are up to 10^{11} stars at the distance $\sim 13\,$Gly, see [4, 5, 17, 18, 28]. We show that the large flux luminosities could be explained by a positive curvature of the physical universe that creates three large independent artificial optical magnification effects introduced in Examples 1–3 below.

Already in 1900, Karl Schwarzschild [23] conjectured that the physical universe can roughly be described by a huge three-dimensional sphere in the Euclidean space $\mathbb{E}^4$

$$\mathbb{S}^3_a = \{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 + w^2 = a^2\}, \tag{1}$$

where $a > 0$ stands for its radius. Also Albert Einstein in his famous 1917-paper on cosmology [8, p. 152] assumes that the entire universe can be modeled by the sphere $\mathbb{S}^3_a$ with fixed a. In this way he could avoid initial and boundary conditions for his field equations. If $a = 1$ we write only $\mathbb{S}^3$.

For the spherical model (1) Schwarzschild derived that the corresponding radius a should be at least $\approx 1.5 \cdot 10^{19}$ m. At present, this lower bound can be made much larger. The mean density of the universe in our neighborhood is $\rho \approx 10^{-26}$ kg/m^3, i.e., approximately 6 protons per m^3. Moreover, over 10^{12} galaxies are currently observed each having on average about 10^{11} stars. Thus, the total mass M of the entire universe can roughly be bounded from below by $M > 10^{12} \cdot 10^{11} \cdot M_\odot = 2 \cdot 10^{53}$ kg. Since the volume of 3-sphere (1) is $V = 2\pi^2 a^3 = M/\rho$, we get $\pi^2 a^3 > 10^{53+26}$ m^3. Hence, for the present value of the radius we obtain the following lower bound

$$a > \sqrt[3]{10^{78}} = 10^{26} \text{ [m]}. \tag{2}$$

In [25], Tuomo Suntola introduces his own model, called the *Dynamic Universe* (DU), in which the radius of 3-sphere (1) increases with time as $a(t) \sim t^{2/3}$. This model describes the physical universe in many parameters better than the standard cosmological ΛCDM model which is based on the theory of relativity, see [24] for details.

2 The space-lens principle for a non-expanding 3-sphere

To introduce the main idea of the space-lens principle, let a in (1) be a fixed constant. For clarity, consider only the cross section of the 3-sphere (1) by the equatorial hyperplane $w = 0$. Suppose that the corresponding 2-sphere $\mathbb{S}^2_a$ is perfectly transparent.

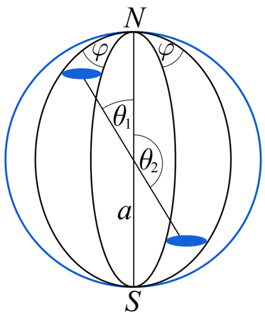

Figure 1: Two galaxies having the same diameter and the same bolometric luminosity are seen from the North Pole N of the sphere $\mathbb{S}^2_a$ under the same angle φ when the radius a is fixed. Their observed flux will also be the same although their comoving distances expressed in the spherical coordinates $\theta_1 = 30°$ and $\theta_2 = 150°$ differ five times. Photons travel along geodesics (i.e. great circles) to N. The viewing angle $\varphi > 0$ can be arbitrarily small. The magnification takes place along the whole curved trajectory, i.e. at any point. It works like a standard converging glass lens.

Example 1. Let an observer be located at the North Pole N of the 2-sphere, cf. Figure 1. Let there exist two galaxies of the same size and the same absolute bolometric luminosity. Assume that they are disc galaxies and that they are oriented facing the observer. In this case their observed flux intensity is proportional to the square of their diameter. Further assume that the comoving spherical angle of one galaxy is $\theta_1 \in (0°, 90°)$ and for the second galaxy is $\theta_2 = 180° - \theta_1$. Then the observer will see these two galaxies at the same angular size $\varphi > 0$ and also possessing the same flux even though their distances to N can be radically different (see Figure 1). This means that the observed flux intensity does not decrease with square of the distance like in Euclidean space.

Example 2. The second magnification effect is sketched in Figure 2. Consider two galaxies in $\mathbb{E}^2$ and $\mathbb{S}^2_a$ with the same diameter $D > 0$, the same luminosity, and the same distance d along geodesics from the point N. We see that $d = a\theta$ for $k = 1$. Then the angular size $\alpha = D/d$ for $k = 0$ is always smaller than the angular size φ for $k = 1$ and $\theta \in (0, \pi)$, since (see Figure 3)

$$\alpha = \frac{D}{d} = \frac{D}{a\theta} < \frac{D}{a \sin\theta} =: \varphi. \tag{3}$$

This represents another artificial magnification effect than that in Example 1.

The above-described two nonlinear optical magnification effects lead to the so-called *space-lens principle*. Furthermore, by means of (3) we can define the *reduction factor*

$$R(\theta) = \frac{\theta^2}{\sin^2\theta} \tag{4}$$

which is the ratio between areal sizes of a given galaxy for the manifolds $\mathbb{S}^3$ and $\mathbb{E}^3$. In Figure 4, we see that $R = R(\theta)$ is large if the comoving distance $\theta \in (0, \pi)$ is also large. Note that most of present papers on astrophysics and cosmology assume that $k = 0$. So their conclusions can be largely distorted if z is large and $k = 1$.

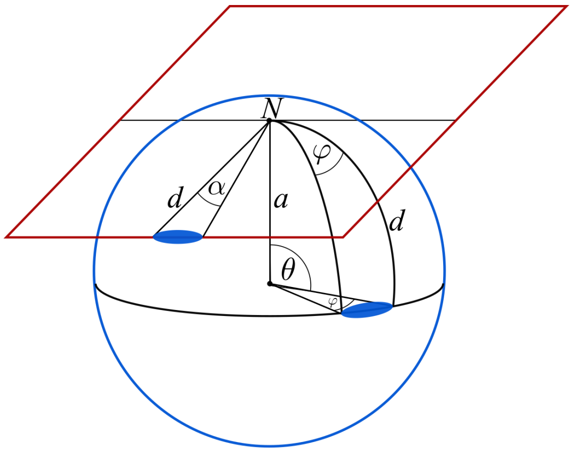

Figure 2: Angular size of a galaxy (except for MW) is indirectly proportional to its distance d if the space is modeled by $\mathbb{E}^3$ for a small viewing angle α measured in radians. This dependency is completely different for the 3-sphere with the viewing angle φ, see (3). The space dimensions are reduced to two.

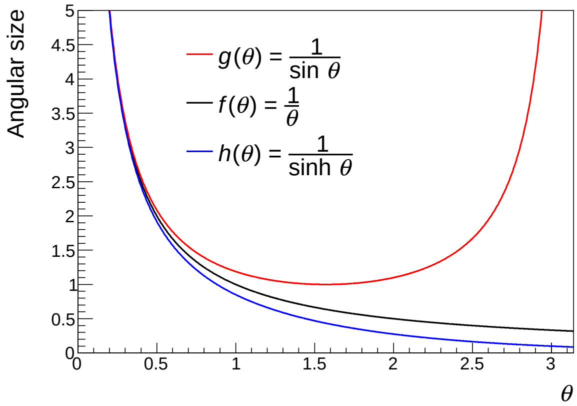

Figure 3: Angular size of a galaxy depends quite differently (up to a multiplicative constant D/a from (3)) on the comoving distance $\theta \in (0, \pi)$ on each of the manifolds $\mathbb{S}^3, \mathbb{E}^3$ and the hyperbolic pseudosphere $\mathbb{H}^3$.

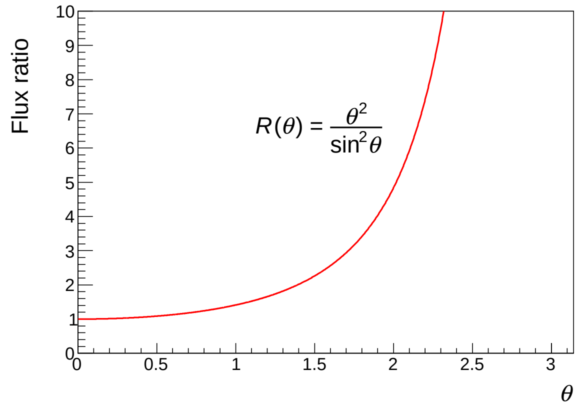

Figure 4: The ratio $R(\theta)$ between total flux intensities (areal sizes) of a given reference galaxy for the manifolds $\mathbb{S}^3$ and $\mathbb{E}^3$ is large if the comoving distance θ is also large: $0 \ll \theta < \pi$.

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Remark 1. Albert Einstein in [8] did not suppose that our universe could expand. His model (called *Einstein's static universe*) is described by the maximally symmetric manifold (1). However, there is a serious problem with time dilation effect in this model. Consider two spacetime travelers Adam and Bob. They do not have to be twins or brothers. They can even be replaced by accurate clocks. Let us fix Adam as indicated in the left part of Figure 5. For a moment assume that Bob travels from Adam along a given great circle (geodesic in $\mathbb{S}^3$) with a nonzero constant speed. If there were a time dilation of moving objects, then Bob's clock would tick slower than Adam's clock. Hence, after one orbit of Bob when they meet again, their proper time intervals would satisfy

$$\Delta t_{\mathrm{Adam}} > \Delta t_{\mathrm{Bob}}.$$

On the other hand, when Bob is fixed, we get the opposite inequality

$$\Delta t_{\mathrm{Adam}} < \Delta t_{\mathrm{Bob}},$$

see the right part of Figure 5. Therefore, $\Delta t_{\mathrm{Adam}} = \Delta t_{\mathrm{Bob}}$, the principle of relativity (see [7]) does not hold and there is no time dilation effect, see also [27]. The corresponding systems are not inertial. Anyway, if there does not exist time dilation globally, then there is also no time dilation locally, e.g., in an arbitrarily small neighborhood of Adam (resp. Bob) in the system, where he is fixed and where the space is locally almost flat. If $k \in \{0, 1\}$, then a freely moving clock cannot know how fast it should tick in the physical universe when moving uniformly straight ahead along a geodesic over a distance of, say, one meter. Should the clock tick regardless of time dilation or obey it?

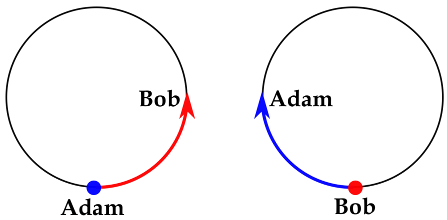

Figure 5: The twin paradox in Einstein's static universe. If there would exist a time dilation effect, then this would lead to a mathematical contradiction with the principle of relativity. In particular, the inequalities $\Delta t_{\mathrm{Adam}} > \Delta t_{\mathrm{Bob}}$ and $\Delta t_{\mathrm{Adam}} < \Delta t_{\mathrm{Bob}}$ should hold simultaneously. Hence, $\Delta t_{\mathrm{Adam}} = \Delta t_{\mathrm{Bob}}$.

We should bear in mind the above considerations, because the standard cosmological ΛCDM model is primarily based on the theory of relativity.

3 The spacetime-lens principle for an expanding 3-sphere

Now we describe the third artificial magnification effect caused by the expansion of the universe in time. Let us realize that the universe cannot change its geometry during its continual evolution, i.e., a closed spherical universe modeled by the bounded manifold $\mathbb{S}^3_a$ with time dependent radius $a = a(t)$ (see Figure 6) cannot be continuously deformed into the Euclidean or hyperbolic model.

According to [22],

$$\frac{a(t_0)}{a(t_1)} = z + 1, \tag{5}$$

where t_1 is the time instant when a photon was emitted and t_0 when it was received, i.e. $0 < t_1 < t_0$ and

$$t_0 \approx 13.8 \text{ Gyr} \tag{6}$$

is the *estimated age of the universe* by the standard ΛCDM model, and z is the corresponding redshift.

Furthermore, let L be the intrinsic bolometric luminosity of a galaxy (i.e., the total luminosity integrated over all frequencies and measured in Watts). The *luminosity distance* for the 3-sphere can be derived from the Friedmann-Lemaître-Robertson-Walker metric

$$d_{\mathrm{L}} = a(t_0)(z + 1) \sin \theta \tag{7}$$

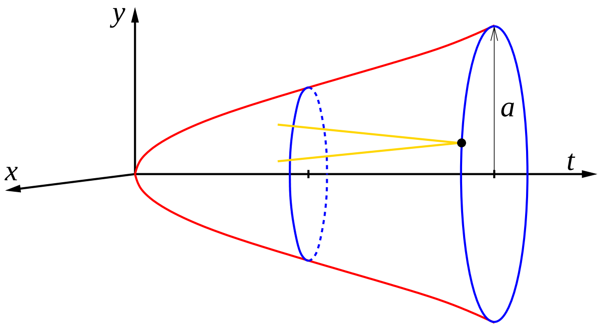

Figure 6: Three different manifolds that are used to model our physical universe for $k = 1$. The number three of space dimensions is reduced to one. Thus, the 3-sphere with radius $a = a(t) > 0$ at a fixed time instant t is replaced only by its great circle $\mathbb{S}^1_a$ for $z = w = 0$. This model expresses the homogeneity and isotropy of the universe. The model of spacetime can be obtained by rotation of the graph of the expansion function $a = a(t)$ about the time axis t. The observable universe is marked by the yellow manifold.

and for the associated measured flux ℓ we obtain by [22, p. 328] that

$$\ell = \frac{L}{4\pi a^2(t_0)(z+1)^2 \sin^2\theta}.$$

Notice that there is a serious ambiguity (non-uniqueness) in establishing the correct comoving distance $\theta \in (0, \pi/2] \cup (\pi/2, \pi]$, cf. also Figure 1. We will investigate this problem in the next section.

The younger the objects which are observed, the larger the magnification appears. Thus, by angular measurements we paradoxically see a very distant object as being larger. We will demonstrate how this artificial magnification works with the following two examples.

Example 3. According to [9], the redshift of CMB is $z = 1089$, i.e., by (5) the associated magnification is given by

$$z + 1 = 1090 = \frac{a(t_0)}{a(t_2)}, \tag{8}$$

where $t_2 \approx 380\,000$ yr. Hence, by (2) we see the radius of $\mathbb{S}^3_a$ modeling our universe, when the CMB was created, can be estimated as follows

$$a(t_2) = \frac{a(t_0)}{z+1} \sim \frac{10^{26}}{1090} \text{ m} \approx 3 \text{ Mpc}.$$

However, astronomers only can observe CMB on a very small part of $\mathbb{S}^3_a$ for $a = a(t_2)$, see (1). Its extremely magnified picture is observed on the celestial sphere $\mathbb{S}^2_{\overline{a}}$ with center on Earth and $\overline{a} \gg a(t_2)$.

Example 4. An enormous magnification is connected with the Big Bang itself, which appeared roughly 13.8 Gyr ago by (6). Although it happened in a minimal volume, its present position is on the possibly greatest 2-sphere (the so-called *horizon*) with an unimaginable large radius.

Thus, the farther we look, the corresponding sphere seems to be bigger and bigger, even though the universe was smaller and smaller, cf. Figure 7. This is the main reason of functioning of the *time-lens principle*.

4 The main theorem

Let the curvature index $k = 1$. For simplicity assume that the expansion function $a = a(t)$ is linear over some long time interval (t_1, t_0). This assumption is not too restrictive, since the expansion function is almost linear during the last 12 Gyr according to the standard cosmological model and observations, see Figure 8.

Theorem 1. *Let $\mathbb{S}^3_a$ expand at a constant velocity $V > 0$ over an interval (t_1, t_0), i.e., $\dot{a}(t) = V$ for all $t \in (t_1, t_0)$. Then the trajectory of a photon towards the observer at the North Pole N of $\mathbb{S}^3_a$ can be*

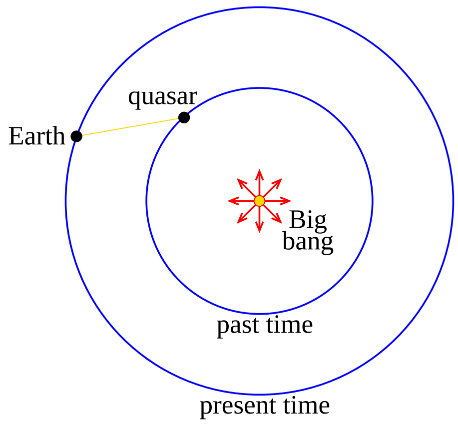

Figure 7: A schematic illustration of an expanding universe. We must consistently distinguish between "at that time" and "actual dimensions", i.e., the size of the universe, when observed photons left a quasar and the universe was much smaller, and "today's" dimensions when "ancient" photons arrived at Earth.

described by a logarithmic spiral (in space variables) with slope angle $\phi = \arctan(c/V)$ *and the comoving distance*

$$\theta = (\tan\phi)\ln(z+1) = \frac{c}{V}\ln(z+1)$$

corresponds to the redshift $z \geq 0$.

P r o o f. Since a photon traveling to N moves along geodesics in a plane passing through the center of $\mathbb{S}^3_a$, we can choose without loss of generality the Cartesian coordinate system in (1) so that $w = z = 0$. Since the space expands in the radial direction in this plane at the constant velocity V and since the photon moves in the tangential direction at the constant velocity c, the total velocity $\sqrt{c^2+V^2}$ is also constant and thus the corresponding slope angle

$$\phi = \arctan\frac{c}{V}$$

is constant, too (see Figure 9). Therefore, the trajectory of the photon is described by the logarithmic spiral in the standard polar coordinates (r, θ) as follows:

$$r(\theta) = a_1 \exp(\theta \operatorname{cotan}\phi), \tag{9}$$

where $a_1 = a(t_1) > 0$ and $\phi > 0$ are given constants.

From (9) and (5) we find the searched relation between z and θ, namely (cf. Table 1),

$$\theta = \frac{1}{\operatorname{cotan}\phi}\ln\frac{a_0}{a_1} = (\tan\phi)\ln(z+1) = \frac{c}{V}\ln(z+1), \tag{10}$$

where $a_0 = a(t_0)$. *QED*

In Figure 9, a similar trajectory can be constructed symmetrically with respect to the axis y. This produces the third kind of an artificial magnification effect in time, see also Suntola [25, p. 277].

Example 5. So let $V = c$ and consider a galaxy with redshift $z = 13$, see [28]. From the relation $\tan\phi = c/V$ we find that the slope angle is $\phi = 45°$, see Figure 9. Using (5), we get

$$14a_1 = a_0$$

(i.e., the volume of $\mathbb{S}^3_{a_1}$ is $14^3 = 2744$ times smaller than the present volume of $\mathbb{S}^3_{a_0}$). According to Theorem 1,

$$\theta \approx \ln 14 = 2.639 > \frac{\pi}{2}$$

and thus the galaxy with $z = 13$ is below the equatorial hyperplane $w = 0$ and $\theta = 151°$, cf. Figure 1. This explains why some very distant galaxies and quasars seem to be so luminous. Therefore, the actual

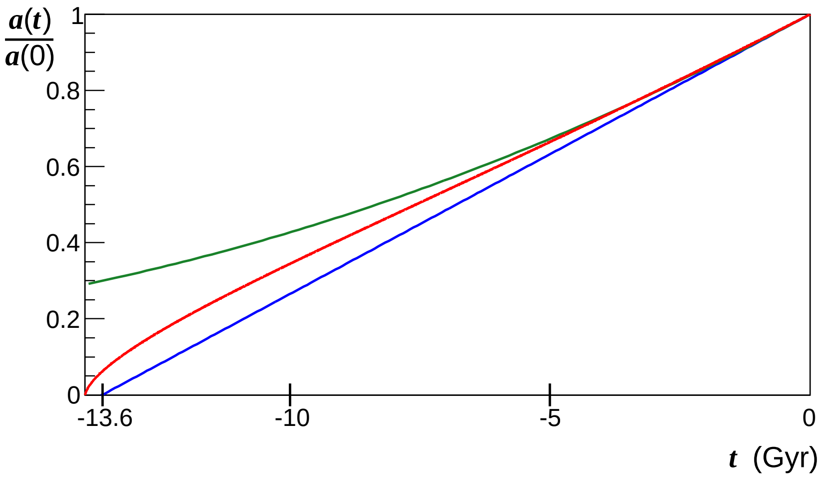

Figure 8: The middle red graph illustrates the behavior of the normalized expansion function $a(t)/a(0)$ calculated numerically from the Friedmann equation (11) for $k = 1$, $\Omega_{\rm M} \doteq 0.3$, $\Omega_\Lambda \doteq 0.7$, and the Hubble-Lemaître constant $H_0 = 70$ km/(s Mpc). The time variable is shifted for simplicity so that $t_0 = 0$ corresponds to the present time. The lower blue graph corresponds to the linear Taylor polynomial $1 + H_0 t$ on the interval $[-1/H_0, 0]$. The upper green graph shows the quadratic Taylor polynomial $1 + H_0 t - \frac{1}{2} q_0 H_0^2 t^2$ with deceleration parameter $q_0 = -0.6$.

bolometric luminosities of these objects are at least one order of magnitude smaller than the luminosities calculated from flux measurements. As of 2024 the most distant known galaxy JADES-GS-z14-0 has even $z = 14.18$ (established spectroscopically).

For $V = c$ the equatorial hyperplane of $\mathbb{S}^3_a$ with angle $\theta = \pi/2$ corresponds by Theorem 1 to the redshift

$$z = e^{\pi/2} - 1 = 3.81,$$

where V is the expansion rate of the radius $a = a(t)$. The equator associated with the comoving angle $\theta = 90°$ is represented by a 2-sphere.

Let us still note that the assumption $V = c$ in Theorem 1 is quite realistic, because by (2) and (6)

θ	z	r/a_0	relative volume
0°	0	1.000	1.00000
30°	0.69	0.592	0.20788
60°	1.85	0.351	0.04321
90°	3.81	0.208	0.00898
120°	7.12	0.123	0.00187
150°	12.71	0.073	0.00039
180°	22.14	0.043	0.00008

Table 1: Theoretical redshifts $z = \exp(\theta V/c) - 1$ corresponding to several comoving distances θ given by (10) for the expanding 3-sphere and $V = c$. In this special case, the third column shows relative radii r/a_0, where r is given by (9), and the last column shows relative volumes of the corresponding 3-spheres.

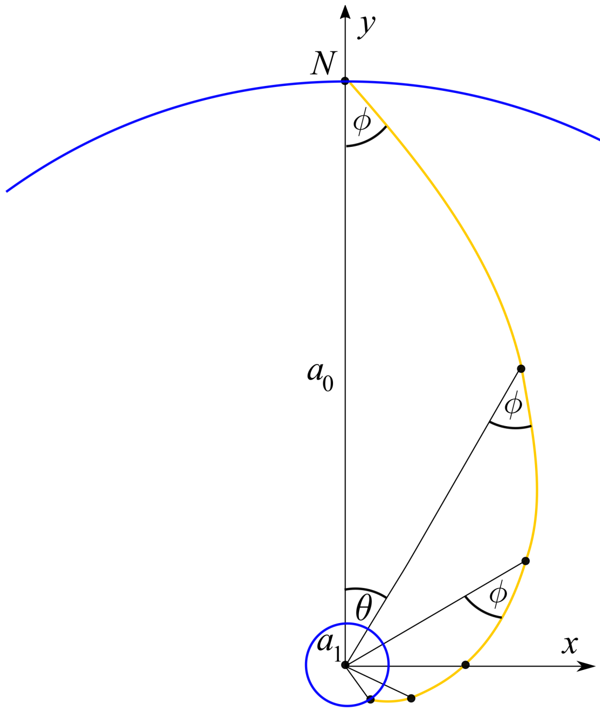

Figure 9: Yellow trajectory of a photon in an expanding universe modeled by the blue sphere $\mathbb{S}^3_{a(t)}$ is described by the logarithmic spiral (10) in the plane $z = w = 0$ with constant slope angle $\phi = \arctan(c/V)$. Redshifts corresponding to various comoving distances are presented in Table 1 for $V = c$ yielding $\phi = 45°$, i.e., the expansion rate is $\dot{a}(t) = c$, cf. also Suntola [25, pp. 97, 253].

the expansion rate satisfies the following inequality

$$V \approx \frac{a(t_0)}{t_0} > \frac{10^{26}}{13.8 \cdot 10^9 \cdot \pi \cdot 10^7} \text{ m/s} = 0.75c,$$

where 1 yr $\approx \pi \cdot 10^7$ s. If $V \neq c$ then the above relations have to be appropriately modified.

Example 6. According to [1], the total mass after coalescence of two black holes was 150 $M_\odot$ at the luminosity distance 5.3 Gpc and the redshift of the associated event GW190521 is $z \approx 0.82$ and $k = 0$. If $k = 1$, then by Theorem 1 the corresponding comoving distance is about $\theta = 0.6$ for $V = c$. In [1] nothing is mentioned about the curvature of the universe (the authors probably implicitly assume that $k = 0$). Thus the proposed mass of the resulting black hole could be reduced, cf. (4), Figure 4 and Table 1. In [13], we present five other arguments why masses of such calculated stellar black holes are largely overrated.

5 Arguments against unbounded manifolds

The hyperbolic pseudosphere $\mathbb{H}^3$ cannot be isometrically imbedded to $\mathbb{E}^4$ like $\mathbb{S}^3$ in (1). However, $\mathbb{H}^3$ can be isometrically imbedded to $\mathbb{E}^7$ (see [21]) and it is not known whether the exponent 7 can be reduced. Hence, $\mathbb{H}^3$ is quite exceptional manifold.

Moreover, the fact that an infinite universe for a fixed time instant would have at each point almost the same curvature, density, pressure, temperature, etc., on large scales is very unlikely. This would require an infinite speed of information transfer. Therefore, the unbounded manifolds $\mathbb{E}^3$ and $\mathbb{H}^3$ are not good models of our universe. An expanding unbounded universe would have an infinite energy.

The normalized Friedmann differential equation is of the form (see e.g. [14, 20, 22])

$$1 = \Omega_{\rm M} + \Omega_\Lambda + \Omega_k, \tag{11}$$

where $\Omega_{\rm M} = \frac{8}{3}\pi G\rho/H^2$, $\Omega_\Lambda = \frac{1}{3}\Lambda c^2/H^2$, ρ is the mean mass density, G is the gravitational constant, Λ is

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the cosmological constant,

$$H = H(t) = \frac{\dot{a}(t)}{a(t)}$$

is the *Hubble parameter*, and the measured value of the *density curvature parameter*

$$\Omega_k = -\frac{kc^2}{\dot{a}^2}$$

is very small, cf. inequalities (12) below. From the above definition of Ω_k it is incorrectly deduced that $k = 0$.

Example 7. For $t_2 = 380\,000$ yr (corresponding to CMB) we obtain from (5), (6), and (8) that

$$\frac{a(t_2)}{t_2} = \frac{t_0}{(z+1)t_2}\frac{a(t_0)}{t_0} = \frac{13.8 \cdot 10^9}{1090 \cdot 380\,000}\frac{a(t_0)}{t_0} = 33.3 \cdot \frac{a(t_0)}{t_0}$$

and thus the mean expansion rate on the interval $(0, t_2)$ was 33.3 times larger than that on $(0, t_0)$. Hence, according to (2), we get

$$\frac{a(t_2)}{t_2} > 33.3 \frac{10^{26}}{13.8 \cdot 10^9 \cdot \pi \cdot 10^7} \text{ m/s} \doteq 25\,c.$$

By the Mean Value Theorem of differential calculus there exists a positive $t_3 < t_2$ such that $\dot{a}(t_3) > 25\,c$. From this we find that density curvature parameter can be really very small for $k \neq 0$, namely,

$$0 < |\Omega_k(t_3)| = \Big|\frac{kc^2}{\dot{a}^2(t_3)}\Big| < \frac{1}{625}. \tag{12}$$

Therefore, the case $k = 1$ is possible for the time instant $t = t_3$ and the value of the parameter Ω_k can really be very small but positive, see also [6].

In [14, p. 167], we show that the CMB radiation (the cosmological horizon) might be just the image of the antipodal point of our neighborhood ≈ 14 Gyr ago.

Example 8. Kroupa et al. [16] investigate a very rapid emergence of supermassive black holes with high redshifts, e.g. when $z = 9.1$. By Theorem 1 the corresponding comoving distance is $\theta = 2.3 > \pi/2$ rad for $V \approx c$. Thus these early black holes are perhaps not so supermassive as assumed when we replace $k = 0$ by $k = 1$.

Example 9. Baryonic acoustic oscillations are fluctuations in the density of the visible baryonic matter caused by acoustic waves of the primordial plasma in the time period t_2. The power spectrum of CMB indicates that the angular size of the most frequent fluctuations is $\varphi \approx 1^\circ = 0.017$ rad, see the right half of the sphere in Figure 1. Let us estimate its actual size D at time t_2 for $k = 1, 0, -1$ by means of the angular distance (see [14, p. 149]). By (7),

$$D = \begin{cases} \varphi a(t_2) \sin\theta & \text{if } k = 1, \\ \varphi a(t_2)\, \theta & \text{if } k = 0, \\ \varphi a(t_2) \sinh\theta & \text{if } k = -1. \end{cases}$$

From this, (5), and Example 3 we have

$$\varphi a(t_2) = \varphi \frac{a(t_0)}{z+1} = 0.017 \frac{a(t_0)}{1090} \geq 0.157 \text{ Mly}.$$

Unfortunately, we cannot use Theorem 1 to establish more precisely the comoving distance θ of CMB, since the expansion function is not linear near $t_2 = 380\,000$ yr, see Example 7. Anyway, we shall assume that $\theta \approx 3$ rad, i.e., the origin of CMB is close to the South Pole of the expanding 3-sphere. In this case, from the above expression of the actual diameter D we find that

$$D \geq \begin{cases} 0.022 \text{ Mly} & \text{if } k = 1, \\ 0.471 \text{ Mly} & \text{if } k = 0, \\ 1.574 \text{ Mly} & \text{if } k = -1. \end{cases}$$

We see that the actual physical size of the most frequent fluctuations is dramatically different for spherical, flat, and hyperbolic universe. Since the period, when CMB had appeared, was about 10^4 yr, the most probable case is again $k = 1$. The case $k \leq 0$ contradicts the causality principle, since acoustic waves could not travel such a long distance greater than 471 kly during 10^4 years. This also enables us to exclude flat and hyperbolic geometries.

Is the global curvature of the universe positive? In Example 7, we saw that

$$\Omega_{\mathrm{M}} + \Omega_\Lambda \doteq 1 \not\Rightarrow k = 0.$$

In Sections 2 and 3, we showed how the spherical geometry magnifies the observed flux intensity leading to the *spacetime-lens principle* which could explain the large observed flux intensity of galaxies at $z \gtrsim 13$. These distant galaxies contained a lot of super-massive stars which could increase their total luminosity.

The fact that we do not observe too many quasars and galaxies with $z > 10$ also indicates that $k = 1$, cf. Figure 1. Their number is proportional to $\sin^2\theta$. For $k \leq 0$ we would observe a large amount of galaxies with $z > 10$ which is not the case. Hence, several other results should be revised from the case $k = 0$ to $k = 1$, too.

A positive curvature index $k = 1$ allows us to explain why in the early universe we observe:

1) supermassive stars,
2) too large stellar black holes,
3) supermassive black holes,
4) superluminal velocities of jets produced by distant quasars,
5) too large early galaxies [17, 18, 28],
6) too large galaxy superclusters, see [2],
7) large size of the most frequent fluctuations in CMB,
8) super energetic quasars,
9) giant γ-ray bursts,
10) very luminous supernovae,...

All these phenomena could be mere apparent optical effects due to the spacetime-lens principle for the expanding 3-sphere.

6 Local Hubble expansion of the Solar system

Most cosmologists believe that the universe expands globally, but not locally, see e.g. [20, p. 719]. However, this immediately leads to a mathematical contradiction, see [14, p. 183]. In this and the following section, we present several arguments for local overall expansion of the universe. Most of them are due to the finite speed of gravitational interaction which produces apparent tiny repulsive forces that act permanently [12].

Now we will briefly show that our Solar system expands at a similar rate to the Hubble-Lemaître constant $H_0 = H(t_0) \approx 70$ km/(s Mpc) ≈ 10 m/(yr au). Such a substantial expansion cannot be explained by the reduction of the Solar mass (as a result of nuclear reactions, ejection of plasma jets, and solar wind), tidal forces, magnetic fields, and so on, see [12, p. 203].

Relative luminosity of the Sun increases approximately linearly, see Figure 10. Thus, if the expansion rate of the Earth's orbital radius were approximately H_0 over the last 3.5 Gyr, then the Earth would receive an almost constant energy flux from the Sun, see [12, 14] for the proof. Therefore, the Earth-Sun distance could only be $130 \cdot 10^6$ km approximately 3.5 Gyr ago.

To ensure favorable conditions for life on Earth it is necessary at present that the Sun's luminosity be at most 5 % larger or smaller than the *solar constant*

$$L_0 = 1.361 \text{ kW/m}^2.$$

Such a ring (resp. spherical layer) is called an *ecosphere*. Since the relative luminosity decreases with the square of the distance, the present radii of the ecosphere are $\sqrt{0.95}$ au and $\sqrt{1.05}$ au, which corresponds to a very narrow interval of 145.8–153.3 million km.

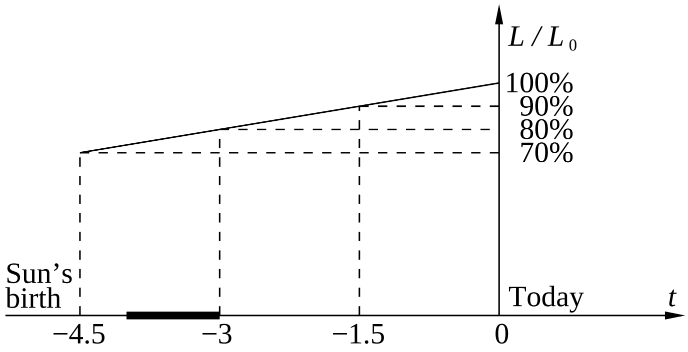

Figure 10: Relative luminosity L/L_0 of the Sun from the origin of the Solar system up to the present increases approximately linearly. The bold interval on the time axis indicates the origin of life on Earth in Gyr and also the existence of liquid water on Mars.

Higher concentration of CO_2 contributed to a higher surface temperature on Mars, but cannot fully explain liquid water there 3.75 Gyr ago because of the huge 67 %-decrease of the luminosity:

$$L_{\mathrm{Mars}} = 0.75 L_0 \Big(\frac{150}{225}\Big)^2 = \frac{L_0}{3},$$

where the coefficient 0.75 is due to Figure 10 and the number in parentheses is the ratio between the semi-major axes of orbits of Earth and Mars. Thus, Mars must have been much closer to the Sun to have liquid water on its surface 3–4 Gyr ago.

The local Hubble expansion of the Solar system can explain many paradoxes, e.g.:

1) the faint young Sun paradox, see Suntola [25],
2) the large recession speed 3.84 cm/yr of the Moon from the Earth, see Styrman [24],
3) the paradox of the large orbital angular momenta of the Moon, Triton, and Charon,
4) the slow rotation of Mercury and the absence of its moons,
5) the existence of dry riverbeds on Mars, see [12],
6) the paradox of the large recession speed 11 cm/yr of Titan from Saturn, see [11],
7) the tidal locking of the distant moon Iapetus of Saturn,
8) the formation of Uranus and Neptune,
9) the long existence of rapidly-moving satellites below the stationary orbit of Uranus and Neptune,
10) the long existence of life on Earth, see [14].

7 Expansion of galaxies

There is no reason to assume that tiny repulsive forces would somehow be not present in the interior of galaxies, since its manifestations are observed not only at large cosmological distances, but also locally inside the Solar (see Section 6). In spite of that it is generally claimed that galaxies do not expand, because they are gravitationally bound and that only the space between them expands. Galaxies are usually included in clusters that should also not expand, because they are gravitationally bound as well. Galaxy clusters are gravitationally bound again in superclusters. So where does the universe expand?

According to [19], the observed expansion rate of the Milky Way is approximately 0.6–1 km/s. This expansion rate nicely fits to the Hubble-Lemaître constant recalculated on the radius $R = 50\,000$ ly of our Galaxy, namely (cf. Figure 11)

$$H_0 \approx 1 \text{ km/(s}\,R).$$

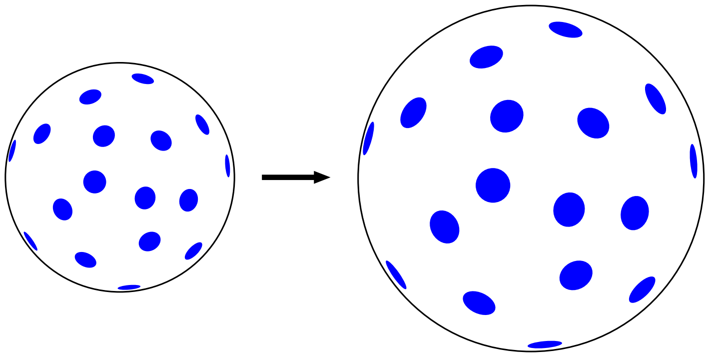

Figure 11: An expanding universe $\mathbb{S}^2_{a(t)}$ with comoving and swelling galaxies.

According to [10], superdense galaxies were quite common in the early universe with redshift $z > 1.5$. At present they are quite sparse.

Another reason for the expansion of galaxies is given in Figure 12. It shows what we would observe if galaxies were not growing, see also [12].

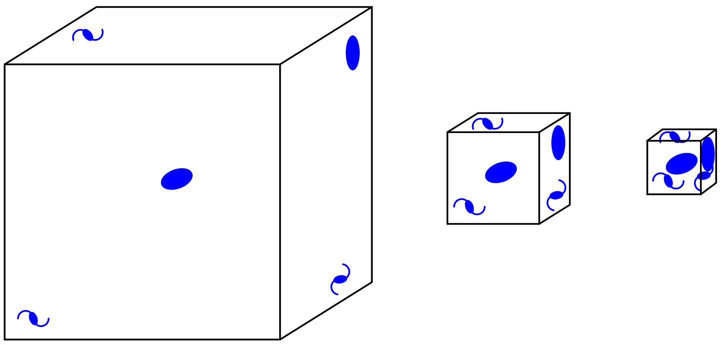

Figure 12: A schematic illustration of nongrowing galaxies of constant size over time in an expanding universe. The unit cube is on the left. It contains several galaxies in our neighborhood for the redshift $z = 0$. The distribution of galaxies at cosmological distance $z = 2$ is in the middle and for $z = 4$ on the right. Such a picture of tightly crowded galaxies is not observed by astronomers.

Note also that according to Suntola [26, p. 74], galaxies in DU expand, while by the standard cosmological ΛCDM model they do not expand. It is also estimated that the mean diameter of early galaxies was about 1 kpc, while at present it is at least one order of magnitude larger.

8 Final remarks

The current standard cosmology ΛCDM model is based on the normalized Friedmann equation (11). However, this equation was derived under excessive extrapolations from Einstein's field equations that were applied to the entire universe. Note that nonlinear Einstein's equations are not scale invariant and are "verified" on much smaller scales of orders of astronomical units. However, the entire universe is by (2) at least 15 orders of magnitude larger than 1 au $= 1.5 \cdot 10^{11}$ m. In [14], we explain why these extrapolations are incorrect, why the unrestricted use of the term "verified" is questionable, and why dark matter may exist only by definition.

Cosmologists often claim that our universe has no center. Nevertheless, in Figure 6 we observe that the blue manifold has its center on the time axis t even though this center does not belong to $\mathbb{S}^1_a$. (The circle $x^2 + y^2 = 1$ also has a center which does not belong to it.) The observable universe (the yellow manifold in Figure 6) is centered on Earth and the center of spacetime (represented by the red manifold) corresponds to the Big Bang at the origin of the spacetime coordinates, see also Figure 7. One way to imagine $\mathbb{S}^3$ is to tile it with curved congruent regular polyhedral cells [3].

Einstein formulated his special theory of relativity (STR) under the assumption that the laws of nature have the same mathematical expression in all inertial systems. This is the so-called *principle of relativity*. Therefore, in Section 2 we presented Remark 1 showing that the principle of relativity does not hold in 3-sphere which is locally flat. Thus, there is also no time dilation, cf. [15] for accelerated systems. Einstein himself did not perform any physical experiments with STR, but restricted himself only to theoretical speculations which do not describe reality well for large relativistic velocities. For example, a fast-moving traveler in any direction will see the spectrum of galaxies on one half of the celestial sphere shifted to blue colors and on the opposite half to red colors, i.e., the relativity principle does not apply in the physical universe for any relativistic velocity $v \gtrsim 0.05c$. Another reference system can be connected with the ubiquitous CMB relative to which the Sun moves at a non-relativistic velocity of 370 km/s. It is estimated that each cubic meter contains on average 411 000 000 CMB photons. A 3rd such system may be assigned to intergalactic gas (or dust) between galaxies. Thus, there exist a kind of preferred "frame of the Universe" which cannot be deleted, excluded, or ignored under any circumstances. Therefore, inertial systems are not equivalent, i.e., indistinguishable.

Of course, Einstein could not have known in 1905 about the existence of other galaxies or CMB. However, he could have guessed that a fast-moving traveler in the Milky Way would see the spectrum of stars in one half of the celestial sphere shifted to blue colors and in the other half to red colors, since the Doppler effect was well known in 1905. As far as we know, there are no papers criticizing the theory of numbers, theory of graphs, theory of groups, theory of probability, theory of matrices, etc. On the other hand, there exist dozens of works continually criticizing the special theory of relativity. Since the principle of relativity and time dilation are fundamental pillars of General Relativity and thus also of the ΛCDM model, it is highly desirable that some important statements should be improved.

Acknowledgments. The author is indebted to Jan Brandts and Lawrence Somer for useful suggestions. Supported by the Czech Academy of Sciences (RVO 67985840).

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