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Twin paradox in accelerated systems and the Equivalence Principle

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Abstract. We present arguments supported by several examples showing that the relativistic time dilation observed in frames affected by gravitational and non-gravitational forces are different being thus inconsistent with the Equivalence Principle. This inconsistency questions validity of the standard interpretations of the Equivalence Principle or of the twin paradox in accelerated frames.

1 Introduction

At the beginning of the 20th century, Albert Einstein formulated his famous General Principle of Relativity (see [1, p. 71–72]): "All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics". This is the basic principle of General Theory of Relativity (GTR) and is closely related to the Equivalence Principle [2], [3, p. 220], which is at present usually formulated as follows: "Inertial mass and gravitational mass are the same thing" or "there is no way of distinguishing between the effects of a uniform gravitational field and of constant acceleration on an observer". Roughly speaking, Einstein [2] claimed that an observer in a closed box is not able to distinguish whether he stays on the Earth or whether he is in a rocket flying with a constant acceleration a equal to the Earth's gravitational acceleration g,

$$a = g = 9.81 \text{ m/s}^2.$$

In other words, no physical experiment can distinguish between these two cases, because they are fully equivalent. According to this, Einstein [2] deduces that time dilation predicted for accelerated frames treated in the Special Theory of Relativity (STR) must also be present in gravitational fields; hence, gravity must also affect clocks. Consequently, Einstein came to the conclusion that light propagating out of a gravitational potential must experience a redshift.

Since these principles form fundamentals of Einstein's theory of relativity, it is highly desirable to probe, whether they do not violate some other physical principles and/or whether they can be proved experimentally. Various forms of the Equivalence Principle (EP) have been tested in several experiments [4]–[6]. The Weak Equivalence Principle (WEP), stating that all the laws of motion for freely falling systems are the same as in an unaccelerated reference frame, was tested at 5×10^{-9} level by Eötvös and collaborators [7]. Later, it has been extensively tested to a precision of 2×10^{-13}, see [8]–[11], and recently to a precision of 10^{-15} by Touboul et al. [12], using the MICROSCOPE mission. The EP has also been tested by the Pound-Rebka-Snider gravitational redshift experiments that measured the frequency shift of gamma-ray photons from ^{57}Fe due to the changes in the gravitational potential at two different heights [13, 14]. Other experiments measured the redshift due to the gravitational field of Sun and the change in rate of atomic clocks transported on aircrafts, rockets and satellites (for review see [15] and also [16, pp. 40–46]).

In this paper, we focus on another aspect of the General Principle of Relativity and the Equivalence Principle. We know that EP predicts time dilation for both accelerated relativistic systems and systems in gravitational fields. However, we try to probe, whether these effects are really equivalent also quantitatively as predicted by EP. We analyze the twin paradox (called also the clock paradox) in systems with acceleration, when relativistic time dilation in one frame is affected by non-gravitational forces, and time dilation in another frame is influenced by gravity. We show that both systems behave in a different way, the time dilations being distinct. We present numerical examples comparing both types of time dilation and discuss their discrepancy.

2 The twin paradox in accelerated systems

For simplicity, assume that the Earth with time variable $\tilde{t}$ is completely isolated from other gravitational sources like a free-floating non-rotating lonely planet. First, we recall that the gravitational field of the Earth is weak and a clock on the Earth ticks with a very similar speed as the same clock in the zero gravity field with time variable t. According to GTR, their difference after 1 year will be only (see e.g. [17, p. 35], [18, p. 659]),

$$\delta = (1 - \sqrt{1 - r/R})T = 6.5 \times 10^{-10}T \approx 0.02 \text{ s}, \tag{1}$$

where $R = 6373$ km is the mean radius of the Earth, $r = 8.87$ mm is its Schwarzschild radius, and

$$T = 31\,558\,149.45 \text{ s} \tag{2}$$

is the sidereal year. Hence, the difference between the increase of times t and $\tilde{t}$ after one year can be treated as negligible in our further analysis, since the clock on Earth will be delayed by approximately 1 second after 50 years due to Eq. (1). Hence, the time dilation due to the Earth's gravitational field will not play any significant role in the investigated clock paradox.

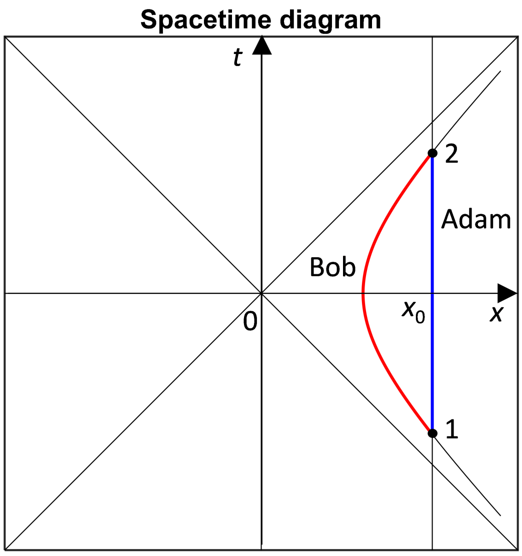

Figure 1: The spacetime diagram for $c = 1$ (ly/yr). The world line of Adam is vertical, whereas Bob's word line is a hyperbola defined by Eq. (6).

Next, we shall deal with the following time-dependent problem, which is spatially only one-dimensional. Consider two spacetime travelers Adam and Bob in space with no gravitational field. They do not have to be twins or brothers. They can even be replaced by accurate clocks equipped with transmitters and receivers. Adam stays at an inertial reference frame fixed with Earth. As derived above, its time rate is practically the same as when being directly on Earth and feeling a constant acceleration $a = g$. Further assume that Bob flies from a distant universe in a rocket in a negative direction (←) along the x-axis. Throughout the flight, he will be subjected to the same positive constant acceleration $a = g$ as Adam feeling on the Earth's surface. At the moment of the first encounter of Bob with Adam (see the event 1 in Figure 1), they will exchange time data on their clocks. After this encounter, Bob first slows down to a complete stop and then accelerates in the opposite direction (→) to the Earth, where he again will exchange his time data with Adam (see the event 2 in Figure 1).

Further, we assume without loss of generality that the spatial x-coordinate of Adam is a fixed constant

$$x_0 > \frac{c^2}{a}$$

(which comes from Eq. (4) below), where $c = 299\,792\,458$ m/s is the speed of light in a vacuum. This inequality will guarantee that the world lines of Adam and Bob intersect twice. Since the acceleration of Bob is the same as if Adam were on the Earth's surface, $a = g$, we expect that both frames – experiencing the gravitational acceleration and the acceleration in a rocket – are physically equivalent due to EP, and all physical laws should hold in the same way. Consequently, we deduce that time for Adam and Bob should also run at the same rate. It is easy to check whether this property is true or not.

Next, we describe the world lines of Adam and Bob in detail. As regards Bob, we use the STR approach for accelerated observers according to [18, §6.1]. Denote by τ the proper time of Bob and recall that $\tilde{t}$ and t stand for Adam's proper coordinate time on Earth and in zero gravity, respectively. Then by Rindler [20], these time variables can be suitably shifted so that

$$t = \tilde{t} = \tau = 0$$

on the x-axis, see Figure 1 and also [18, p. 166], i.e. in the middle between the events 1 and 2. Here Bob's velocity is zero for $\tau = 0$ and his distance from Adam is maximal. Then Bob's world line in Adam's coordinate system satisfies the so-called *Rindler coordinates:*

$$t = t(\tau) = \frac{c}{a} \sinh \frac{a\tau}{c}, \tag{3}$$

$$x = x(\tau) = \frac{c^2}{a} \cosh \frac{a\tau}{c}. \tag{4}$$

From Eq. (3) we can calculate by how many seconds is Bob's clock late to Adam's clock. In particular, Adam would be

$$\Delta = 2(t - \tau) \tag{5}$$

older than Bob.

Multiplying (3) by c and then squaring and subtracting equations (3) and (4), we find that

$$x^2 - c^2t^2 = \frac{c^4}{a^2}. \tag{6}$$

Hence, Bob's world line is a hyperbola in the coordinate system (x, t) in which Adam has a fixed coordinate x_0, see Figure 1.

Since $\Delta > 0$, we proved that due to the clock paradox and time dilation, Bob's time runs slower than Adam's clock, cf. also Examples 1–6 below. Comparing their proper times, they can easily distinguish, who was flying and who stayed on the Earth. Hence, the frames connected to Adam and Bob are not fully physically equivalent, because time runs at different rate in their frames. The main reason is following:

First recall that the shortest distance between two distinct points is given in Euclidean space by the straight line segment joining them. However, a very counter-intuitive property holds in the Minkowski spacetime. Assume that the first event lies at the vertex of the corresponding future light cone and the second event lies inside this cone. Then the *straight line segment between these two events is, in contrast, the longest.* It is called the *Lorentzian distance* and is measured in seconds. Moreover, any other continuous causally admissible world line between these two events is shorter, cf. Figure 1. Its slope in absolute values is greater than 1 everywhere for $c = 1$ ly/yr.

3 Examples of weak accelerations

Further, we demonstrate numerically that EP does not hold on various time scales, assuming Adam and Bob feeling the same acceleration $a = g$ in their frames. In particular, we get an invalid relation $\tau < t = \tau$, where the first inequality is due to time dilation and the second equality is due to EP.

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Example 1. First, let us consider the upper part of Bob's world line lying in the first quadrant of the system (x, t) for $a = g$, see Figure 1. For the proper half-flight time of Bob $\tau = 86\,400$ s (i.e. one day), we get by Eq. (3) that $t = 86\,400.115$ s and by Eq. (4) that $x_0 = x(\tau)$. For the lower part in the fourth quadrant, we get the same values due to symmetry. Thus, by Eq. (5) we find that Adam would be

$$\Delta = 2(t - \tau) = 0.23 \text{ s}$$

older than Bob, which is a perfectly measurable value, see Table 1. The velocity of Bob during the first encounter is

$$v(-\tau) = -c \tanh \frac{a\tau}{c} = -8.45 \times 10^5 \text{ m/s}$$

and during the second encounter $v(\tau) = -v(-\tau)$. Moreover, $x(\tau) - x(0) = 3.66 \times 10^{10}$ m.

Example 2. Again, we assume $a = g$. According to (4), we have $x(0) = c^2/a = 9.16 \times 10^{15}$ m which is about 1 ly. Assume that the proper half-flight time of Bob is $\tau = 1$ year, see (2). The argument of hyperbolic functions in (3)–(4) corresponding to the second event equals $a\tau/c = 1.032$. Then from (4) we get the fixed position $x_0 = x(\tau) = 14.49 \times 10^{15}$ m of Adam and from (3) the corresponding proper time of Adam

$$t = 3.747 \times 10^7 \text{ s} = 1.187 \text{ yr}. \tag{7}$$

On the other hand, from EP we get

$$\tilde{t} = \tau = 1 \text{ yr}\,, \tag{8}$$

which differs from (7) by more than 2 months, since the difference $t - \tilde{t} = 0.02 \text{ s} \approx 6 \times 10^{-10}$ yr is clearly negligible value by (1), i.e.,

$$\tau \ll t \doteq \tilde{t} = \tau$$

or more precisely using (7)–(8),

$$0.187 = |t - \tau| \le |t - \tilde{t}| + |\tilde{t} - \tau| = |t - \tilde{t}| = 6 \times 10^{-10}$$

in years, which is a contradiction. Note also that the velocity of Bob during the first and second encounter was $\mp c \tanh(a\tau/c) = 0.775c$, i.e., Bob felt the Earth's gravitational field for just a second.

So where is the mistake? If the total flight time 2τ of the experiment increases linearly for Bob, then according to (3), the total time $2t$ of Adam increases almost exponentially. This is the main reason of the controversy (7)–(8).

Consequently, the systems connected to Adam and Bob are not physically equivalent, because time runs at different rate in their frames. Analogously we find that Adam will always be older than Bob also for infinitely many other constant accelerations $a > 0$ and infinitely many other proper flight times $2\tau > 0$ of Bob. For instance, taking $a = 20.69$ m/s^2 and again $\tau = 1$ yr, we obtain by (3) that $t = 2$ yr and by the Equivalence Principle (8) that

$$2 = t = \tau = 1$$

in years, i.e. from this we may get "$1 + 1 = 3$". The value of δ in (1) changes again only negligibly.

Example 3. If Bob would fly 10 + 10 years of his proper time with $a = g$, Adam would be by equations (3) and (5) almost 30 000 years old (see Table 1), while Bob will be only 20 years older after their second encounter. We see that relative precision $(t - \tau)/\tau$ can indeed be very large.

Table 1. Time dilation discrepancy, where Δ is the total time difference defined by Eq. (5) for various values of the non-gravitational acceleration a and the proper half-flight time of Bob is denoted by τ. The last column shows the relative precision.

Example	a	τ	Δ	$(t - \tau)/\tau$
1	g	1 day	0.23 s	1.3×10^{-6}
2	g	1 yr	0.374 yr	0.187
3	g	10 yr	29 540 yr	1477
4	$10g$	1 hour	1.67 ms	2.3×10^{-7}
5	$10g$	1 day	23 s	1.3×10^{-4}
6	$10g$	10 days	2311 s	0.013

If we consider Adam staying at a massive planet with the gravitational acceleration on the surface $10g$ instead of g and Bob flying in a rocket with the same acceleration $a = 10g$, the discrepancy between the age of Adam and Bob is even more evident than in Examples 1–3, even though this is still only a weak acceleration. This means that the identification of the two frames can be realized in a shorter time.

Example 4. If $a = 10g$, then the corresponding value δ in (1) is still negligible. For one hour flight there and one hour back, i.e., with half-flight time $\tau = 3600$ s the corresponding Δ equals 1.67 ms that is still a perfectly measurable value. According to (4), the maximum distance of Bob from Earth would be

$$x(\tau) - x(0) = x_0 \frac{c^2}{a} = 6.355 \times 10^8 \text{ m}.$$

Examples 5 and 6. For $\tau = 1$ day, we obtain 100 times larger value $\Delta = 23$ s than in Example 1, provided $a = 10g$. For $\tau = 10$ days, we can derive even 10 000 times larger value $\Delta = 2300$ s than in Example 1 that is a consequence of the Taylor expansion of function sinh in Eq. (3). The relative precision raises similarly, see Table 1.

So, the flight time can be considerably shortened, provided the acceleration in the Adam's and Bob's frames is several times higher than g. The violation of EP is thus better visible. Still, we can consider the gravitational field characterized by such acceleration as weak. We intentionally exclude the case of strong gravitational fields, for which a more restrictive Strong Equivalence Principle should be applied.

The above examples indicate that Adam and Bob are able to distinguish between these two time periods without leaving their closed boxes and they can decide whether they stay in a gravitational field or in a uniformly accelerating rocket. This clearly violates either the General Principle of Relativity (and) or the Equivalence Principle.

4 Conclusions

The Einstein's basic postulate of General Relativity and the Equivalence Principle claim that physical laws in accelerating systems should be equivalent regardless the acceleration to be caused by gravitational or other forces. These postulates unify our view on physical systems with and without gravity and have fundamental consequences for physical measurements and their interpretations. However, a detailed inspection of the behavior of non-inertial systems shows that the equivalence is not valid, in general.

We compared relativistic time dilation in frames affected by gravitational forces and by other forces and found that they might significantly differ. Moreover, a contradiction similar to (7)–(8) can be made on an arbitrarily short time interval with positive length. This seriously questions the commonly accepted concept of the equivalence of the gravitational and non-gravitational effects.

These findings suggest that the Special Theory of Relativity and General Theory of Relativity may lack full internal consistency. We propose that either the Equivalence Principle may not hold universally, or the twin paradox in STR requires a different interpretation, or both concepts may provide incomplete representations of physical reality. Our critical perspective is further supported by the extensive body of literature that scrutinizes STR for more than 100 years (see e.g. [17, 21, 22, 23, 24, 25, 26, 27, 28]), often challenging its alignment with physical reality due to the ambiguity of the selection of the rest frame and destroying an understandable concept of time. A model that leads to contradictions cannot fully capture the complexities of the universe. Given that both the Principle of Relativity and the Equivalence Principle are foundational to GTR, it is essential to reassess certain core assumptions to ensure a more accurate description of fundamental physical phenomena.

Finally note that the Andromeda galaxy M31 is 2 537 000 light years away. However, according to STR, an astronaut traveling from the Earth with constant $a = g$ would need only $\tau = 15$ yr to reach a distance

$$x(\tau) = \frac{c^2}{g}\Big(\cosh\frac{g\tau}{c} - 1\Big) = 2\,583\,607 \text{ ly}. \tag{9}$$

For $\tau = 25$ yr we would even get by (9) that $x(\tau) \approx 8 \times 10^{26}$ m, which is roughly the circumference of Einstein's static universe with radius $1/\sqrt{\Lambda} \approx 10^{26}$ m, where Λ is the cosmological constant. This is very counter intuitive.

Hence, *if the phenomenon of time dilation* (3) *does not exist in the physical universe, then our optimistic outlook for travel to distant exoplanets or even galaxies will be significantly limited.*

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