# Which effects of galaxy clusters can reduce the amount of dark matter

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**Abstract.** In 1933, Zwicky observed velocities of galaxies in the Coma cluster which were too high. By means of the Virial Theorem considered in Euclidean space he proposed the existence of dark matter to keep the cluster stable. In this paper we collect several phenomena and facts not taken into account by Zwicky that can partly explain the high calculated mass of the cluster, for instance, dark energy, finite speed of gravitational interaction, gravitational aberration, curved spacetime, the selflensing effect, relativistic effects, decreasing Hubble parameter, etc. Thus, the calculated amount of dark matter in the Coma cluster may be substantially reduced. This also applies for dark matter inside galaxies.
**Key words:** dark matter, gravitational lensing, Virial Theorem

## Introduction

Eighty years ago, Fritz Zwicky published the groundbreaking paper (Zwicky, 1933) that essentially changed the development of astronomy and cosmology for many decades. Using the Virial Theorem, he found that in order to explain fast movements of galaxies in the giant galaxy cluster Abell 1656 in Coma Berenices, he had to assume the existence of a 400 times larger amount of nonluminous matter than luminous to keep the cluster together. Sinclair Smith (Smith, 1936, p. 27) independently reached a similar conclusion for the Virgo cluster. Later the huge factor 400 was reduced to 10. The term *dark matter* (in German *dunkle Materie*) was used by Zwicky in his paper (Zwicky, 1933) on page 125.

In (Zwicky, 1937) a new approach of gravitational lensing by means of an intervening galaxy is presented. Zwicky found that the probability of an alignment of two galaxies is much higher than for the alignment of two stars. He also introduced a gravitational lens formed by a cluster of galaxies, see (Zwicky, 1937, p. 238).

In this paper, we present several facts and phenomena which should be taken into account to estimate the amount of dark matter, and which have not been treated by other authors since Zwicky's introduction of the concept of dark matter. In Section 2 we briefly recall the Virial Theorem which establishes the equilibrium of the total kinetic and potential energy of gravitationally interacting bodies of a stable system. In Section 3 we present Zwicky's method and comment on it. In Section 4 we point out several effects that were not taken into account by Zwicky and that may reduce the calculated mass of the whole cluster A1656 (called *Coma cluster*). We also survey various methods that have been applied to determine the total mass of the Coma cluster by other authors. In Section 5 we introduce our own analysis based on existing data. We examine the influence of dark energy, the selflensing effect, relativistic effects, gravitational redshift, decreasing Hubble parameter, etc., showing that the amount of dark matter is overestimated. We also present a simple example illustrating whether it is necessary to assume extragalactic dark matter in the Coma cluster center. Finally, in Section 6 we give some concluding remarks on the existence of dark matter and present another example reducing the amount of dark matter in the Milky Way and M31.

## 1. The Virial Theorem

Consider $N$ mass points with masses $m_1, \dots, m_N$ which interact only gravitationally. Denote their positions by $r_1, \dots, r_N$, i.e., for each time instant $t$ the vector $r_i(t) \in \mathbb{R}^3$ is a point of the trajectory of the $i$th point. Then the kinetic and potential energy of this system are given by

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \dot{r}_i \cdot \dot{r}_i, \quad V = -\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{G m_i m_j}{|r_j - r_i|}, \tag{1}$$

where $\dot{r}_i = \mathrm{d}r_i/\mathrm{d}t$ stands for the time derivative, $\cdot$ is the scalar product in $\mathbb{R}^3$, $|\cdot|$ is the norm in $\mathbb{R}^3$ and $G = 6.674 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$ is the gravitational constant. For simplicity, we do not indicate the dependence of $T$, $V$, $r_i$, ... on time $t$.

Applying Newton's second law $F_i = m_i \ddot{r}_i$ and the gravitational law, we obtain

$$\ddot{r}_i = \sum_{j \neq i}^{N} \frac{G m_j (r_j - r_i)}{|r_j - r_i|^3}. \tag{2}$$

From this and (1) we have

$$V = -\frac{1}{2} \sum_{i=1}^{N} m_i \sum_{j \neq i}^{N} \frac{G m_j (r_j - r_i) \cdot (r_j - r_i)}{|r_j - r_i|^3} =$$

$$= \frac{1}{2} \sum_{i=1}^{N} m_i \sum_{j \neq i}^{N} \frac{G m_j (r_j - r_i) \cdot r_i}{|r_j - r_i|^3} + \frac{1}{2} \sum_{i=1}^{N} m_i \sum_{j \neq i}^{N} \frac{G m_j (r_i - r_j) \cdot r_j}{|r_i - r_j|^3} =$$

$$= \frac{1}{2} \sum_{i=1}^{N} m_i \ddot{r}_i \cdot r_i + \frac{1}{2} \sum_{j=1}^{N} m_j \ddot{r}_j \cdot r_j = \sum_{i=1}^{N} F_i \cdot r_i. \tag{3}$$

Denoting the polar moment of inertia by $I = \sum_i m_i r_i \cdot r_i$ (see Zwicky, 1937, p. 228), from (1) and (3) it follows that

$$\ddot{I} = 2 \sum_{i=1}^{N} m_i (\dot{r}_i \cdot \dot{r}_i + \ddot{r}_i \cdot r_i) = 4T + 2V. \tag{4}$$

If $\ddot{I} = 0$ then

$$V = -2T \tag{5}$$

for stabilized systems. The term *virial* is the original denotation for the potential energy $V = \sum_i F_i \cdot r_i$ (see (3)). In 1870 Rudolf Clausius derived the **Virial Theorem** (cf. (1), (3) and (5)) for $E = T + V < 0$ in the form

$$\Big\langle \sum_{i=1}^{N} m_i v_i^2 \Big\rangle + \Big\langle \sum_{i=1}^{N} F_i \cdot r_i \Big\rangle = 0, \tag{6}$$

where angular parentheses denote mean values over a very long time interval and $v_i = |\dot{r}_i|$.

## 2. Zwicky's application of the Virial Theorem to the Coma cluster

Zwicky examined curious redshifts of galaxies discovered by Slipher, Hubble, Humason, etc. He was wondering why redshifts of individual galaxies from the Coma cluster (see Fig.1) have such a large dispersion from the average redshift of the whole cluster. The cluster moves away from us due to the expansion given by the present value of the Hubble constant (Planck, 2013)

$$H_0 \approx 68 \text{ km}\,\text{s}^{-1}\text{Mpc}^{-1}. \tag{7}$$

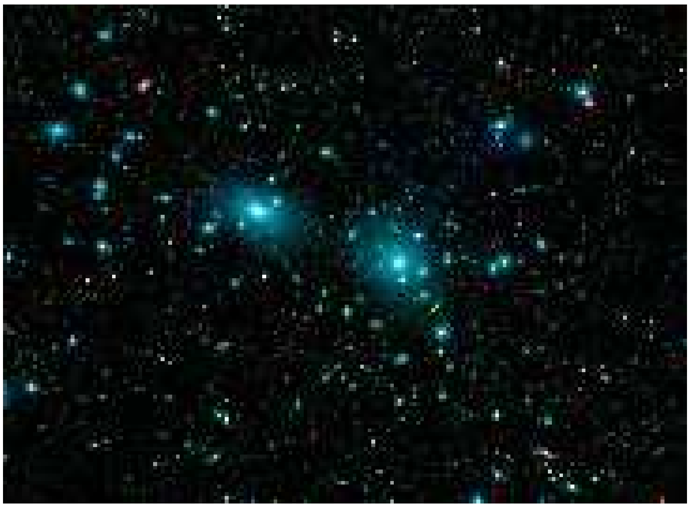

**Fig. 1.** Giant galaxy cluster Abell 1656 in the constellation Coma Berenices. In the middle there are two supergiant elliptic galaxies NGC 4889 and NGC 4874 (photo NASA).

In 1933 Zwicky found that some galaxies orbit the center much faster than would follow from the Virial Theorem. He assumed $N = 800$ galaxies in the cluster and estimated that each galaxy has on average $10^9$ stars. In this way he got the following estimate of the total mass (see Zwicky, 1933, p. 124),

$$\mathcal{M} = 800 \times 10^9 \times M_\odot = 1.6 \times 10^{42}\,\text{kg}, \tag{8}$$

where $M_\odot = 2 \times 10^{30}$ kg is the mass of Sun. However, from the Virial Theorem he obtained a 400 times higher mass $\mathcal{M}$ of the cluster, see (Zwicky, 1933, p. 125). Four years later he published a more detailed analysis (Zwicky, 1937), where a similar factor is reduced to 150 (cf. (8) and (19)). To explain this paradox he assumed that there exists a large amount of some invisible dark matter that has gravitational influence.

Let us take a closer look at Zwicky's method for establishing the total mass of galaxy clusters by means of the Virial Theorem. We try to follow his original notation from the papers (Zwicky, 1933) and (Zwicky, 1937). Denote the total mass of the investigated galaxy cluster by

$$M = \sum_{i=1}^{N} m_i, \tag{9}$$

where $m_i$ is the mass of the $i$th galaxy, and let $\overline{v}_i$ be the time independent mean heliocentric speed of the $i$th galaxy. The Coma cluster is very close to the North Galactic pole. Therefore, $\overline{v}_i$ is equal to the recession speed of the $i$th galaxy from our Galaxy, although the speed $v_\odot = 230$ km/s of the Sun about the Galactic center is relatively high. Then the center of gravity of the cluster recedes with the mean speed

$$\overline{v} = \frac{1}{M} \sum_{i=1}^{N} m_i \overline{v}_i. \tag{10}$$

Zwicky then approximated the total kinetic energy of galaxies with respect to the center of the cluster as follows:

$$\overline{T} = \frac{1}{2} M \overline{\overline{v}}^2 = \frac{1}{2} \sum_{i=1}^{N} m_i \overline{\overline{v}}^2 := \frac{1}{2} \sum_{i=1}^{N} m_i (\overline{v} - \overline{v}_i)^2, \tag{11}$$

where the root-mean-square speed $\overline{\overline{v}}$ of all galaxies with respect to the center of mass of the cluster is defined by the last equality in (11).

To estimate the potential energy of the cluster, Zwicky assumed that galaxies are uniformly distributed in a sphere with radius $R$. The corresponding constant density is

$$\rho = \frac{3M}{4\pi R^3}.$$

The force acting on the galaxy with mass $m_i$ whose position is given by the radius-vector $r_i$ can thus be approximated by

$$F_i \approx -\frac{4\pi |r_i|^3 \rho\, m_i r_i}{3 |r_i|^3} = -\frac{G M m_i r_i}{R^3} \tag{12}$$

taking into account that

$$M - m_i \approx M. \tag{13}$$

The corresponding potential energy of the $i$th galaxy is then

$$V_i = F_i \cdot r_i \approx -\frac{GMm_i|r_i|^2}{R^3}. \tag{14}$$

Then Zwicky estimated the root-mean-square distance $\overline{\overline{r}}$ from the cluster center for a typical galaxy (see Zwicky, 1937, p. 230) as

$$\overline{\overline{r}}^2 = \frac{1}{M}\sum_{i=1}^{N} m_i|r_i|^2 \approx \frac{3}{4\pi R^3\rho}\int_0^R r^2 \times 4\pi r^2 \rho \,\mathrm{d}r = \frac{3}{5}R^2, \tag{15}$$

where the density $\rho$ of the cluster is assumed to be constant. By (3), (14), (9), and (15) we have

$$V = \sum_{i=1}^{N} F_i \cdot r_i \approx -\frac{GM}{R^3}\sum_{i=1}^{N} m_i|r_i|^2 \approx -\frac{GM\overline{\overline{r}}^2}{R^3}\sum_{i=1}^{N} m_i = -\frac{3GM^2}{5R}.$$

From this, the Virial Theorem (5), and estimate (11), we get the resulting relation

$$M = \frac{5R\overline{\overline{v}}^2}{3G} \tag{16}$$

for the total mass (Zwicky, 1933, p. 124). We shall call it the *virial mass*.

Zwicky used the following data to establish the radius $R$. He assumed that the Coma cluster in the celestial sphere subtends the angle $\beta = 1.7°$. At that time Hubble with Humason estimated the distance to the cluster to be 13.8 Mpc. Since 1 pc $= 3.086 \times 10^{16}$ m, it follows

$$R = 13.8 \times 10^6 \times 3.086 \times 10^{16} \times \sin\frac{1}{2}\beta = 6.318 \times 10^{21} \text{ m}, \tag{17}$$

i.e. $R \approx 0.2$ Mpc.

Radial components of velocities of individual galaxies can be well-determined from the Doppler effect. From data available at that time, Zwicky found that their redshifts have a large dispersion from the mean value of the whole cluster, even though by (Sanders, 2010, p. 14), he considered only eight largest galaxies. From this he calculated the square of the mean radial velocity $\overline{\overline{v}}^2_{\mathrm{radial}} = 5 \times 10^{11}\ \mathrm{m^2s^{-2}}$ with respect to the cluster center. Due to expected isotropic distribution of velocities and spherical symmetry, for the mean value $\overline{\overline{v}}$ he obtained

$$\overline{\overline{v}}^2 = 3\,\overline{\overline{v}}^2_{\mathrm{radial}} = 1.5 \times 10^{12}\,(\mathrm{m/s})^2 \quad \text{and} \quad \overline{\overline{v}} = 1.22 \times 10^6\,\mathrm{m/s}. \tag{18}$$

Substituting (17) and (18) into (16), we find the amount of the virial mass

$$M = 2.367 \times 10^{44}\,\mathrm{kg}, \tag{19}$$

which is about a 150 times higher value than found in (8). This discovery by Zwicky was ignored for many decades.

## 3. Problem of missing matter

Zwicky made many groundbreaking discoveries including the first postulation of dark matter, the existence of neutron stars as a result of supernova explosions and origin of cosmic rays, gravitational lensing by galaxies and their clusters. In spite of that, Zwicky's approach that led him to propose the existence of dark matter requires a more detailed discussion. Zwicky had to make many simplifying assumptions to estimate the total mass of the cluster A1656.

**1)** According to the Hubble law and current measurements of velocities, the distance to the Coma cluster is not 13.8 Mpc but about 100 Mpc. Substituting the distance $d = 100$ Mpc into (17), we get for the angular diameter $\beta = 1.7^\circ$ a much bigger radius

$$R = d \sin \frac{\beta}{2} = 1.48\,\mathrm{Mpc} = 4.58 \times 10^{22}\,\mathrm{m} \approx 5 \times 10^6\,\mathrm{ly}, \tag{20}$$

and thus also a larger mass than in (19). Altogether from (16), (18), and (20) the Virial Theorem yields

$$M = 1.71 \times 10^{45}\,\mathrm{kg},$$

which is about 1000 times larger than the value given in (8).

**2)** The angular diameter $\beta = 1.7^\circ$ from (17) is in reality somewhat greater. By (Biviano et al., 1995) the Coma cluster is in a region of dimensions $2.7^\circ \times 2.5^\circ$ with a boundary that is not well defined. Some other sources give smaller values.

**3)** Zwicky in (8) supposes that the mass of each galaxy is on average $10^9$ Sun masses. These data are, on the other hand, considerably underestimated. A great amount of light from stars is absorbed by interstellar gas. For comparison, our Galaxy has over $500 \times 10^9$ stars and its total mass is approximately $M_{\mathrm{G}} = 10^{12} M_\odot$, see (Lang, 2006, p. 127), which is even more than the total mass $\mathcal{M}$ of all 800 galaxies from (8) estimated by Zwicky. Nevertheless, the Milky Way belongs to a class of considerably larger galaxies. Therefore, at present physicists think that there is one order of magnitude more dark matter than luminous baryonic matter.

Zwicky made a lot of further approximations which have an essential influence on the resulting calculation of the mass:

**4)** The estimate of the total number of galaxies $N = 800$ (see (8)) is slightly underestimated, although Zwicky in (Zwicky, 1937, p. 244) admits $N \geq 1500$. At present we know more than one thousand galaxies in the Coma cluster. Each $\mathrm{Mpc}^3$ thus contains at least 70 galaxies on average. In 1933 Zwicky could not observe many dwarf galaxies in the cluster.

**5)** Relation (9) does not consider intergalactic matter inside the cluster. Its density is quite high in the central region of the cluster. There are more gas, dust, and solitary stars which were ejected by various gravitational collisions. By X-ray analysis from (Böhringer, Werner, 2010) and (Hughes, 1989) we know that the intracluster medium contains a lot of invisible baryonic matter.

**6)** Zwicky considered a uniform distribution of galaxies inside the cluster A1656, see (Zwicky, 1937, p. 229). However, the central region is much denser than the region around the boundary (see Fig.7) and larger galaxies are closer to the center, in general. In Section 5 we show that the coefficient $\frac{5}{3}$ from (16) is probably overestimated for the observed nonuniform and almost radially symmetric mass distribution. Since the cluster is seen only in projection, it can also be elongated or flattened.

**7)** The kinetic energy $\overline{T}$ with respect to the center of gravity is not entirely exact, since the mean recession velocity $\overline{v}_i$ from (11) over long time intervals is replaced by the current value $v_i(t) = |\dot{r}_i(t)|$ for a given $i \in \{1, \ldots, N\}$. At present we know the values of the heliocentric radial components of velocities of individual galaxies more precisely, but we cannot establish their mean values over long time intervals as required by (6). From the data presented in (Adami et al., 2005), (Biviano et al., 1995), and (Colless, Dunn, 1996) for galaxies belonging to the Coma cluster we get radial components of velocities (see Fig.2) and

$$\overline{\overline{v}} \approx 1.686 \times 10^6 \,\mathrm{m/s}, \tag{21}$$

which is even more than $\overline{\overline{v}}$ from (18). The potential energy (14) is also not stated exactly due to the approximation (13).

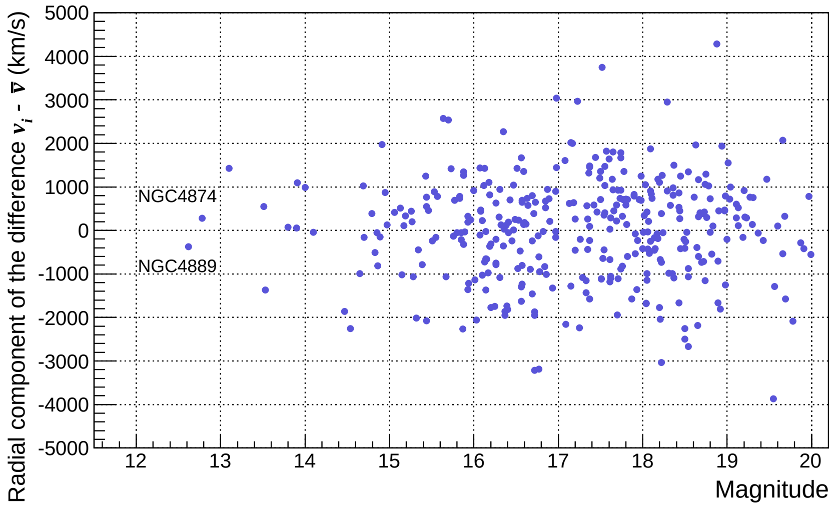


**Fig. 2.** Dependence of the radial component of the difference of recession velocities $v_i - \overline{v}$ in the Coma cluster on magnitude, where $v_i = v_i(t)$ corresponds to the present time.

**8)** Zwicky assumed an isotropic distribution of velocities. However, we can see a certain asymmetry of the histogram in Fig.6 with respect to the Gaussian curve (see also (Neumann et al., 2003)). A giant tidal tail of stars whose mass is 20 % of the mass of NGC 4874 was detected by (Gregg, West, 1998, p. 551).

**9)** Dark energy might also contribute to large velocities of galaxies inside the Coma cluster. There is no reason to assume that dark energy would somehow avoid the Coma cluster which is part of the expanding "cosmic web". The linear growth rate of large cosmic structures was observed by (Rapetti et al., 2010). Moreover, as shown in (Dumin, 2003, 2008), (Křížek, 2009, 2012), (Křížek et al., 2012), (Zhang, Li, Lei, 2010) dark energy acts also locally. This could magnify the calculated mass. In addition, the speed in (16) is squared! Therefore, a reliable knowledge of radial velocities is essential.

**10)** Zwicky restricted himself to the case in which all galaxies have the same time independent mass (Zwicky, 1937, p. 231). Nevertheless, galaxies exchange mass with the intergalactic environment and their magnitudes differ by eight orders of magnitude.

Let us put forward some other facts which should be taken into account for a thorough error analysis.

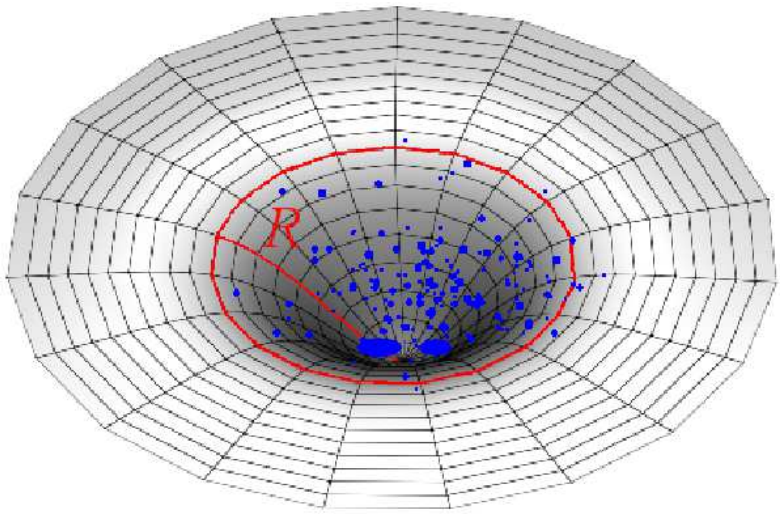


**Fig. 3.** Deformation of spacetime due to high density of galaxies in a cluster of radius $R$. The circumference of the circle with radius $R$ is smaller than $2\pi R$.

**11)** Zwicky assumed that the Coma cluster is in equilibrium and that the Virial Theorem holds exactly. However, by (Łokas, Mamon, 2003) most of the spiral galaxies are infalling on to the cluster, while ellipticals are mainly concentrated close to the center. Since their formation, galaxies could orbit the cluster center only a few times by the average velocity $\overline{\overline{v}}$ from (21), since one period takes about

$$2\pi\overline{\overline{r}}/\overline{\overline{v}} = 4.11 \times 10^9\,\mathrm{yr}, \tag{22}$$

where $\overline{\overline{r}} = \sqrt{3}R/\sqrt{5}$ is the mean distance from (15) and $R$ is given by (20). Although $\overline{\overline{r}}$ and $\overline{\overline{v}}$ are slightly overestimated with respect to the mean values, we probably cannot talk about the relaxed system and thus one has to defend whether the mechanical use of the Virial Theorem is well-founded.

**12)** Zwicky used Newtonian mechanics with an infinite speed of gravitational interaction, whereas the real speed is surely finite. In the cluster whose diameter is ten million light years, gravitational aberration effects are not negligible, see (Křížek, 2009), (Křížek et al., 2012). From Fig.6 we shall see that the velocities of some galaxies with respect to the cluster center are more than 1 % of the speed of light, i.e. long-term relativistic effects influence the evolution of the system. We know well the functioning of Newtonian mechanics on short time scales and low velocities in the Solar System. However, by (20) the cluster has diameter approximately 3 Mpc $> 6 \times 10^{11}$ au, where 1 au $\approx 150 \times 10^9$ m is the mean Earth-Sun distance. It is not clear whether we are allowed to employ Newton's laws to objects many orders of magnitude larger. This is similar to the application of the laws of quantum mechanics to objects of scale on the order of meters.

**13)** Zwicky replaced galaxies of diameter $10^{10}$ au by mass points. This does not allow one to consider angular moments of rotating galaxies which surely contribute to the total angular momentum. We also cannot include tidal forces that influence the dynamics of the system. For instance, an isolated binary system of two galaxies that orbit closely to each other is not stable, since galaxies will merge due to tidal friction, whereas the classical two-body problem has a periodic solution.

Zwicky restricted himself to the case in which $N$ is constant. However, sometimes galaxies merge due to tidal forces or various collisions in the densely populated space (see Fig.1).

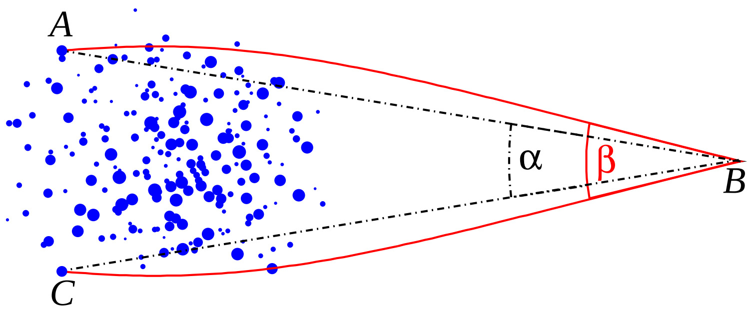


**Fig. 4.** A schematic illustration of the selflensing effect. The observation angle $\beta = \tilde{\sphericalangle} ABC$ is larger than the angle $\alpha = \sphericalangle ABC$ due to the bending of light caused by the gravitation of a galaxy cluster itself.

**14)** Zwicky substituted the spacetime curved by a thousand of galaxies (see Fig.3) with a total mass about $10^{45}$ kg by Euclidean space. Deformation of the space containing a galaxy cluster causes gravitational lensing. In fact, we need not consider an intervening galaxy as Zwicky suggested, since the cluster itself magnifies the perception of distances between its own objects. Note that the "index of refraction" of a gravitational lens represented by a galaxy cluster increases toward the center. Nevertheless, magnification of angular sizes by gravitational lensing remains. The selflensing effect (see Fig.4) modifies the observed density profile so that the cluster seems to be more sparse. It also enlarges the observed radius $R$ in (16), (17), and (20) which then yields a greater calculated virial mass $M$ than the cluster has in reality.

The volume of a sphere in such a deformed space is not $4\pi R^3/3$ (cf. e.g. (15) and Fig.3), but is smaller due to the Bishop-Gromov inequality (Petersen, 2006, Chap. 9), cf. also (Misner et al. 1997, p. 1099). Analogously, the surface of a sphere integrated in (15) is smaller than $4\pi r^2$.

The curved spacetime brings other effects. The total redshift is caused not only by a cosmological expansion of the Universe, but also partly by a gravitational redshift. Photons have to overcome the potential well of a star, as well as the potential well of the corresponding galaxy and the potential well of the whole cluster of galaxies.

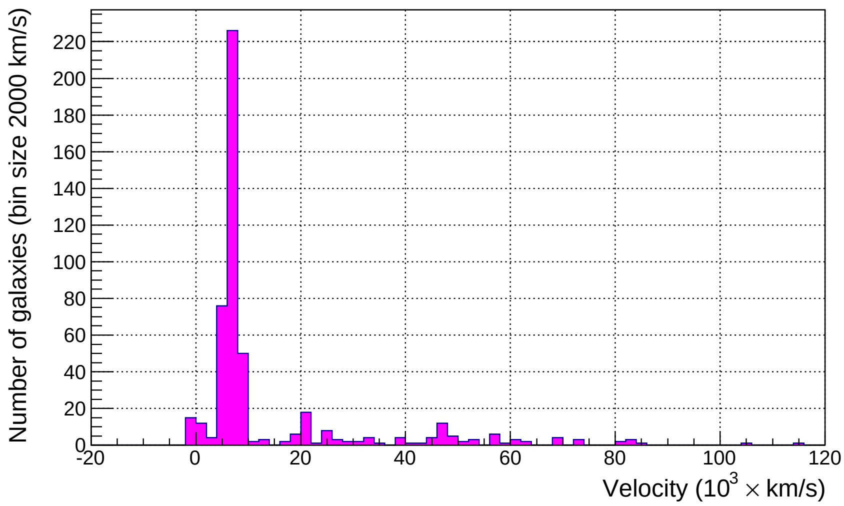


**Fig. 5.** Histogram of radial velocities of galaxies whose magnitude does not exceed 20 which are projected to the region about the cluster A1656.

**15)** Further sources of uncertainities are in input data and rounding errors (e.g. (11) represents a large sum with many terms of various orders of magnitude). According to (Biviano et al., 1995) the right ascension of the cluster center is $\alpha = 12\,\mathrm{h}\ 57.3\,\mathrm{m}$ and the declination $\delta = 28^\circ\,14.4'$. Other sources give different data, for instance, $\alpha = 13\,\mathrm{h}\ 00\,\mathrm{m}\ 00.7\,\mathrm{s}$ and $\delta = 27^\circ\,56'\,51''$ by (Rines et al., 2001). It is also not clear how to define the center, since the cluster is seen only in projection and its boundary is fuzzy. Moreover, we do not know the speed of gravitational interaction which is necessary to determine the "center".

A large amount of small errors of various origins may essentially distort the resulting calculation of the mass. Zwicky became well aware that he committed a lot of various simplifications and rough approximations that are treated in points **2), 4), 6), 11), 15)**, see (Zwicky, 1937, pp. 230, 231, 233, 242, 244), otherwise he could not compute anything. However, he did not consider several important facts that are surveyed e.g. in points **9), 12), 13), 14)**.

There exists a large number of different methods that have been applied to determine the total mass of the Coma cluster. They are based on the Virial Theorem (Łokas, Mamon, 2003), gravitational lensing (Gavazzi, Adami et al., 2009), (Kubo, Stebbins et al., 2007), and X-ray emitting gas (Allen et al., 2011), (Böhringer, Werner, 2010), (Broadhurst, Scannapieco, 2000), (Evrard et al., 2008), (Hughes, 1989), and (Voit, 2005). From these papers we know that there is approximately five times more nonluminous baryonic matter than luminous matter. Clusters of galaxies are also used to evaluate the cosmological parameters $\Omega_{\rm b} \approx 0.05$, $\Omega_{\rm M} \approx 0.25$, and $\Omega_{\Lambda} \approx 0.7$ indicating the density of baryonic matter, dark matter, and dark energy, respectively, see (Allen et al., 2011), (Evrard et al., 2008), and (Voit, 2005). In the next two sections we show that the value $\Omega_{\rm M}$ seems to be somewhat overestimated.

## 4. Analysis of current data

The galaxy cluster Abell 1656 contains in its center two supergiant elliptic galaxies NGC 4889 and NGC 4874 which are 10 times larger than the Milky Way and contribute essentially to the dynamics of the whole cluster (see Fig.1). Unfortunately, Zwicky in (Zwicky, 1933) and (Zwicky, 1937) does not present any input data on velocities and magnitudes of individual galaxies from the Coma cluster. He only gives $R$ and $\overline{\overline{v}}$ from (17) and (18), respectively.

Now let us show what Zwicky would get by his method for the present data. To reconstruct Zwicky's calculation, we employ data given in (Adami et al., 2005), (Biviano et al., 1995), and (Colless, Dunn, 1996). They contain some galaxies that do not belong to the Coma cluster, although they are in the considered region of the celestial sphere (Ledoux et al., 1999). There are for instance more than 50 galaxies whose radial velocities exceed 40 000 km/s. One galaxy (see Fig.5) has speed 114 990 km/s, which is more than one third of the speed of light! By the relativistic relation $z = \sqrt{(c+v)/(c-v)} - 1$ its redshift is $z \approx 0.5$. Note also that a galaxy whose radial speed is 40 000 km/s would fly through the distance corresponding to the radius (20) of the cluster within less than 50 million years. Thus, galaxies from the right part of Fig.5 cannot be in the cluster.

The nonuniform distribution of velocities with respect to the cluster center is well visible from the histograms in Figs.5 and 6. Note that some galaxies with blueshift or very small redshift form another independent cluster[4] in the left part of Fig.6. Thus to calculate (21) and (23), we restrict ourselves only to velocities from the interval $2\,000 - 12\,000$ km/s. A slightly different choice is given in (Kent, Gunn, 1982), (Łokas, Mamon, 2003), and (Rines et al., 2001).

Since the cluster is 100 Mpc from the Earth and has a relatively small radius (20), all its components are approximately at the same distance from us. We may therefore assume that the mass of each galaxy $m_i$ is by the Pogson relation proportional to $10^{-0.4 {\rm mag}_i}$, where ${\rm mag}_i$ is the measured magnitude of the $i$th galaxy. This trick enables us to calculate the mean speed $\overline{v}$ (resp. $\overline{\overline{v}}$) from relation (10) (resp. (11) and (21)) without knowing the real masses $m_i$.

[4] This small cluster of about 30 galaxies acts as a weak supplementary converging lens.

In this way we get

$$\overline{v} \approx 6877\,\mathrm{km/s}. \tag{23}$$

By (16), (20), and (21) the total virial mass of the cluster is[5]

$$M = 3.25 \times 10^{45}\,\mathrm{kg}. \tag{24}$$

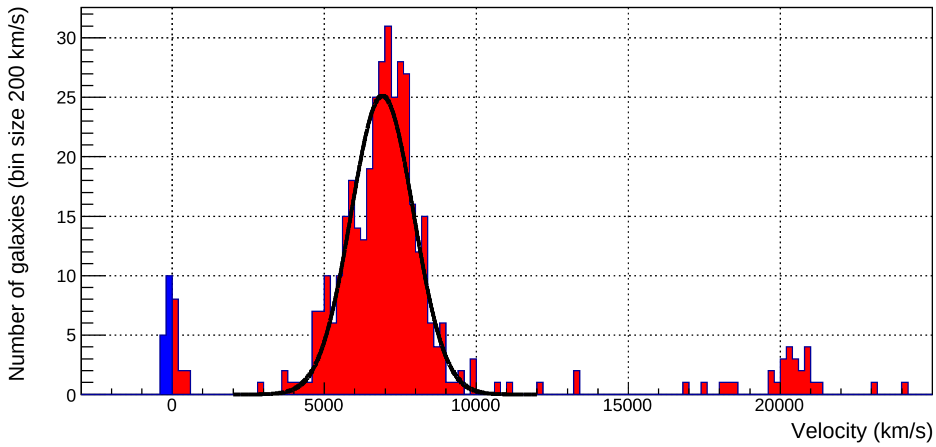


**Fig. 6.** A detail of the histogram from Fig.5 for radial velocities less than 25 000 km/s. Galaxies possessing blueshift are on the left. The dark line represents the Gaussian curve fitted to the considered data.

For comparison (see also (8)) let us give a lower bound of the total mass of the cluster based on the Pogson relation and the measured luminosities of galaxies

$$\mathcal{M} > C \sum_i 10^{-0.4\mathrm{mag}_i} = 3.3 \times 10^{44}\ \mathrm{kg}, \tag{25}$$

where the sum is taken over the 352 most luminous galaxies with known reference magnitudes not exceeding 20, $C = m\,10^{0.4\mathrm{mag}}$ is the scaling constant, and $\mathrm{mag} = 12.78$ is the magnitude of the comparative galaxy NGC 4874, which is by `http://en.wikipedia.org/wiki/NGC_4874` ten times more massive than our Galaxy, i.e.

$$m = 10 M_{\mathrm{G}} = 10^{13} M_{\odot} = 2 \times 10^{43}\ \mathrm{kg}, \tag{26}$$

where the total mass of our Galaxy $M_{\mathrm{G}} = 10^{12} M_{\odot}$ is given in (Lang, 2006, p. 127). We see that the total mass $M$ derived from the Virial Theorem is

[5] From (20) we get the associated mean density $\rho = 8 \times 10^{-24}$ kg/m$^3$ of the cluster, which is substantially bigger than the present mean density $\approx 10^{-27}$ kg/m$^3$ of space. For comparison, by (Bovy, Tremaine, 2012) the density of dark matter in our Galaxy is 0.008 $M_{\odot}\mathrm{pc}^{-3} = 5.444 \times 10^{-22}$ kg/m$^3$. By (Moni Bidin et al., 2012) this density is at least one order of magnitude smaller.

one order of magnitude greater than the lower bound for $\mathcal{M}$. Nevertheless, it is a question how much additional mass is formed by the approximately one thousand existing dwarf galaxies which are not included in the above sum, what is the contribution of nonluminous intergalactic baryonic matter, etc. For instance, by examination X-ray bands it was found that the intracluster medium contains more baryonic mass than the mass of all stars in galaxy clusters (Böhringer, Werner, 2010) and (Hughes, 1989).

When claiming that dark matter exists, we should first reliably estimate all possible errors from points **1)–15)** in Section 4 and perhaps some other. In particular, errors in **6), 9)**, and **11)–15)** may be quite large. Now, let us take a closer look on some of them.

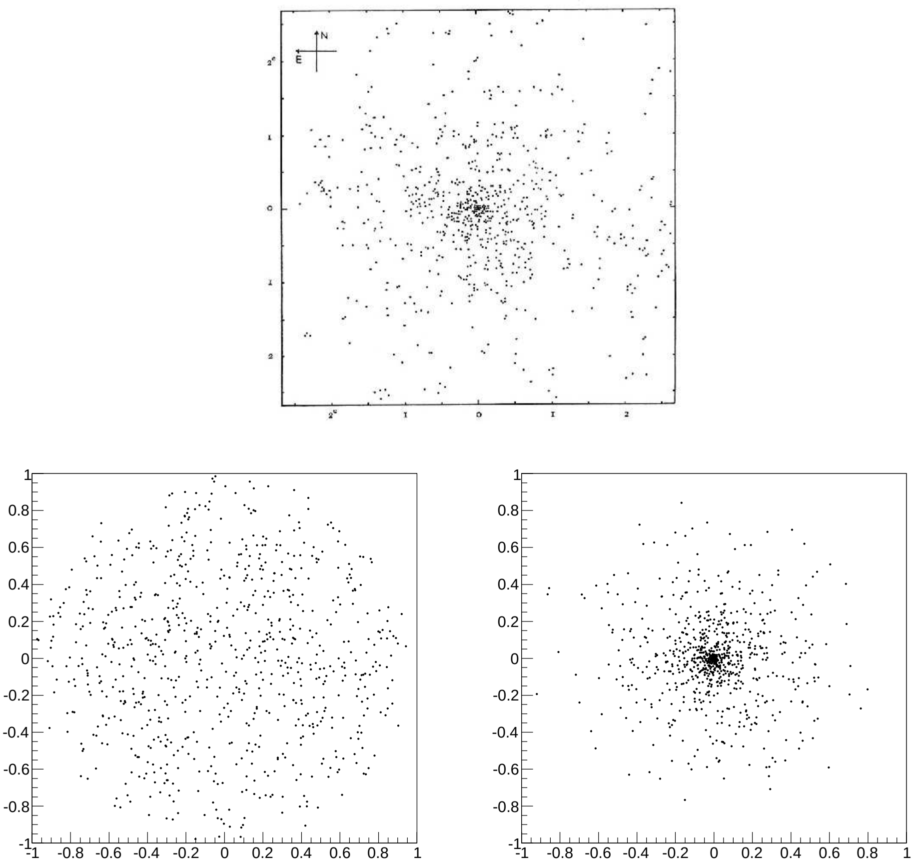


**Fig. 7.** The upper figure shows the distribution of galaxies in the Coma cluster from Zwicky's original paper (Zwicky, 1937, p. 227). The left lower figure illustrates a randomly generated uniform distribution of the same number of points inside a three-dimensional sphere projected to the plane and the right lower figure shows projected mass distribution (28) with $b = 1$ which is similar to the actual distribution from the upper figure.

**Nonuniformity of mass distribution.** We show that the coefficient $\frac{5}{3}$ appearing in (16) should be smaller. From Fig.7 we observe that the distribution of galaxies in the Coma cluster is far from being uniform as was assumed by Zwicky. Let us suppose that the mass density $\rho = \rho(r)$ is spherically symmetric in the sphere of radius $R$ given by (20). Then the whole mass of the Comma cluster can be expressed as

$$M = \int_0^R \rho(r) 4\pi r^2 \, \mathrm{d}r. \tag{27}$$

Zwicky assumed that $\rho$ is independent on $r$. Consider a more general density distribution which enables us to find a closer approximation of the observed density profile. Namely,

$$\rho(r) = a(R^b - r^b), \quad 0 \le r \le R, \tag{28}$$

where $b > 0$ and the parameter

$$a = \frac{3(b+3)M}{4\pi b R^{b+3}} \tag{29}$$

is chosen so that

$$\int_0^R a(R^b - r^b) 4\pi r^2 \, \mathrm{d}r = 4\pi a\Big(\frac{R^{b+3}}{3} - \frac{R^{b+3}}{b+3}\Big) = \frac{4\pi a b R^{b+3}}{3(b+3)} = M.$$

So now we have to replace the force $F_i$ acting on the $i$th galaxy and corresponding to a uniform mass density distribution given in (12) by another relation that takes into account the spherical symmetry of $\rho$. For the position given by the radius-vector $r_i$, we have by (28)

$$\begin{aligned} F_i &= -\frac{G m_i r_i \int_0^{|r_i|} 4\pi \rho(r) r^2 \, \mathrm{d}r}{|r_i|^3} = -\frac{4\pi G a m_i r_i}{|r_i|^3}\Big(\frac{R^b |r_i|^3}{3} - \frac{|r_i|^{b+3}}{b+3}\Big) \\ &= -4\pi G a m_i r_i \Big(\frac{R^b}{3} - \frac{|r_i|^b}{b+3}\Big). \end{aligned}$$

Hence, the total potential energy is

$$V = \sum_{i=1}^N F_i \cdot r_i = -4\pi G a \sum_{i=1}^N m_i \Big(\frac{R^b |r_i|^2}{3} - \frac{|r_i|^{b+2}}{b+3}\Big). \tag{30}$$

Further, we need to evaluate the mean value (cf. (15)) of the power $r^e$ for $e = 2$ and $e = b+2$. By (27) and (28) we get

$$\begin{aligned} \langle r^e \rangle &= \frac{\int_0^R r^e \rho(r) 4\pi r^2 \, \mathrm{d}r}{\int_0^R \rho(r) 4\pi r^2 \, \mathrm{d}r} = \frac{4\pi a}{M} \int_0^R (R^b - r^b) r^{e+2} \mathrm{d}r \\ &= \frac{4\pi a}{M}\Big(\frac{R^{b+e+3}}{e+3} - \frac{R^{b+e+3}}{b+e+3}\Big) = \frac{4\pi a b R^{b+e+3}}{M(b+e+3)(e+3)}. \end{aligned}$$

Using this equality for the exponents $e = 2$ and $e = b+2$, we find by (30), (9), and (29) that

$$
\begin{aligned}
V &\approx -4\pi G a\Big(\frac{R^b 4\pi a b R^{b+5}}{3M\,5(b+5)} - \frac{4\pi a b R^{2b+5}}{M(b+3)(b+5)(2b+5)}\Big)\sum_{i=1}^{N} m_i \\
&= -(4\pi a)^2 G R^{2b+5}\Big(\frac{b}{15(b+5)} - \frac{b}{(b+3)(b+5)(2b+5)}\Big) \\
&= -\Big(4\pi\frac{3(b+3)M}{4\pi b R^{b+3}}\Big)^2 G R^{2b+5}\frac{b^2(2b+11)}{15(b+3)(b+5)(2b+5)} \\
&= -\frac{3GM^2}{5R}\frac{(b+3)(2b+11)}{(b+5)(2b+5)}.
\end{aligned}
$$

From this, the Virial Theorem (5), and (11) we obtain a new relation for the reduced virial mass

$$
M = \frac{5R\overline{\overline{v}}^2}{3G}\frac{(b+5)(2b+5)}{(b+3)(2b+11)} \tag{31}
$$

which converges to Zwicky's original estimate (16) for $b \to \infty$. The best fit of the parameter $b$ of mass density distribution to Zwicky's data from the upper part of Fig.7 seems to be close to the value $b \approx 1$ (see Fig.7). The corresponding coefficient $\frac{35}{26}$ from (31) is only 80 % of the value $\frac{5}{3}$ from (16) proposed by Zwicky. However, since larger galaxies are closer to the center (see Figs.1 and 2), formula (16) may overestimate the total virial mass by about 20–25 %.

**Relativistic effects of high velocities.** From the Hubble law

$$
v = H_0 d \tag{32}
$$

with $v = \overline{v}$, and from formulae (7) and (23), it is estimated that the Coma cluster is at a distance of $d \approx 100$ Mpc from us (cf. (20)). The corresponding redshift $z$ is assumed to be linearly proportional to the distance $d$, i.e.,

$$
z = \frac{H_0}{c}d = 0.023, \tag{33}
$$

where $c = 3 \times 10^8$ m/s is the speed of light in vacuum. However, the distance $d$ above is slightly overestimated, since relativistic effects of the large speed (23) have to be taken into account. The classical formula accounting for the increase in wavelength $\lambda$ of electromagnetic radiation

$$
z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c},
$$

where $\lambda_0$ is the wavelength measured in earthly laboratory, has to be replaced by the relativistic relation

$$\lambda = \lambda_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}.$$

From this for $z = 0.023$ we get

$$v = c\frac{z(z+2)}{z^2+2z+2} = 6820 \quad \mathrm{km/s},$$

which is 1 % smaller than the speed in (23). Hence, the distance $d$ is by (32) about 1 % smaller and thus also the radius $R$ should be about 1 % smaller.

By (Weinberg, 1972, p. 485), the proper motion distance is $(z+1)$ times smaller than the luminosity distance (see also (Beijersbergen at al., 2002) and (Mobasher et al., 2003)) for the discussion about the luminosity function of the Coma cluster in an expanding universe).

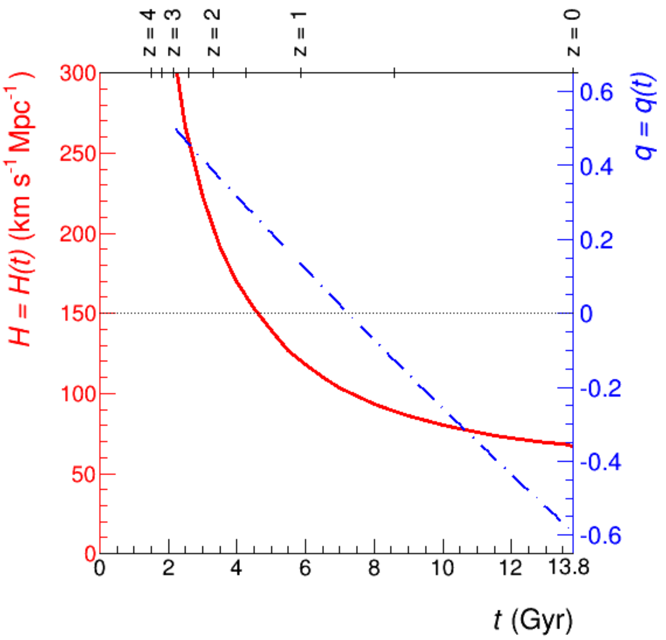


**Fig. 8.** The lower horizontal axis shows time in Gyr since the Big Bang. In the upper horizontal axis we see the corresponding redshift $z$.The behavior of the Hubble parameter $H = H(t)$ is sketched by the solid line (Pilipenko, 2013). The dashed-dotted line stands for the corresponding deceleration parameter $q = -1 - \dot{H}/H^2$.

**Gravitational redshift.** The above distance $d$ is also slightly overestimated due to the gravitational redshift of the Coma cluster, which has to be subtracted from the total measured redshift. According to (Cappi, 1995, p. 10), the total gravitational redshift of the two large central galaxies of the Coma cluster is about 61 km/s, which is cca 1 % of the speed given in (23). Although the redshifts of galaxies near the boundary of A1656 are about 20 km/s, this also leads to the overestimation of the distance of A1656 from us and thus also of its radius (20), speed (21), and total mass (see (16) and (31)). Similar gravitational redshifts of galaxy clusters are given also in (Broadhurst, Scannapieco, 2000), (Kim, Croft, 2004), and (Wojtak et al., 2011).

**The selflensing effect.** The selflensing effect of the Coma cluster may be estimated by means of (20), (24), and the famous formula for the bending angle (see e.g. (Roulet, 2002) and (Stephani, 1990))

$$\phi = \frac{4GM}{c^2R} \approx 2 \times 10^{-4} \text{ rad } \approx 0.7',$$

where $\phi = (\beta - \alpha)/2$ (see Fig.4). This value represents about 1 % of 1° corresponding to the angular radius $\beta/2$ of the Coma cluster (see (17)). Hence, $R$ in (16) should be again about 1 % smaller.

**Decreasing Hubble parameter.** The expansion speed of the Universe, which is characterized by the Hubble parameter $H = H(t)$, depends essentially on the total mass density and the density of dark energy. It is decreasing with time (see Fig.8). According to (Pilipenko, 2013), its value for $z = 0.023$ is more than 1 % larger than the present value $H_0$. Hence, the distance $d$ in (33) is again slightly overestimated.

**Contribution of dark energy.** The effect of local expansion of the Universe has a long history dating back to the paper (McVittie, 1933). Such an expansion of a similar rate as the Hubble constant has been observed even on scales of the Solar System, see e.g. (Dumin, 2003, 2008), (Křížek, 2009, 2012), and (Zhang, Li, Lei, 2010), and of galaxies or even larger structures, see (Křížek et al., 2012), (Křížek, Somer, 2013), and (Rapetti, et al., 2010). By (7) and (20) the value of the Hubble constant rescaled on the radius of the cluster is $RH_0 \approx 10^5$ m/s, which is more than 5 % of the speed from (21). This, of course, yields a larger mean quadratic speed $\overline{\overline{v}}$ than would occur if dark energy did not act on the cluster. Since the speed $\overline{\overline{v}}$ in (31) is squared, the contribution of dark energy could seemingly increase $M$ by about 10 % (see Table 1).

**Reduction of the root-mean-square speed.** Now, let us introduce one more quadratically nonlinear effect which has a nonnegligible influence on the total mass estimate. Above we saw that the mean recession speeds $\overline{v}$ and $\overline{v}_i$ were overestimated by several percent. If it were, say 8 %, then the square $\overline{\overline{v}}^2$ defined by formula (11) would be overestimated by about $100(1 - 0.92^2)\,\% \approx 15\,\%$. This essentially reduces the estimated mass (24) with respect to the virial mass (16) or (31).

In summary, the seven effects analyzed above can essentially reduce the total mass (24) obtained from the Virial Theorem by a factor of about two. Hence, the total mass of the cluster is at most five times larger than its luminous mass (25).

According to (Kent, Gunn, 1982), the possible distribution of dark matter in the Coma cluster cannot be significantly different from that of the galaxies inside the cluster. Thus, the distribution of dark matter approximately follows the distribution of galaxies.

Finally, we shall present a "back of the envelope" calculation illustrating whether it is necessary to assume extragalactic dark matter in the center of the Coma cluster.

**Table 1.** Effects and the corresponding percentage that reduce the viral mass $M$, observed radius of the Comma cluster $R$, and the root-mean-square speed of galaxies $\overline{\overline{v}}$.

| Effect | $M$ | $R$ | $\overline{\overline{v}}$ |
|---|---|---|---|
| Nonuniformity of galaxy distribution | 20 − 25 | 0 | 0 |
| Relativistic effects of high velocities | 3 | 1 | 1 |
| Gravitational redshift | 3 | 1 | 1 |
| The selflensing effect | 1 | 1 | 0 |
| Decreasing Hubble parameter | 3 | 1 | 1 |
| Contribution of dark energy | 10 | 0 | 5 |

**Example 1.** For simplicity assume that the two supergiant elliptic galaxies NGC 4889 and NGC 4874 (see Fig.1) have the same mass $m$ and that they orbit along a circular trajectory with center $O$, radius $r$, and velocity $v$. If one of these two galaxies were to be smaller, it would orbit the larger one by a higher velocity and along a longer path. Then it would absorb more additional galaxies than the larger one. By this mechanism the masses of both galaxies are balanced.

Since the gravitational potential inside a homogeneous spherical layer is constant, external galaxies and possible dark matter outside the sphere with center $O$ and radius $r$ have almost no influence on this motion. From Newton's laws we get

$$\frac{Gm^2}{4r^2} = \frac{mv^2}{r}. \tag{34}$$

The distance of both galaxies on the celestial sphere is $8.15'$ which in projection on the distance 100 Mpc gives $7.32 \times 10^{21}$ m. Thus for the radius $r$ we have

$$r \geq 3.66 \times 10^{21}\,\mathrm{m}. \tag{35}$$

According to (Adami et al., 2005, p. 19), the measured radial velocities of both supergiant galaxies are 6472 km/s and 7189 km/s. Their average velocity $\tilde{v} =$ 6830.5 km/s nicely corresponds to the mean recession speed of the whole cluster (23). For the radial velocity $v_{\mathrm{radial}}$ with respect to $\tilde{v}$ we get by (26), (34), and (35)

$$\begin{aligned} 3.585 \times 10^5 = \frac{7\,189\,000 - 6\,472\,000}{2} &= v_{\mathrm{radial}} \leq v = \sqrt{\frac{Gm}{4r}} \\ &\leq \sqrt{\frac{6.673 \times 10^{-11} \times 2 \times 10^{43}}{4 \times 3.66 \times 10^{21}}} = 3.02 \times 10^5\ (\mathrm{m/s}). \end{aligned} \tag{36}$$

Comparing the left-hand and the right-hand sides, we find a small discrepancy. This simplified example shows that Newtonian mechanics does not describe reality correctly or the masses or radial velocities of both the elliptic galaxies are wrongly estimated or we have to assume the existence of invisible matter between galaxies. The different velocities 6505 km/s and 7108 km/s of the two giant galaxies as given in (Biviano et al., 1995) would even yield a smaller value $v = 3.015 \times 10^5$ m/s on the left-hand side of (36). Moreover, considering the gravitational influence of other matter that is inside the sphere with center $O$ and of radius $r$, the right-hand side of (36) would be larger. By (Allen et al., 2011), (Böhringer, Werner, 2010), and (Voit, 2005) clusters of galaxies contain five times more baryonic matter in the form of hot gas producing X-rays than baryonic matter contained in galaxies. Therefore, it does not seem that there should be ten times more invisible (baryonic and dark) matter than visible matter. Also the lower bound in (35) is smaller, since the cluster magnifies the angular distances due to the abovementioned effects (see e.g. Fig.4). Hence, the right-hand side of (36) should be larger. □

## 5. Current status of the knowledge of dark matter

At present we do not know what is the composition of nonluminous matter. It surely consists partly of known particles. Nevertheless, it is difficult to estimate experimentally which portion is due to interstellar gas and dust, fossil neutrinos, and massive compact halo objects (known as MACHOs). In addition, new particles are searched for, e.g. axions or weakly interacting massive particles (known as WIMPs). Results of experimental searches for particles of dark matter were up to now negative.

The influence of dark matter in the Solar System is also negligible (Moni Bidin et al., 2012) even though our Sun is a large gravitational attractor. This indicates that dark matter (if it exists) almost certainly cannot dissipate its inner energy. Oscillations of stars in the direction perpendicular to the galactic plane can be well explained by classical Newtonian mechanics (without dark matter), see (Moni Bidin et al., 2012).

In the gravitational field of a central force of a mass point, the speed $v$ of particles in circular orbits is proportional to $r^{-1/2}$, where $r$ is the distance to the center. Such orbits are called *Keplerian* (see Fig.9). According to (Rubin, 2003) and (Rubin et al., 1962), the stellar (rotation) curve is flat, and does not decrease as is expected for Keplerian orbits. However, spiral galaxies do not possess a field of central force except for the very center (e.g., in the Milky Way the stars S1, S2, … orbiting around the central black hole obey Kepler's laws). Denote by $m(r)$ the mass inside a ball of radius $r$ around the galactic center. When $r$ increases, $m(r)$ also increases, and thus the speed $v$ of stars on circular orbits should be higher than that for Keplerian orbits. For simplicity, assume that this speed is proportional to $r^\alpha$ with $\alpha > -1/2$. Vera Rubin (Rubin, 2003, p. 7) observed almost constant speeds of order $v \approx 200$ km/s for $r > r_0$, where $r_0$ is typically a few kpc (see Fig.9). This implies that $\alpha \approx 0$. For an almost constant speed

$$v = \sqrt{\frac{m(r)G}{r}} \tag{37}$$

we find that $m(r)$ should be linearly proportional to $r$, provided the mass outside the sphere of radius $r$ has only negligible influence on $v$. Relation (37) is, of course, very rough. However, the speed corresponding to a flat disk is even higher. In more detail, the speed of a particle orbiting a uniform sphere of radius $r$ is smaller than it would be if it were to orbit around a rotationally symmetric disk of the same mass and radius $r$.

By cosmological models based on the Friedmann equations and by extrapolation from the recent measurements of the 13.8 Gyr old microwave background radiation by the Planck satellite (Planck, p. 11) it was deduced that the present Universe consists of 5 % baryonic matter, 27 % dark matter, and 68 % dark energy. It is said that high velocities of stars orbiting in external parts of spiral galaxies can only be explained by adding five times more dark matter than baryonic matter. We give one more "back of the envelope" calculation investigating if it is really necessary to assume that much dark matter in our Galaxy.

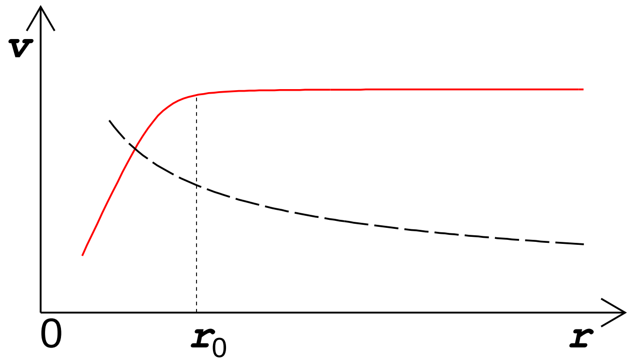


**Fig. 9.** The dashed line illustrates the decrease of velocityof Keplerian orbits with respect to the distance $r$.The solid line shows an idealized flat rotational curve of a general galaxy proposed by V. Rubin.

**Example 2.** For flat rotational curves with a constant speed $v$ for $r > r_0$, we may define by (37) the corresponding *equivalent central point mass* (i.e. mass concentrated to a point)

$$M(r) := \frac{v^2 r}{G}.$$

The total mass (including eventual dark matter) of our Galaxy is equal to

$$M_{\rm G} = 10^{12} M_{\odot} = 2 \cdot 10^{42} \text{ kg}$$

see (Lang, 2006, p. 127) and its radius is $r_{\rm G} = 16$ kpc $= 4.936 \cdot 10^{20}$ m. For a star orbiting the Galaxy on the boundary with constant speed $v = v_{\odot} = 230$ km/s corresponding to the flat rotational curve (cf. Fig.9) the equivalent central point mass

$$M(r_{\rm G}) = \frac{v^2 r_{\rm G}}{G} = \frac{230^2 \cdot 10^6 \cdot 4.938 \cdot 10^{20}}{6.674 \cdot 10^{-11}} = 3.912 \cdot 10^{41} \approx \frac{1}{5} M_{\rm G} \qquad (38)$$

is surprisingly very small. Baryonic matter inside the disk and the bulge is just about $M_{\mathrm{G}}/5$.

A much worse disproportion appears for the Andromeda galaxy M31 whose radius is $r_{\mathrm{A}} \approx 2r_{\mathrm{G}}$ and whose total mass is $M_{\mathrm{A}} \approx 3M_{\mathrm{G}}$. Since the orbital speed is very low – again about 230 km/s outside the central region (Rubin, 2003, p. 7), we get

$$M(r_{\mathrm{A}}) \approx M(2r_{\mathrm{G}}) = 2M(r_{\mathrm{G}}) \approx \frac{2}{5}M_{\mathrm{G}} \approx \frac{2}{15}M_{\mathrm{A}}. \tag{39}$$

Thus, 2/15 of the total mass may explain the speed 230 km/s measured by (Rubin, 2003, p. 7) of a star orbiting M31 on its boundary.

Although relations (38) and (39) are only approximate, the existence of at least five times more dark matter than baryonic matter to hold galaxies together is exaggerated. Moreover, a particle with circular trajectory of radius $R$ orbiting a mass point has a smaller speed than if it would orbit a flat disk of the same mass with arbitrary rotationally symmetric density distribution of mass and radius not exceeding $R$. $\square$

Note that each galaxy curves spacetime, too, and thus effects from **9), 11), 12), 14)** must also be taken into account. Ten times more nonluminous matter than luminous matter can hardly explain sharp contours of galaxy arms, their prefect point-symmetric shapes, etc.

Douglas Clowe (Clowe et al., 2006) presents the collision of two galaxy clusters, where dark matter is revealed by gravitational lensing. Both clusters have approximately the same size and they coincidentally lie together with clouds of dark matter in one line.

The discrepancy between the used model and observations does not imply the existence of dark matter, since the model need not be correct. Newton's law of gravitation probably does not describe reality well at long distances (see e.g. (Carroll et al., 2004), (Milgrom, 1983)). Consequently, various MOND (Modified Newtonian Dynamics) models are developed, e.g. with retarded potentials. Correct interpretation of measured data is a key problem. For instance, dark energy can be only a consequence of incorrect and mechanical use of current physical models for extremely long time intervals (see (Křížek, 2012), (Křížek, Somer, 2013)) Analogously, dark matter can be only a consequence of the use of Newtonian mechanics (valid in Euclidean space) over very long distances in a curved spacetime (see Fig.3).

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