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Manifestations of Dark Energy in the Solar System

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Abstract—We give more than 10 examples based on astronomical observations showing that dark energy acts not only on large scales but also on small scales. In particular, we present several independent arguments that the average Earth-Sun distance increases by about 5 m/yr. Such a large recession speed cannot be explained by the solar wind, tidal forces, plasma outbursts from the Sun, or by the decrease of the Solar mass due to nuclear reactions. We also discuss possible reasons for disagreement with other authors, who propose much smaller values.

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1. INTRODUCTION

1.1. Paradoxes in the Solar System. The Solar System can at present be assumed to be sufficiently isolated from the influence of other stars. For instance, the gravitational force of the nearest known star Proxima Centauri on Earth is one million times smaller than the maximum gravitational force of Venus.

In this paper we give more than 10 examples showing that some repulsive (antigravity) forces can be detected in the Solar System by means of a wide interdisciplinary approach. We present several geophysical, heliophysical, climatological, geochronometrical, paleontological, astrobiological, mathematical, and astronomical observational arguments to support this conjecture. Introducing dark energy (DE) into the Solar System, a number of classical paradoxes can be easily explained, such as the Faint Young Sun Paradox, the very large orbital momentum of our Moon and Triton, the formation of Neptune and the Kuiper belt, an unexplained residual in the orbit of Neptune and migration of other planets, rivers on Mars, the Tidal Catastrophe Paradox of the Moon, the existence of fast satellites and Saturn's rings below the stationary orbits, etc.

No model describes reality absolutely exactly. Therefore, an extremely small deviation $\varepsilon > 0$ of the real position of some body (comet, planet, star, etc.) from Newtonian mechanics or from general relativity during one year may cause after one billion years quite a large and detectable value of order $10^9\varepsilon$. All small deviations are generally not cancelled (like, e.g., rounding errors are statistically annulled), but accumulated and then possibly observed. They appear if deviations are cumulative, i.e. secular, which means that they act always towards one and the same "direction". At present, a substantial portion of these cumulative deviations could be interpreted as DE effects, which might explain (at least partly) the origin of DE.

1.2. The Hubble parameter. DE is spread almost uniformly everywhere in the Universe. Thus it has an essential influence on the Hubble parameter $H = H(t)$ which characterizes the expansion rate and can be written as

$$H(t) = \frac{a'(t)}{a(t)}, \quad (1)$$

where the prime stands for the time derivative and $a = a(t)$ is the cosmological scale factor. According to Planck Collaboration [70], the present (model-dependent) value of the Hubble parameter is about

$$H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (2)$$

and

$$t_0 \doteq 13.8 \text{ Gyr} \doteq 4.355 \cdot 10^{17} \text{ s} \quad (3)$$

is the age of the Universe. The behaviour of the real Hubble parameter $H = H(t)$ is sketched in Fig. 1. The corresponding data were taken from [65], which is also based on recent Planck satellite results [70]. Integrating (1), the associated expansion function can be expressed as

$$a(t) = a(\bar{t}) \exp \int_{\bar{t}}^{t} H(\tau) d\tau \quad \text{for} \quad 0 < \bar{t} \le t. \quad (4)$$

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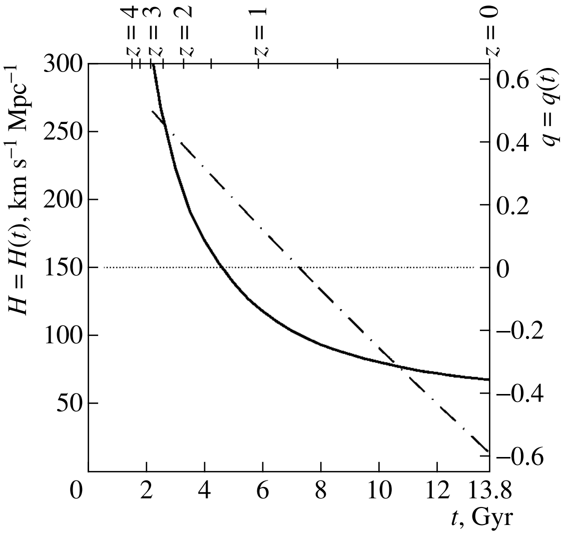

Fig. 1. The behavior of the Hubble parameter $H = H(t)$ is sketched by the solid line. The dashed-dotted line stands for the corresponding deceleration parameter $q = q(t) = -1 - H'(t)/H^2(t)$. The lower horizontal axis shows time in Gyr since the Big Bang. In the upper horizontal axis we see the corresponding redshift z.

Note that measurements in the neighborhood of our Galaxy with redshift $z \approx 0$ yield a larger value $H_0 = 72$ km s^{-1} Mpc^{-1} (see [57, 84]) than that in (2) obtained by fearless extrapolation methods of the microwave cosmic background with $z \approx 1100$.

The current mean distance between the Earth and the Sun is called the *astronomical unit* and is denoted by au. Its present value was redefined by the IAU in 2012 as follows:

$$1 \text{ au} = 149\,597\,870\,700 \text{ m} \approx 150 \cdot 10^9 \text{ m}, \quad (5)$$

and it is almost equal to the present value of the semimajor axis of the Earth's elliptic orbit. The previously accepted value of 1 AU = 149 597 870 691 was thus raised by 9 meters.

Now we will rescale H_0 from (2) to 1 au. Since 1 pc = 206 265 au and one sidereal year has 31 558 149.54 s, an equivalent value of H_0 given by (2) is 10.3 ± 0.2 m yr^{-1}au^{-1}, i.e.,

$$H_0 \approx 10 \text{ m yr}^{-1}\text{au}^{-1}. \quad (6)$$

The value H_0 as stated in (6) is so large that some local manifestations of DE should be detected in our own Solar System. In particular, in the next section we show by several independent methods that the average recession speed of the Earth from the Sun is very roughly about 5 m/yr.

2. LARGE MEAN RECESSION SPEED OF THE EARTH FROM THE SUN

From now on, 0 will stand for the present time, i.e., $H(0) = H_0$.

2.1. Mean DE effect on the Solar System. DE is distributed almost uniformly in the Universe. There is no reason to assume that DE would somehow avoid the Solar System. Thus, it should also be present in the Solar System. In this section we will show that DE has a substantial effect even on scales as small as astronomical units,

$$H_0^{(\text{loc})} \approx 0.5 H_0. \quad (7)$$

This does not mean that the average recession speed of the Earth from the Sun (whose distance is 1 au) should be 5 m per year by (6), but some large speed of several meters per year can be deduced, as we shall see below. We illustrate that the local expansion rate of the Solar System is smaller than the global expansion rate of the Universe, i.e.,

$$H_0 > H_0^{(\text{loc})},$$

but it is of the same order (see e.g. (9), (24), (30), (34)).

Modern levels of accuracy do not allow us to determine directly the secular changes of the Earth-Sun distance ([68], p. 78). The reason is that the center of gravity of the Solar System moves circa 1000 km per day with respect to the Sun as shown in [4], p. 542 or [50], p. 5. Local effects of DE in the Solar System are also examined in [1, 3, 10–13, 23, 25, 41, 42, 44]. Local and global measurements of the Hubble parameter are compared in [35] and [77].

2.2. Stable conditions for life on Earth due to almost constant solar flux. Life on Earth has existed continually for at least 3.5 Gyr, and this requires relatively stable conditions during this very long time period. However, since the Sun is a star on the main sequence of the HR diagram, its luminosity has increased approximately linearly within the last 4.5 Gyr (Fig. 2). The initial value of the luminosity was only 70% of its present value [72]. This leads to the paradox usually referred as the *Faint Young Sun*, see, e.g., [15, 53, 73, 74]. The mean temperature on the Earth's surface would have been much below the freezing point, in contrast to the absence of glaciation in the first 2.7 Gyr (see [4], p. 177). G. Feulner [14] tries to find geochemical constraints on the composition of Earth's early atmosphere to explain the Faint Young Sun problem. However, it is generally believed that the greenhouse effect, higher levels of radioactivity, impacts of comets, and more volcanism 3.5 Gyr ago are not able to explain this paradox.

The Faint Young Sun paradox is, in fact, more severe due to the ice-albedo feedback of the frozen ocean. To prevent the Earth from freezing over, a much higher concentration of CO_2 than today is assumed in [45]. Nevertheless, the Faint Young Sun paradox can also be easily explained by antigravity

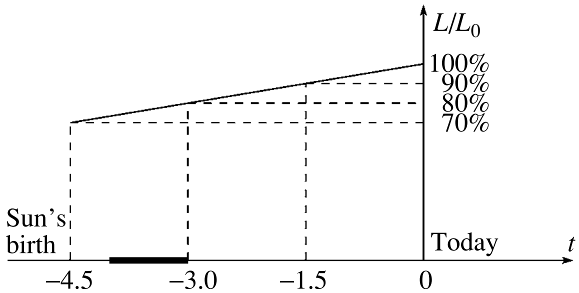

Fig. 2. Linearized relative luminosity L/L_0 of the Sun from the origin of the Solar System up to the present. The time t is given in Gyr. The thick segment along the horizontal axis indicates the period when Mars had liquid water on its surface. In the middle of this time interval, also life on Earth appeared.

forces produced by DE, as was first presented in [50] and then further developed in [34].

Assume for a moment that the recession speed of the Earth from the Sun during the last 3.5 Gyr was equal to the constant value

$$\overline{v} = 5.2 \text{ m/yr} \tag{8}$$

which is in good agreement with (7), since by (6) we get

$$H_0^{(\text{loc})} \approx 0.52 H_0 \tag{9}$$

for the expansion of the Earth–Sun system. We claim that in this case the Earth would receive an almost constant flux density of energy comparable with the *solar constant* (i.e., the total solar power incident per unit area of 1 m^2 perpendicularly to rays at the top of the Earth's atmosphere corrected to 1 au)

$$L_0 = 1.36 \text{ kW m}^{-2} \tag{10}$$

over a very long period of the last 3.5 Gyr.

To prove this (see Proposition 2.1 below), we put $\tau = -3.5$ Gyr. Since the luminosity of the Sun increases approximately linearly with time and it was only about 77% of its present value 3.5 Gyr ago (see Fig. 2), we set

$$L(t) = \Big(1 - 0.23\frac{t}{\tau}\Big) L_0 \tag{11}$$

for every $t \in [\tau, 0]$, i.e. $L(0) = L_0$. As the luminosity decreases with the square of the distance, we can state the following:

Proposition 2.1 (Optimal recession speed of the Earth from the Sun). *Set*

$$L_{\text{opt}}(t) = \frac{L(t)R^2}{(R + \overline{v}t)^2}, \quad t \in [\tau, 0], \tag{12}$$

where $R = 1$ *au and* $\overline{v}$ *is given by (8). Then*

$$L_{\text{opt}}(t) \approx 1.36 \pm 0.005 \text{ kW m}^{-2} \quad \forall\, t \in [\tau, 0]. \tag{13}$$

Proof. The very small dispersion of luminosity ± 0.005 kW m^2 on the right-hand side of Eq. (13) can be easily derived analytically by investigating the rational function $L_{\text{opt}}(t)$ (see Fig. 3). It is concave on the whole interval $[\tau, 0]$,

$$1.3573 < L_{\text{opt}}(\tau) = \min_{t\in[\tau,0]} L_{\text{opt}}(t) < \max_{t\in[\tau,0]} L_{\text{opt}}(t) < 1.3646,$$

and $L_{\text{opt}}(0) = L_0$. □

The luminosity (12) would, of course, guarantee very stable conditions (13) for the development of intelligent life on Earth over a very long period of 3.5 Gyr. In particular, the amount of DE seems to be just right for an almost constant influx of solar energy and thus also for the appearance of mankind.

DE thus represents further support for the (weak) Anthropic Principle, which states that the basic physical constants are favorable to the emergence of life only if they are in very narrow intervals [50]. Moreover, the speed in (8) is optimal in the sense that any other slightly different speed would not yield an almost constant flux expressed by the rational function in (12) on the time interval of 3.5 Gyr. Thus probably the real mean recession speed of the Earth from the Sun was close to the value 5.2 m/yr (see Propositions 2.2 and 2.3 and Remarks 2.1–2.4 below).

The real mean speed of the Earth from the Sun could be even slightly higher than (8) since the temperature of the oceans 3.5 Gyr ago was about 80°C, see [55]. It is known that a decrease of the luminosity by only a few percent caused ice ages in the past. A decrease larger than 5% would cause total glaciation of the whole planet.

A decrease or increase of the solar constant (10) up to 5% corresponds to a ring—popularly called the *ecosphere (habitable zone)*—with radii $(0.95)^{1/2}$ au and $(1.05)^{1/2}$ au which represents a very narrow range of 145.8–153.3 million km from the Sun.

Now, for a variable continuous recession speed $v \in C([\tau, 0])$, we define similarly to (12) the associated luminosity

$$\mathcal{L}(v, t) = \frac{L(t)R^2}{\Big(R - \int_t^0 v(\theta)d\theta\Big)^2}, \quad t \in [\tau, 0], \tag{14}$$

where $\tau = -3.5$ Gyr, $R = 1$ au, and $L(t)$ is defined by (11).

Proposition 2.2 (Two-sided bounds). *If the recession speed* $v = v(t)$ *of the Earth from the Sun lies in the interval* $[4.26,\ 6.14]$ *m/yr for every*

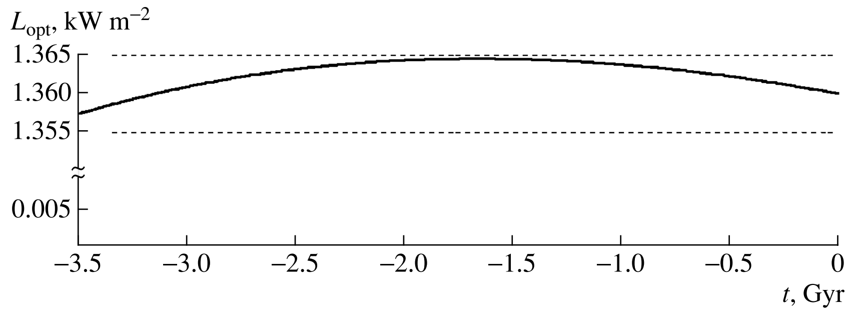

Fig. 3. Plot of the almost constant function $t \mapsto L_{\rm opt}(\tau)$ over the interval $[\tau, 0]$. The vertical axis is substantially shortened for clarity of the presentation.

$t \in [\tau, 0]$*, then the luminosity determined by (14) changes at most about* 5% *from* L_0*, namely,*

$$0.95L_0 \le \mathcal{L}(v, t) \le 1.05L_0 \quad \forall t \in [\tau, 0].$$

Proof. For the constant velocity $v_1 \equiv 4.26$ m/yr the rational function $t \mapsto \mathcal{L}(v_1, t)$ is increasing on $[\tau, 0]$, and thus by (14)

$$0.95L_0 = \frac{L(\tau)R^2}{(R + v_1\tau)^2} = \mathcal{L}(v_1, \tau) \le \frac{L(t)R^2}{(R + v_1 t)^2} = \mathcal{L}(v_1, t) \qquad (15)$$

for any $t \in [\tau, 0]$. Analogously, for the constant velocity $v_2 \equiv 6.14$ m/yr, we find that $t \mapsto \mathcal{L}(v_2, t)$ is decreasing, and therefore,

$$\mathcal{L}(v_2, t) \le \mathcal{L}(v_2, \tau) = \frac{L(\tau)R^2}{(R + v_2\tau)^2} = 1.05L_0. \qquad (16)$$

Putting (15) and (16) together, we find by (14) that for any $t \in [\tau, 0]$

$$0.95L_0 \le \frac{L(t)R^2}{(R + 4.26t)^2} \le \frac{L(t)R^2}{\left(R - \int_t^0 v(\theta)d\theta\right)^2} \le \frac{L(t)R^2}{(R + 6.14t)^2} \le 1.05L_0. \quad \square$$

A more important converse proposition has stronger assumptions on velocities.

Proposition 2.3 (Additional two-sided bounds). *If the mean recession speed* $\overline{v}$ *lies outside the interval* [4.26, 6.14] *m/yr, then there exists a nonempty subinterval* $I \subset [\tau, 0]$ *such that the luminosity* $\mathcal{L}(\overline{v}, t)$ *is less than* 95% *or greater than* 105% *of* L_0 *for all* $t \in I$.

Proof. If $\overline{v} < v_1 \equiv 4.26$ m/yr, then similarly to (15) we get

$$\mathcal{L}(\overline{v}, \tau) = \frac{L(\tau)R^2}{(R + \overline{v}\tau)^2} < \frac{L(\tau)R^2}{(R + v_1\tau)^2} = \mathcal{L}(v_1, \tau) = 0.95L_0.$$

From the continuity of the rational function $t \mapsto \mathcal{L}(\overline{v}, t)$ it follows that there exists a nonempty time interval I_1 such that $\mathcal{L}(\overline{v}, t) < 0.95L_0$ for all $t \in I_1$.

Analogously to (16) we find that for $\overline{v} > v_2 \equiv 6.14$ m/yr there exists a nonempty interval $I_2 \subset [\tau, 0]$ such that $\mathcal{L}(\overline{v}, t) > 1.05L_0$ for all $t \in I_2$. $\square$

Remark 2.1. The statement of Proposition 2.3 would not guarantee suitable conditions for the development of life. From Propositions 2.2 and 2.3 we find that a probable secular expansion rate of Earth's trajectory lies in the range

$$[0.426H_0, \ 0.614H_0].$$

Such a local expansion is therefore perfectly tuned (see [50]).

Remark 2.2. The recession speed (8) also guarantees very stable conditions on the Earth for several Gyr in the future. For instance, during the next 3.5 Gyr from now the flux density of energy from the Sun will be in the interval 1.33–1.36 kW m^{-2} if the luminosity behaves as in (12).

Remark 2.3. The linear function $L(t) = (1 - 0.23t/\tau)L_0$ (cf. Fig. 2) is in some models replaced with a rational function (see e.g. [4], p. 177)

$$\hat{L}(t) = \frac{L_0}{1 + 0.3t/\tau_0}, \quad t \in [\tau, 0],$$

where $\tau_0 = -4.5$ Gyr. In this case the optimal mean recession speed (guaranteeing an almost constant energy flux from the Sun) is $\overline{v} = 4.36$ m/yr, and the mean recession speed should be in the interval [3.27, 5.21] m/yr to keep variations of the solar energy flux below 5% as in Propositions 2.2 and 2.3.

Remark 2.4. From Fig. 1 we observe that $H = H(t)$ is almost constant over the last 4.5 Gyr when the Solar System was formed. It lies in the interval $[H_0, \frac{5}{4}H_0]$. Assuming that $H(t)$ is constant, i.e., $H(t) \equiv H_0$, we get by (4)

$$a(t) \approx a(0)\exp(H_0 t). \qquad (17)$$

Proposition 2.1 can be modified for an exponential expansion as follows. If the mean recession speed of the Earth from the Sun is $\tilde{v} = 5.014$ m/yr, then $L_{\rm opt}(t) = 1.36 \pm 0.008$ kW m^{-2} for all $t \in [\tau, 0]$, which is analogous to (8) and (13). Also Propositions 2.2–2.3 can also be modified only slightly. However, in Section 5.2 we show that the expansion function $a = a(t)$ is almost linear during the last 4.5 Gyr.

The paper [89] suggests to test the existence of dark matter and DE on the scale of the Solar System (see also [40, 49]).

2.3. Analysis of growth patterns on fossil corals from solar data. The present value of the sidereal year is

$$Y = Y(0) = 31\,558\,149.54 \text{ s} \\ = 365.25636 \cdot 24 \cdot 3600 \text{ s}. \tag{18}$$

However, the length of the sidereal year in seconds in ancient times was

$$Y(t) = n(t)(24 \cdot 3600 + f(t)t), \quad t \le 0, \tag{19}$$

where $(-t)$ is the number of revolutions of the Earth about the Sun, $t = 0$ corresponds to the present time, $f = f(t) > 0$ characterizes the day length increase per year (i.e., slowdown of the Earth's rotation), and $n(t)$ is the number of days per year which is known from paleontological data by calculating the number of layers deposited during one year in fossil corals. Hundreds of patterns were examined in [90], pp. 4013–4014. One should have at least three or four consecutive years of data to reduce the error in the calculations. In particular, for the Devonian era Zhang et al. [90] found that $n(\tau) \approx 405$ days for $\tau = -370 \cdot 10^6$ years ago. A similar value of about 400 days can be found in the classic paper by Wells [85] from the seventies. Due to larger tidal forces when the Moon was closer to the Earth and the Earth closer to the Sun, the function f is decreasing. Note that tidal forces decrease cubically with distance, see [4], p. 96. According to [90], p. 4014, $f(\tau) = 2.6 \cdot 10^{-5}$ s per year, whereas the present value is

$$f(0) = 1.7 \cdot 10^{-5} \text{ s/yr}. \tag{20}$$

It was measured with respect to some fixed quasars at cosmological distances (see also Subsection 4.2 for a different approach). The Earth's rotational history (paleorotation) is examined, e.g., in [62, 86]. Substituting the above data into (19), we get (cf. (18))

$$Y(\tau) = 405(86400 - 2.6 \cdot 10^{-5} \cdot 370 \cdot 10^6) \\ = 405 \cdot 76780 = 31\,095\,900 \text{ (s)},$$

i.e., the day in the Devonian era had about 76780 seconds ($\approx$21.327 hours).

Now denote by $R(t)$ the Earth's semimajor axis at time t. For a very short time, Kepler's third law

$$\frac{R^3(t)}{Y^2(t)} = \frac{GM_\odot}{4\pi^2} \tag{21}$$

describes reality quite well. Here

$$G = 6.674 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \tag{22}$$

is the gravitational constant and

$$M_\odot = 1.989 \cdot 10^{30} \text{ kg} \tag{23}$$

is the Sun's mass which can be assumed to be constant as we shall derive in Subsection 3.4. Note that Kepler's laws are not reliable over long time periods, especially due to DE. Thus, applying (19)–(23) for $t = \tau$, we get the length of the major semiaxis of Earth's orbit in the Devonian era

$$R(\tau) = \left(\frac{Y^2(\tau)GM_\odot}{4\pi^2}\right)^{1/3} = 148.1 \cdot 10^9 \text{ m}.$$

This together with (5) yields the following mean recession speed, which has the same order of magnitude as that in (8) or Remark 2.3:

$$\overline{v} = \frac{R(\tau) - R(0)}{\tau} = \frac{(149.6 - 148.1) \cdot 10^9}{370 \cdot 10^6} \\ = 4.01 \text{ (m/yr)}.$$

In this case from (6) we get

$$H_0^{(\rm loc)} \approx 0.4 H_0. \tag{24}$$

A drawback of this method is that $\overline{v}$ is sensitive to the particular choice of $n(\tau)$ and $f(\tau)$. Therefore, Zhang et al. ([90], p. 4016), used hundreds of paleontological data from various epochs starting from the Cambrian era. They derived a somewhat higher mean recession speed $\sim 0.57 H_0$, which is again in good agreement with (8). From Fig. 4 of [90] we find that during the last 500 Myr the Earth-Sun distance increased by about 3 million km. This implies an average recession speed of 6 m/yr.

Remark 2.5. By (8) the semimajor axis $R = 1$ au of Earth's orbit increases on the average by about $\Delta R = 5.2$ m per sidereal year. However, such a small change cannot be reliably detected since the Newtonian barycenter of the Solar System travels hundreds of thousands km per year due to the influence of large planets. From Kepler's third law

$$\frac{(R + \Delta R)^3}{(Y + \Delta Y)^2} = \frac{R^3}{Y^2}$$

we can easily find that the increase of the orbital period of the Earth after one year would be only $\Delta Y = 1.6$ ms. In particular,

$$Y^2(R^3 + 3R^2 \Delta R + \cdots) = R^3(Y^2 + 2Y\Delta Y + \cdots).$$

Neglecting higher-order terms, we get by (18) and (5)

$$\Delta Y \approx \frac{3Y}{2R}\Delta R = 0.0016 \text{ s.} \tag{25}$$

Such a small time change also cannot be reliably detected, since one or two additional leap-seconds are usually added every year to compensate for the slowing of Earth's rotation. The increase of the orbital period by about $\Delta Y = 1.6$ ms would require one additional second after 35 years, since after two years we have to add $2\Delta Y$ to the orbital period, after 3 years $3\Delta Y$, etc. By properties of the triangular numbers we get

$$(1 + 2 + \cdots + 35)\Delta Y$$
$$= (1 + 35)\frac{35}{2} \cdot 0.0016 \approx 1 \text{ (s)}.$$

This makes the evidence of a slightly increasing orbital period very difficult to obtain.

2.4. Analysis of growth patterns on fossil corals from lunar data. Let $P = P(t)$ be the length of the sidereal month and $s = s(t)$ the number of sidereal months per year. At present it is $P(0) = 27.322$ days, and $s(0) = 13.368$. The number $s(t)$ is known from paleontological data for many negative t's, since s equals one plus the number of lunar months. The number of lunar months can be manually calculated from many growth patterns on coral fossils (see [85], p. 4012). Note that in the Cambrian era, the Moon was at least 20 000 km closer to the Earth than it is now, so its angular area was more than 10% larger than it is now and thus lunar patterns are better visible on fossil corals. In particular, $s(\tau) \approx 14.2$ for $\tau = -5 \cdot 10^8$ years according to [90], p. 4013.

Using the generalized Kepler's third law for the Earth-Moon system, we obtain the length of the year

$$Y(t) = s(t)P(t)$$
$$= s(t)\left((D + w(t)t)^3 \frac{4\pi^2}{G(M+m)}\right)^{1/2}, \tag{26}$$

where

$$D = 384.402 \cdot 10^6 \text{ m} \tag{27}$$

is the present mean distance between the Earth and the Moon,

$$M = 5.9736 \cdot 10^{24} \text{ kg}, \quad m = 7.349 \cdot 10^{22} \text{ kg} \tag{28}$$

are their masses, and $w(t)$ is the recession speed of the Moon from the Earth. Due to larger tidal forces when the Moon was closer to the Earth, the function w is slowly decreasing from the past to the present. By laser retroreflectors installed by the Apollo missions 11, 14, 15, and Lunokhod 2 on the Moon more than 40 years ago, it has been found that the present mean distance D between the Earth and the Moon increases at present by about

$$w(0) = 3.84 \text{ cm/yr}. \tag{29}$$

From this, (21), (26), (22), (28), and (27) we get for $t = \tau = -5 \cdot 10^8$ yr the following upper estimate:

$$R(\tau) = \left(Y^2(\tau)\frac{GM_\odot}{4\pi^2}\right)^{1/3}$$
$$= s(\tau)^{2/3}\left(\frac{M_\odot}{M+m}\right)^{1/3}(384.402 \cdot 10^6 + w(\tau)\tau)$$
$$< 14.2^{2/3}328919^{1/3}(384.402 \cdot 10^6 + w(0)\tau)$$
$$= 147.8 \cdot 10^9 \text{ (m)},$$

since the former recession speed $w(\tau)$ is larger than $w(0)$. The previous inequality yields the following guaranteed lower bound for the average recession speed of the Earth from the Sun:

$$\overline{v} = \frac{R(\tau) - R(0)}{\tau} > \frac{(149.6 - 147.8) \cdot 10^9}{5 \cdot 10^8}$$
$$= 3.6 \text{ (m/yr)}.$$

In this case we obtain from (6)

$$H_0^{(\mathrm{loc})} > 0.36 H_0. \tag{30}$$

By a thorough analysis of growth patterns on fossil corals from lunar data (which are independent of solar data), Zhang et al. ([90], pp. 4013–4016), got further values of $s(t)$ for other time epochs t, leading to the local expansion rate

$$H_0^{(\mathrm{loc})} \approx 0.57 H_0.$$

2.5. One more argument. In Subsection 4.1 we present strong arguments that Mars had to be much closer to the Sun since there were rivers in the period of 3–4 Gyr ago when the luminosity of the Sun was only 0.75 $L_\odot$. Hence, the Earth had to be closer to the Sun, since Mars was closer. On the other hand, if the Earth-Sun distance at that time would be 1 au, then the orbits of the Earth, Moon, and Mars would be unstable (which can be easily checked numerically).

3. ELIMINATION OF OTHER POSSIBILITIES FOR A LARGE RECESSION SPEED

3.1. Solar radiation. We first show that the solar radiation is not able to explain a large speed similar to (8). The cross-section area of our Earth is $S = \pi(6.378 \cdot 10^6)^2 \text{ m}^2 = 1.277964 \cdot 10^{14} \text{ m}^2$. From this and (18), the total energy coming from the Sun during one year is

$$E = SYL_0 = 5.4 \cdot 10^{24} \text{ J}, \tag{31}$$

where L_0 is given by (10).

Now denote E_i, λ_i, ν_i, and p_i to be respectively the energy, wave length, frequency, and momentum of the i-th photon. Then we have

$$p_i = \frac{h}{\lambda_i} = \frac{h\nu_i}{c} = \frac{E_i}{c},$$

where h is Planck's constant and $c \doteq 3 \cdot 10^8$ m/s is the speed of light. Summing the above equation over all photons coming to Earth from the Sun during one year, we get by (31) that

$$p = \sum_i p_i = \frac{E}{c} = \frac{5.4 \cdot 10^{24}}{3 \cdot 10^8} = 1.8 \cdot 10^{16} \text{ kg m/s}.$$

However, by (28) and (18) we find that

$$v = \frac{p}{M} = 9.5 \text{ cm/yr},$$

which is much smaller than the speed given in (8).

3.2. Tidal forces. The angular sizes of the Moon and of the Sun are almost the same. However, the Earth's rotation slows down mainly due to tidal forces of the Moon—about 69% and only 31% of the Sun, since the density of the Moon is approximately 2.3 times higher than that of the Sun, see [6]. Note that tidal forces (per 1 kg of the Earth) are equal to $2GM_\odot r/R^3$, where the mass $M_\odot$ of the Sun is given by (23), $R = 1$ au, and r is the Earth's radius. By the above arguments, the Earth-Sun distance increases by about only a few cm per year due to tidal forces (see [61] and [4], p. 606).

3.3. Decrease of the Solar mass due to nuclear reactions. One atom of helium is 0.7% lighter than 4 atoms of hydrogen. This means that at most 0.7% of the Sun's mass changes into energy during 10 Gyr (the life period of the Sun). When the Sun was born, it already contained about 30% of helium. Hydrogen changes into helium only in central parts of the Sun, and by the end of its time on the main sequence, the Sun will still contain a lot of hydrogen. Thus we may assume that only 0.07% of the Sun's mass will change into energy. In this way the Sun loses $0.0007\, M_\odot/(10^{10} \cdot Y(0)) = 4.46 \cdot 10^9$ kg per second due to (23) and (18). This is an essential part of the total mass losses collected in the next Subsection 3.4.

3.4. Plasma outbursts from the Sun. If the speed of a solar plasma outburst is larger than 613 (resp. 434) km/s, then the plasma can escape the Solar System (resp. the Sun), which reduces the Sun's mass as well. At smaller speeds the plasma falls back down to the Sun.

By Noerdlinger [60], the Sun loses every second altogether $5.75 \cdot 10^9$ kg of its mass due to the solar wind, electromagnetic radiation, neutrino losses, and large eruptions. Taking into account that mass losses during one year (see (18)) are $1.815 \cdot 10^{17}$ kg/yr, we find by (23) that (see also [25])

$$\frac{M'_\odot(t)}{M_\odot(t)} = C \ \text{ with } \ C = -9.13 \cdot 10^{-14} \text{ yr}^{-1},$$

where $M'_\odot(t)$ stands for the time derivative and $M_\odot(0) = M_\odot$ is given by (23). Since the planetary orbits expand at the same rate [60], we find by (5) that the mean recession speed of the Earth from the Sun due to the radiative and particle losses of Sun's mass is approximately $9.13 \cdot 10^{-14}$ yr$^{-1} \cdot 149.6 \cdot 10^{11}$ cm $\doteq 1.4$ cm/yr (see also [68]).

Since $M_\odot(t) = M_\odot e^{-Ct}$, changes of the Sun's mass are negligible. For instance, if $t = -370 \cdot 10^6$ yr (which corresponds to the Devonian era), we find that $M_\odot(t) = 1.989067 \cdot 10^{30}$ kg (cf. (23)) if magnetohydrodynamic effects are ignored. By [34], the Sun could have lost mass at an enhanced rate 4.3 Gyr ago.

3.5. Further influences. According to [23, 26, 66, 69], relativistic effects and dark matter also do not seem to have detectable influence on the expansion of the Solar System.

The Earth moves in the Sun's magnetic field. Since the Earth has a large iron core, circulating eddy (Foucalt) currents should appear, and thus the Earth should descend onto lower orbits. By Section 2, this is not observed. The reason is that the magnetic potential decreases as r^{-2} (whereas the gravitational potential decreases as r^{-1}). The interaction between the magnetic fields of the Earth and the Sun occurs mostly in the terrestrial magnetosphere (the plasma envelope around the Earth, whose size is of the order of 10 Earth's radii). The magnetic force again has almost no influence on secular changes of Earth's orbit due to the fast decrease r^{-2} of the magnetic potential. Moreover, the Sun reverses its polarity every 11 years, so the possible errors due to the reversal of magnetic polarity are not accumulated but canceled.

4. FURTHER TESTABLE HYPOTHESES IN THE SOLAR SYSTEM

4.1. Mars was much closer to the Sun when there were rivers. The present mean Mars–Sun distance is about $r = 225 \cdot 10^9$ m. Mars had liquid water on its surface 3–4 Gyr ago, which was deduced from the number of craters in its dry riverbeds. Neither wind nor lava can create such sinuous formations. At that time the Sun's luminosity was about 75% of its present value (see Fig. 2). Since the solar power decreases with the square of the distance from the Sun, the corresponding luminosity would be by (5) only

$$L_{\text{Mars}} = 0.75 L_0 \frac{150^2}{225^2} = \frac{L_0}{3},$$

which corresponds to a 67%-decrease of the solar constant L_0 (see (10)). If Mars were on the same orbit as it is now, the existence of rivers on its surface would be impossible. Note that a decrease of L_0 by only 2% causes ice ages on the Earth, even though there is the greenhouse effect. An ancient atmosphere on Mars 3–4 Gyr ago had one-third to two-thirds of the surface atmospheric pressure as Earth has today (for details see [17]). Higher concentration of CO_2 (as suggested by [4], p. 177) surely contributed to a higher surface temperature on Mars but cannot fully explain liquid water there because of the huge 67% decrease of the luminosity. Therefore, Mars must have been much closer to the Sun to account for liquid water. The associated mean recession speed of about 6–10 m/yr (cf. (7)) could be verified as soon as laser retroreflectors are installed on Mars.

According to Google Mars Maps, there were hundreds of large rivers whose dry riverbeds are now mainly between $-50°$ and $50°$ of Martian latitude. Due to measurements of the missions Viking I and II, Pathfinder, Spirit, etc., we know that the current annual average temperature on Mars is very much below the freezing point of water, about $-60°$C. The present overall mean surface temperature can also be estimated by

$$\mathrm{T}_{\text{equilibrium}} = \left(\frac{(1-A)L_\odot}{16\pi\sigma r^2}\right)^{1/4} = 211\ \mathrm{K}, \qquad (32)$$

neglecting the greenhouse effect, where $A = 0.25$ is the present value of the Bond albedo, $L_\odot = 3.846 \cdot 10^{26}$ W is the total Solar luminosity, and $\sigma = 5.669 \cdot 10^{-8}$ Wm^{-2} K^{-4} is the Stefan–Boltzmann constant. The above relation can be derived from the nonlinear dependence of the equilibrium temperature on the solar luminosity given by the Stefan–Boltzmann law $\sigma \mathrm{T}_{eq}^4 = L_\odot/(4\pi r^2)$, taking into account that the Martian surface is four times larger than the area of its maximum cross-section. Notice that the theoretical temperature (32) is in very good agreement with the above-mentioned measured average temperature of $-60°$C.

When the Sun's luminosity was 0.75 $L_\odot$, then by (32) we would only get $r = 117 \cdot 10^6$ km to reach the freezing point of water $\mathrm{T}_{\text{equilibrium}} = 273.15$ K (cf. Fig. 4). However, this distance is more than 100 million km smaller than the current radius $r = 225 \cdot 10^6$ km. Substituting 0.75 $L_\odot$ instead of $L_\odot$ into (32), the corresponding equilibrium surface temperature would be only 197 K for $r = 225 \cdot 10^6$ km which contradicts the observed survival of liquid water.

Moreover, the Bond albedo of Mars' surface 3–4 Gyr ago was higher than A, since there were water clouds feeding many rivers. Ice and snow were not only present in polar caps, but also in other regions, which increased the albedo, too.

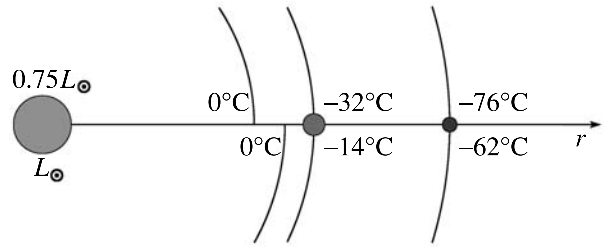

Fig. 4. The equilibrium temperature by the Stefan–Boltzmann law (32) for albedo $A = 0.25$. The temperatures in the upper part correspond to the luminosity 0.75 $L_\odot$ and in the lower part to $L_\odot$. The radii of the above circles are 117, 134, 150, and 225 million km. The Sun is on the left, Mars is on the right, and the Earth is in between.

The above arguments show that Mars must have been closer to the Sun by several tens of million km when it had liquid water. The mere erosion of its atmosphere cannot explain, e.g., the existence of an ancient ocean in the Northern hemisphere [64]. By (7) and (6) recalculated to the Mars–Sun distance, we find that Mars could move further from the Sun by an amount of at least 30 ($= 4 \cdot 5 \cdot 225/150$) million km during the last 4 Gyr. This would explain the Faint Young Sun paradox for Mars proposed in [43, 74, 75]. Secular change rates of Mars's semimajor axis are also investigated in [21] and [28] for a non-Newtonian model of gravity.

4.2. The Earth-Moon distance increases more than can be explained by tidal forces. The first observed discordance between the secular acceleration of the Moon's mean longitude utilizing Atomic Time and the Ephemeris Time scale has been reported in van Flandern [82] in 1975. By laser measurements we know that the present mean distance $D = 384\,402$ km between the Earth and the Moon increases by about 3.8 cm per year, see (29). Tidal forces can explain only 55% of this value, i.e., 2.1 cm per year as we shall observe below. This lunar orbital anomaly is usually referred to as the *Tidal Catastrophe Paradox* (see [83]). However, the remaining part

$$\delta = 0.45 \cdot 3.8 = 1.7\ \mathrm{cm/yr} \qquad (33)$$

could be due to DE that influences the Hubble parameter.

In [90], p. 4016, a similar averaged value $\delta \approx 1.6$ cm/yr during the last 500 Myr is independently obtained by measurements of growth patterns on fossil corals. This method uses geochronometrical techniques introduced in Wells [85].

The large value in (33) is derived from the following facts. The angular frequency of the Earth is

$\omega_1 = 2\pi/Y = 7.292 \cdot 10^{-5}$ s^{-1}. The Earth's rotation slows down mainly due to tidal forces of the Moon (cca 68.5%), see Subsection 3.2. By a thorough analysis of the Ancient Babylonians' records of solar eclipses [76] we know that the length of a day has increased on average by $1.7 \cdot 10^{-5}$ s per year during the last 2700 years (see (20)).

For example, one late Babylonian astronomical tablet containing a record of the total solar eclipse on 15th April in 136 BC is preserved in the British Museum (see [80]). It was found that the rotation of the Earth slowed down about $\Delta T = 4$ hours more than if it would have rotated uniformly. At that time a day was shorter than in the year 2000. This period contains altogether $N \approx 780\,000$ days. Assume for simplicity that the length of every day increases linearly by about the value Δt, i.e., the nth day is about $n\Delta t$ longer than the day of the eclipse. Thus, for the total delay we obtain

$$\Delta T = \Delta t \sum_{n=1}^{N} n = \Delta t \frac{N(N+1)}{2} = 4 \cdot 3600 \text{ s},$$

which yields $\Delta t = 4.734 \cdot 10^{-8}$ s, and after one year $365.25 \cdot \Delta t = 1.7 \cdot 10^{-5}$ s.

This value (see also (20)) is consistent with data measured by the Lageos satellite (see [9, 88]), and it implies that $\omega_1' = -4.56 \cdot 10^{-22}$ s^{-2}. However, in the literature we can find a larger increase of the day of $2.4 \cdot 10^{-5}$ s/yr as well as a smaller increase of $0.9 \cdot 10^{-5}$ s/yr (see, e.g. [78]). So the value (20) is close to the average of these upper and lower limits. The slowdown of Earth's rotation as given in (20) thus represents only some mean value over long time intervals.

By the conservation of the total momentum $\mathcal{M}$ of the Earth-Moon system, the value

$$\mathcal{M} = I_1\omega_1 + I_2\omega_2 + m_1R_1v_1 + m_2R_2v_2$$

must be constant. Here $I_1 = 8.036 \cdot 10^{37}$ kg m^2 and I_2 are the inertial moments of the Earth and Moon, ω_1 and ω_2 are their angular frequencies, $m_1 = M$ and $m_2 = m$ (see (28)), v_1 and v_2 are the speeds of the Earth and Moon, respectively, relative to their center of gravity, and the corresponding distances satisfy $D = R_1 + R_2$.

First assume that the moments I_i are independent of time. Since the decrease of the Moon's angular momentum is negligible, we can derive from (20) that $\mathrm{d}D/\mathrm{d}t = 0.674 \cdot 10^{-9}$ m/s (for a detailed calculation see [49], pp. 1034–1937). However, the observed value corresponding to the real recession speed of 3.8 cm/yr is much higher, namely, $\mathrm{d}D/\mathrm{d}t = 1.2 \cdot 10^{-9}$ m/s. Putting these values together, we find that 1.7 cm/yr $\approx 3.8(1.2-0.674)/1.2$ cm/yr, which is the speed given in (33).

Rescaling H_0 to the Earth-Moon distance D, we easily get by (6), (5), and (27) that $H_0 = 2.57$ cm yr$^{-1}D^{-1}$, and thus for the expansion of the Earth-Moon system we obtain by (33),

$$H_0^{(\mathrm{loc})} \approx \frac{1.7H_0}{2.57} = 0.66H_0. \tag{34}$$

Dumin in [10], p. 2463, derives a very similar value to (34), namely, $H_0^{(\mathrm{loc})} \approx 0.5H_0$ (and $H_0^{(\mathrm{loc})} \approx 0.85H_0$ in [11]). He also shows that Lambda-perturbations of Keplerian orbits enable us to reach the rate of the standard Hubble flow (see [13]).

A time-variable inertial moment $I_1 = I_1(t)$ was observed, e.g., in [6, 9, 88]. Its derivative $I_1' \approx -10^{20}$ kg m^2 s^{-1} indicates that a transport of mass toward the Earth's center has existed at least for 2700 years since the ancient Babylonian observations. However, such a large transport of mass cannot be explained by a simple process such as melting of glaciers, el Nino, or internal processes, since the magnitude of I_1' is too large.

4.3. Neptune was formed closer to the Sun than it is now. It is an open problem how Neptune could be formed as far away as $r = 30$ au from the Sun, where the original protoplanetary disc was very sparse and where all motions are very slow [4]. By Kepler's third law, its mean velocity is only $\sqrt{GM_\odot/r} = 5.43$ km/s.

Standish in [79] observed a small anomalous delay in Neptune's position. Subsequent searches for Planet X have been unsuccessful, but DE can again explain this paradox.

Assuming a similar expansion rate as in (7), Neptune could be formed much closer to the Sun than it is now. Indeed, similarly to (17), if $r = r_0 \exp(\frac{1}{2}H_0t)$, where r_0 stands for the initial radius of Neptune's orbit, then for $t = 4.5$ Gyr we get

$$r_0 = r \exp\Big(-\frac{1}{2}H_0t\Big) = r\mathrm{e}^{-0.15} = 25.82 \text{ au}.$$

For a linear expansion rate we get by (7) an analogous value,

$$r_0 = r - \frac{1}{2}H_0tr$$

$$= 30 \text{ au} - 5 \text{ m/(yr au)} \cdot 4.5 \cdot 10^9 \text{ yr} \cdot 30 \text{ au} = 25.5 \text{ au}.$$

The increase of the distance between the Sun and Neptune could also be due to various resonances, but DE could also play an essential role in this process.

Similarly to (25), we obtain that

$$\Delta P \approx \frac{3P}{2r}\Delta r,$$

where $P = 164.79$ yr is the orbital period of Neptune around the Sun. Thus after one period P Neptune will be delayed by the angle α for which

$$\tan\alpha \approx \frac{\Delta P}{r}\frac{2\pi r}{P} = \frac{2\pi\Delta P}{P} = \frac{3\pi\Delta r}{r}.$$

From this and (8) we find that

$$\alpha \approx 3\pi P \cdot 5.2\ (\mathrm{m/yr})/\mathrm{au} = 0.01''. \qquad (35)$$

Note that such small anomalous unexplained delays on the order of several milliarcseconds per century have already been observed [79].

4.4. Fast satellites. In the Solar System we know 19 satellites of Mars, Jupiter, Uranus, and Neptune that are below the corresponding stationary orbit with radius (cf. Kepler's third law (21))

$$r_i = \Big(\frac{Gm_iP_i^2}{4\pi^2}\Big)^{1/3}, \qquad (36)$$

where m_i is the mass of the ith planet and P_i is its sidereal rotation. We call them *fast*, since their orbital period is smaller than P_i. From a statistical point of view, it is very unlikely that all these satellites would be captured, since all of them move in the same direction on circular orbits with almost zero inclination. Therefore, they have been mostly in their orbits approximately 4.5 Gyr even though some may be parts of larger disintegrating satellites.

By Newtonian mechanics, the tidal bulges continuously reduce the potential energy and orbital periods of these fast satellites to keep the total orbital momentum constant. Due to tidal forces they should approach their mother planets along spiral trajectories. Assuming their approaching speed of 1–2 cm per year (see [49, 50] for details), we find that they should be 45 000–90 000 km closer to their mother planets during the 4.5 Gyr of their existence. However, this contradicts the fact that the radii of the respective stationary orbits of Uranus or Neptune are $r_7 = 82\ 675$ km and $r_8 = 83\ 496$ km. For the time being, their fast satellites are on very high orbits with radii circa 50 000–76 000 km. Moreover, by (36) the radii of stationary orbits were smaller in the past (cf. [4], p. 440), since the rotations of the planets were faster.

It is again DE which acts in the opposite direction than gravity and thus protects the fast satellites against crashing onto their mother planet. The same argument applies for the retrograde moon Triton whose distance from Neptune is quite large: 354 760 km.

4.5. Saturn's rings. Saturn's rings have probably existed for several billion years. Rings B, C, and D are below the stationary orbit of Saturn. Ring B is composed of bodies up to 10 m in diameter subject to tidal forces. Moreover, small collisions of bodies continuously reduce the energy of Saturn's rings, and thus they should be slowly approaching Saturn, which is not observed. DE which acts in the opposite direction can again easily explain this mystery which takes place also at the rings of Jupiter, Uranus, and Neptune.

4.6. Why Mercury and Venus have no moons. Since the Earth probably was 25 million km closer to the Sun 4.5 Gyr ago due to (8), leading to a distance of 125 million km from the Sun, Venus (whose present mean distance from the Sun is 108 million km) also had to be closer to the Sun. Otherwise their orbits would be unstable. Mercury and Venus have no moons since the corresponding lunar orbits would be unstable due to tidal forces when they were closer to the Sun. Moreover, from (36) we see that their stationary orbits are too large since the P_i are relatively large numbers.

4.7. Kuiper belt. There are further arguments for the influence of DE in the Solar System. According to [4], p. 534, there is strong evidence that the Kuiper belt of comets (at 30 to 50 au from the Sun) had been formed much closer to the Sun in a region with larger velocities. Relation (7) can explain a shift of at least 10 au during the last 4.5 Gyr due to DE. A similar argumentation applies to Sedna-like bodies and the Oort cloud whose origins are not easily understood, see [24, 29, 32, 37, 56, 81].

4.8. Large orbital momentum of our Moon. A paradoxically very large orbital momentum of the Earth-Moon system (see [46] and [4], p. 534) can also be explained by DE which causes an additional shift (33) in the recession speed of the Moon from the Earth that is not due to tidal forces.

An anomalous increase in the eccentricity of the Moon's orbit is presented in [2, 30, 31]. For lunar rotational dissipation see [87].

4.9. Further footprints. The DE influence over longer duration on the order of the lifetime of the Sun left further footprints in the Solar System. They are recorded in the physical characteristics of planets. For instance, the rotation of Mercury is very slow (59 days) due to larger tidal forces when the planet was closer to the Sun. We know that tidal forces decrease cubically with distance. Thus, if Mercury were, e.g., 10 million km closer to the Sun 4.5 Gyr ago, then the tidal forces would be twice as large as today. This could essentially slow down Mercury's rotation.

If Mars were much closer to the Sun than it is now, then Jupiter was also closer to the Sun. Otherwise Mars would have a larger mass. It has only 0.1 of Earth's mass since Jupiter captured its constituent material.

5. WHY OTHER AUTHORS OBTAINED MUCH SMALLER VALUES OF RECESSION SPEEDS?

There are some discrepancies between our results and those of other authors. Now we will explain why.

5.1. Classical Newtonian theory of gravitation. Krasinsky and Brumberg [47] derived that the present recession speed of the Earth from the Sun is equal to $v = 15$ cm/yr. Their calculation is based on the assumption that Newtonian gravitation describes all motions in the Solar System absolutely exactly. They solve an algebraic system for 62 unknown Keplerian parameters of all planets and some large asteroids and implicitly assume that modelling, discretization, and rounding errors are negligible. However, the classical Newtonian theory assumes an infinite speed of gravitational interaction, whereas the real speed is surely finite. Hence, the modelling error is not zero. The existence of DE was not taken into account (see also [67]).

5.2. Almost linear expansion function. Cooperstock et al. [8] and many others derive a tiny outward acceleration of the Earth of $3.17 \cdot 10^{-47}$ m/s^2, but the large present value of the Hubble parameter itself $H_0 = H(0) = 10$ m yr^{-1} au^{-1} (see (6)) is not taken into account. The derivative $H'(0)$ is, of course, extremely small. Carrera and Giulini in [7], p. 175, derived that the acceleration of the Universe is negligible on the scales of the Solar System. However, they also did not consider the large value of the linear expansion rate as given in (7) and (8). Their calculation is based on a very small effect of the present value of the deceleration parameter $q_0 = -0.6$ (cf. Fig. 1 and [70]) which appears in the Taylor expansion of the scale factor

$$\begin{aligned} a(t) &= a(0) + a'(0)t + \frac{1}{2}a''(0)t^2 + \cdots \\ &= a(0)(1 + H_0 t - \frac{1}{2}q_0 H_0^2 t^2 + \ldots). \end{aligned} \tag{37}$$

They ignore the large value of H_0 occurring in (6) which appears as the linear term in (37) and concentrate only on the purely quadratic term.

The accelerated expansion of the Universe is absolutely negligible on the scale of the Solar System, but the linear expansion itself is essential (see Fig. 5) since

$$|H_0 t| \gg \frac{1}{2}|q_0| H_0^2 t^2 \tag{38}$$

for t close to 0. Consequently, the accelerated expansion only appears at cosmological distances due to (38). In spite of that, the single quadratic term is so small that the linear term essentially dominates not only in the neighborhood of 0, but for all t in the whole interval $(-1/H_0, 0)$ since we have

$$0.3 \cdot |H_0 t| > \frac{1}{2}|q_0| H_0^2 t^2,$$

where $\frac{1}{2}|q_0| = 0.3$.

6. DISCUSSION

The discussion of the effect of cosmological expansion on local systems (such as the Solar System) has a long history dating back to [58]. In the present paper, we have also shown that DE acts not only on large scales but also on small scales. It essentially contributes to the migration of planets and their moons. DE also causes many star clusters to dissipate and helps to reduce the frequency of collisions of stars and galaxies, see [48, 51]. It has also created suitable habitable conditions on the Earth for several billion years.

To demonstrate the influence of DE in the Solar System, we must either measure very precisely (e.g. the Earth–Moon distance) or consider very long time intervals where all small deviations from Newtonian mechanics are, in general, not cancelled but accumulated and then possibly observed. *An extremely small deviation $\varepsilon > 0$ per year may cause, after one billion years, quite a large and detectable value of order $10^9\varepsilon$ which could be then attributed to DE.*

Modified theories of gravity allowing secular changes of the planetary semimajor axes are examined in many papers (see [22, 36, 40, 54, 89] etc.). For instance, Iorio in [22] examines the impact of a spherically symmetric distribution of dark matter (DM) in our Galaxy on planetary orbits. On the other hand, in [52] we list several effects that essentially reduce the amount of DM.

Note that there exist close binary pulsars whose orbits do not expand with time, but decay. In this case, strong magnetic and gravitational fields are present, and the system loses energy due to electromagnetic and gravitational waves. These effects are much stronger than the weak effects coming from antigravity forces due to DE. Also various resonances may be significantly larger as compared with tiny DE effects (see, e.g., (33) and (35)).

Iorio [39] investigates local secular expansion of a two-body orbit due to the acceleration rate of cosmological expansion. In [63], the authors discuss whether it is possible to directly detect DE on Earth using atomic interferometry through a force hypothetically caused by a gradient in DE density. Possible effects of MOND in the Solar System are discussed in [18–20, 29, 38]. On the other hand, Blanchet and Novak [5] claim that MOND violates

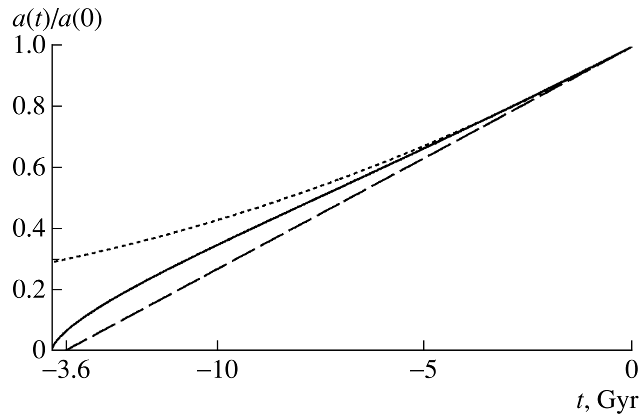

Fig. 5. The assumed behavior of the normalized expansion function scale factor $a(t)/a(0)$. The values on the horizontal axis are given in Gyr. The quantities on the vertical axis are relative with no physical dimensions. The lower plot corresponds to the linear function $1 + H_0t$ from (37) on the interval $[-1/H_0, 0]$, where $1/H_0 =$ 13.6 Gyr is the Hubble time. The upper plot shows the quadratic function $1 + H_0t - \frac{1}{2}q_0H_0^2t^2$ with $q_0 = -0.6$. The middle graph illustrates the normalized scale factor as given in (4). We observe that accelerated expansion differs little from linear expansion during the last few Gyr (see (38)).

the strong equivalence principle known from general relativity.

In [48] and [51] we examined the unknown source of DE that is needed for the accelerated expansion of the Universe. We claim that it may partly come from the finite speed of gravitational influence that causes gravitational aberration, which is much smaller than the aberration of light, but positive due to causality [49]. It increases with distance. The amount of DE that is generated by the Moon and the Earth is estimated in [50], pp. 3 and 5. They are on spiraling orbits which slightly move further and further apart.

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REFERENCES

1. G. S. Adkins and J. McDonnell, "Orbital precession due to central-force perturbations," Phys. Rev. D **75**, 082001 (2007).
2. J. D. Anderson and M. M. Nieto, "Astrometric Solar-System anomalies," in: *Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis*, Proc. IAU Sympos. **261**, 189–197 (2010).
3. H. Arakida, "Note on the perihelion/periastron advance due to cosmological constant," Int. J. Theor. Phys. **52**, 1408–1414 (2013).
4. B. Bertotti, P. Farinella, and D. Vokrouhlický, "Physics of the Solar System" (Kluwer, Dordrecht, 2003).
5. L. Blanchet and J. Novak, "External field effect of modified Newtonian dynamics in the Solar system," Mon. Not. Roy. Astron. Soc. **412**, 2530–2542 (2011).
6. M. Burša and K. Peč, *Gravity Field and Dynamics of the Earth* (Springer, Berlin, 1993).
7. M. Carrera and D. Giulini, "Influence of global cosmological expansion on local dynamics and kinematics," Rev. Mod. Phys. **82**, 169–208 (2010).
8. F. I. Cooperstock, V. Faraoni, and D. N. Vollick, "The influence of the cosmological expansion on local systems," Astrophys. J. **503**, 61–66 (1998).
9. C. M. Cox and B. F. Chao, "Detection of large-scale mass redistribution in the terrestrial system since 1998," Science **297**, 831–833 (2002).
10. Yu. V. Dumin, "A new application of the Lunar laser retroreflectors: Searching for the "local" Hubble expansion," Adv. Space Res. **31**, 2461–2466 (2003).
11. Yu. V. Dumin, "Testing the 'dark-energy'-dominated cosmology via the Solar-System experiments," astro-ph/0507381.
12. Yu. V. Dumin, "Testing the dark-energy-dominated cosmology by the Solar-System experiments," *Proc. of the 11th Marcel Grossmann Meeting on General Relativity*, Ed. by H. Kleinert, R. T. Jantzen, and R. Ruffini (World Sci., Singapore, 1752–1754, 2008), ArXiv: 0808.1302.
13. Yu. V. Dumin, "Lambda-perturbations of Keplerian orbits," ArXiv: 1206.0306.
14. G. Feulner, "The faint young Sun problem," Rev. Geophys. **50**, RG2006, 1–29 (2012).
15. C. Goldblatt and K. J. Zahnle, "Faint young Sun paradox remains," ArXiv: 1105.5425.
16. R. S. Gomes, J. J. Matese, and J. J. Lissauer, "A distant planetary-mass solar companion may have produced distant detached objects," Icarus **184**, 589–601 (2006).
17. W. K. Hartmann, *Mars* (Workman Publ., New York, 2003).
18. A. Hees, W. M. Folkner, R. A. Jacobson, and R. S. Park, "Constraints on MOND theory from radio tracking data of the Cassini spacecraft," ArXiv: 1402.6950.
19. A. Hees et al., "Tests of Gravitation at Solar System scales beyond the PPN formalism," ArXiv: 1403.1365.
20. A. Hees et al., "Radioscience simulations in general relativity and in alternative theories of gravity," Class. Quantum Grav. **29**, 235027 (2012).

21. L. Iorio, "A note on the evidence of the gravitomagnetic field of Mars," Class. Quantum Grav. **23**, 5451–5454 (2006).
22. L. Iorio, "Solar system planetary orbital motions and dark matter," JCAP **5**, 002 (2006).
23. L. Iorio, "Can Solar System observations tell us something about the cosmological constant?," Int. J. Mod. Phys. D **15**, 473–475 (2006).
24. L. Iorio, "Dynamical determination of the mass of the Kuiper belt from motions of the inner planets of the Solar system," Mon. Not. Roy. Astron. Soc. **375**, 1311–1314 (2007).
25. L. Iorio, "Solar System motions and the cosmological constant: A new approach," Adv. Astron., article id. 268647 (2008).
26. L. Iorio, "Classical and relativistic orbital motions around a mass-varying body," SRX Physics, article id. 261249 (2010)
27. L. Iorio, "Orbital effects of Sun's mass loss and the Earth's fate," Natur. Sci. **2**, 329–337 (2010).
28. L. Iorio, "On the Lense-Thirring test with the Mars Global Surveyor in the gravitational field of Mars," Cent. Eur. J. Phys. **8**, 509–513 (2010).
29. L. Iorio, "The perihelion precession of Saturn, planet X/Nemesis and MOND," Open Astron. J. **3**, 1–6 (2010).
30. L. Iorio, "An empirical explanation of the anomalous increases in the Astronomical unit and the lunar eccentricity", Astron. J. **142**, 68 (2011).
31. L. Iorio, "On the anomalous secular increase of the eccentricity of the orbit of the Moon," Mon. Not. R. Astron. Soc. **415**, 1266–1275 (2011).
32. L. Iorio, "Constraints on the location of a putative distant massive body in the Solar System from recent planetary data," Cel. Mech. Dyn. Astron. **112**, 117–130 (2012).
33. L. Iorio, "Perspectives on effectively constraining the location of a massive trans-Plutonian object with the New Horizons spacecraft: a sensitivity analysis," Cel. Mech. Dyn. Astron. **116**, 357–366 (2013).
34. L. Iorio, "A closer Earth and the faint young Sun paradox: Modification of the laws of gravitation or Sun/Earth mass losses?," Galaxies **1**, 192–209 (2013).
35. L. Iorio, "Local cosmological effects of the order of H in the orbital motion of a binary system," Mon. Not. R. Astron. Soc. **429**, 915–922 (2013).
36. L. Iorio, "Exact expressions for the pericenter precession caused by some dark matter distributions and constraints on them from orbital motions in the Solar System, in the Double Pulsar and in the Galactic Center," Galaxies **1**, 6–30 (2013).
37. L. Iorio, "Perspectives on effectively constraining the location of a massive trans-Plutonian object with the New Horizons spacecraft: a sensitivity analysis," Cel. Mech. Dyn. Astron. **116**, 357–366 (2013).
38. L. Iorio, "Constraints on a MOND effect for isolated aspherical systems in the deep Newtonian regime from orbital motions," Class. Quantum Grav. **30**, 165018 (2013).
39. L. Iorio, "Two-body orbit expansion due to time-dependent relative acceleration rate of the cosmological scale factor," Galaxies **2**, 13–21 (2014).
40. L. Iorio, "Modified theories of gravity with nonminimal coupling and orbital particle dynamics," Class. Quant. Gravity **31**, 085003 (2014).
41. L. Iorio and E. N. Saridakis, "Solar system constraints on f(T) gravity," Mon. Not. R. Astron. Soc. **427**, 1555–1561 (2012).
42. P. Jetzer and M. Sereno, "Two-body problem with the cosmological constant and observational constraints," Phys. Rev. D **73**, 044015 (2006).
43. J. S. Kargel, "Mars—a Warmer, Wetter Planet," (Springer, Praxis Books, 2004).
44. A. W. Kerr, J. C. Hauck, and B. Mashhoon, "Standard clocks, orbital precession and the cosmological constant," Class. Quantum Gravity **20**, 2727–2736 (2003).
45. H. Kienert, G. Feulner, and V. Petoukhov, "Faint young Sun problem more severe due to ice-albedo feedback and higher rotation rate of the early Earth," Geophys. Res. Lett. **39**, L23710 (2012).
46. E. Kokubo, S. Ida, and J. Makino, "Evolution of a circumterrestrial disk and formation of a single Moon," Icarus **148**, 419–436 (2000).
47. G. A. Krasinsky and V. A. Brumberg, "Secular increase of astronomical unit from analysis of the major planet motions, and its interpretation," Celest. Mech. Dyn. Astron. **90**, 267–288 (2004).
48. M. Křížek, "Numerical experience with the finite speed of gravitational interaction," Math. Comput. Simulation **50**, 237–245 (1999).
49. M. Křížek, "Does gravitational aberration contribute to the accelerated expansion of the Universe?," Comm. Comput. Phys. **5**, 1030–1044 (2009).
50. M. Křížek, "Dark energy and anthropic principle," New Astron. **17**, 1–7 (2012).
51. M. Křížek, J. Brandts, and L. Somer, "Is gravitational aberration responsible for the origin of dark energy?," in: *Dark Energy: Theory, Implications and Roles in Cosmology*, Ed. by C. A. Del Valle and D. F. Longoria (Nova Sci. Publishers, Inc., New York, 2012), pp. 29–57.
52. M. Křížek, F. Křížek, and L. Somer, "Which effects of galaxy clusters can reduce the amount of dark matter," Bulg. Astron. J. **21**, 1–23 (2014).
53. K. L. Lang, *Cambridge Encyclopedia of the Sun* (Cambridge Univ. Press, Cambridge, 2001).
54. L.-S. Li, "Secular influence of Solar dark-matter accretion upon the evolution of orbits of planets," Publ. Astron. Soc. Japan **65**, 107 (2013).
55. C. H. Linewaver and D. Schartzmann, "Cosmic thermobiology," in: *Origins*, Ed. by J. Seckbach (Kluwer, Dordrecht, 2003), pp. 233-248.
56. P. S. Lykawka, "Trans-Neptunian objects as natural probes to the unknown Solar System," Monographs on Environment, Earth and Planets **1**, 121–186 (2012).

57. V. Marra, L. Amendola, I. Sawicki, and W. Valkenburg, "Cosmic variance and the measurement of the local Hubble parameter," Phys. Rev. Lett. **110**, 241305 (2013).
58. C. G. McVittie, "The mass-particle in expanding universe," Mon. Not. R. Astron. Soc. **93**, 325–339 (1933).
59. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (23rd edition, W. H. Freeman and Company, New York, 2000).
60. P. D. Noerdlinger, "Solar mass loss, the astronomical unit, and the scale of the Solar system," ArXiv: 0801.3807.
61. O. Novotný, *Motions, Gravity Field and Figure of the Earth* (Lecture Notes, Univ. Federal da Bahia, Salvador, 1998).
62. G. Pannella, "Paleontological evidence on the Earth's rotation history since early precambrian," Astrophys. Space Sci. **16**, 212–237 (1972).
63. M. L. Perl and H. Mueller, "Exploring the possibility of detecting dark energy in a terrestrial experiment using atom interferometry," ArXiv: 1001.4061.
64. J. T. Perron et al., "Evidence for an ancient Martian ocean in the topography of deformed shorelines," Nature **447**, 840–843 (2007).
65. S. V. Pilipenko, "Paper-and-pencil cosmological calculator," ArXiv: 1303.5961.
66. N. P. Pitjev and E. V. Pitjeva, "Constraints on dark matter in the Solar System," Astron. Lett. **39**, 141–149 (2013).
67. E. V. Pitjeva, "EPM ephemerides and relativity," in: *Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, 2009, Proc. IAU S261* (Cambridge Univ. Press, 2010), pp. 170–178.
68. E. V. Pitjeva and N. P. Pitjev, "Changes in the Sun's mass and gravitational constant estimated using modern observations of planets and spacecraft," Solar System Res. **46**, 78–87 (2012).
69. E. V. Pitjeva and N. P. Pitjev, "Relativistic effects and dark matter in the Solar system from observations of planets and spacecrafts," Mon. Not. R. Astron. Soc. (2013).
70. Planck Collaboration, "Planck 2013 results. XVI. Cosmological parameters," ArXiv: 1303.5076.
71. P. Rosenblatt, "The origin of the Martian moons revisited," Astron. Astrophys. Rev. **19**, #44 (2011).
72. I. J. Sackmann, I. A. Boothroyd, and K. E. Kraemer, "Our Sun. III. Present and future," Astrophys. J. **418**, 457–468 (1993).
73. C. Sagan, "Reducing greenhouses and the temperature history of Earth and Mars," Nature **269**, 224–226 (1977).
74. C. Sagan and G. Mullen, "Earth and Mars: Evolution of atmospheres and surface temperatures," Science **177**, 52–56 (1972).
75. C. Sagan, O. B. Toon, and P. J. Gierasch, "Climatic change on Mars," Science **181**, 1045–1049 (1973).
76. S. S. Said and F. R. Stephenson, "Solar and lunar eclipse measurements by medieval Muslim astronomers," J. Hist. Astronom. **27**, 259–273 (1996).
77. X. Shi and M. S. Turner, "Expectations for the difference between local and global measurements of the Hubble constant," Astrophys. J. **493**, 519–522 (1998).
78. N. S. Sidorenkov, *Physics of the Earth's Rotation Instabilities* (Nauka-Fizmatlit, Moscow, 2002) [in Russian].
79. E. M. Standish, "Planet X: no dynamical evidence in the optical observations," Astron. J. **105**, 2000–2006 (1993).
80. F. R. Stephenson, "Historical eclipses and Earth's rotation," Astron. & Geophys. **44**, 22–27 (2003).
81. Ch. A. Trujillo and S. S. Sheppard, "A Sedna-like body with a perihelion of 80 astronomical units," Nature **507**, 471–474 (2014).
82. T. C. van Flandern, "A determination of the rate of change of G," Mon. Not. R. Astron. Soc. **170**, 333–342 (1975).
83. F. Verbund, "The Earth and Moon: from Halley to lunar ranging and shells," preprint, Utrecht Univ. (2002).
84. L. Verde, R. Jimenez, and S. Feeney, "The importance of local measurements for cosmology," ArXiv: 1303.5341.
85. J. W. Wells, "Coral growth and geochronometry," Nature **197**, 948–950 (1963).
86. G. E. Williams, "Geological constraints on the Precambrian history of Earth's rotation and the Moon's orbit," Rev. Geophys. **38**, 37–60 (2000).
87. J. G. Williams, D. H. Boggs, C. F. Yoder, J. T. Ratcliff, and J. O. Dickey, "Lunar rotational dissipation in solid body and molten core," J. Geophys. Res. **106**, 27933–27968 (2001).
88. C. F. Yoder et al., "Secular variation of Earth's gravitational harmonic J_2 coefficient from Lageos and nontidal acceleration of Earth rotation," Nature **303**, 757–762 (1983).
89. W. J. Zhang and N. Kelley, "Multiplicative-generated dark matter accelerated cosmic expansion," Adv. Sci. Lett. **4**, 574–576 (2011).
90. W. J. Zhang, Z. B. Li, and Y. Lei, "Experimental measurements of growth patterns on fossil corals: Secular variation in ancient Earth-Sun distance," Chinese Sci. Bull. **55**, 4010–4017 (2010).