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NEGLECTED GRAVITATIONAL REDSHIFT IN DETECTIONS OF GRAVITATIONAL WAVES

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Abstract: In 2016, the letter [1] about the first detection of gravitational waves was published. They were generated by two merging black holes that had approximately 36 and 29 Sun's masses. However, the authors have not taken into account a large gravitational redshift of this binary system, which is a direct consequence of time dilation in a strong gravitational field. Thus the proposed masses are overestimated. In our paper we also give other arguments for this statement.

Keywords: gravitational redshift, time dilatation, black holes, wavelets

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1. Introduction

A century ago Albert Einstein [13], [14] predicted the existence of gravitational waves. He assumed only very weak gravitational fields which enabled him to linearize his field equations of general relativity. He considered only small perturbations of Minkowski spacetime and after some further simplifications he got a nonhomogeneous partial differential equation with the d'Alembert operator for plane gravitational waves (see e.g. [6, p. 24]).

Gravitational waves were first detected on September 14, 2015. According to [1], two black holes with masses

$$m_1 = 36^{+5}_{-4} M_\odot \quad \text{and} \quad m_2 = 29^{+4}_{-4} M_\odot, \tag{1}$$

merged and the generated gravitational waves GW150914 were independently intercepted by two LIGO detectors. We show that these masses are not too trustworthy. First of all, we would like to emphasize that our criticism does not concern the

LIGO detectors themselves, which are highly sophisticated and remarkable instruments. It concerns the methodology that was used to process and then interpret the measured data. The used post-Newtonian model, which neglects the infinite gravitational redshift of each single black hole, can only very roughly approximate reality. Therefore, from this heuristic model we cannot make any definite conclusion about the real masses of the considered binary black hole system and derive any reliable error estimates. In our opinion, the post-Newtonian model is not applicable in this case.

In Section 2, we introduce several drawbacks of the used formula for the so-called chirp mass. In Section 3, we show that the masses (1) including also the associated error bars need not correspond to reality, because the gravitational redshift was neglected. Section 4 contains further arguments supporting our hypothesis that the masses (1) are overestimated. We also present some conclusions in Section 5.

2. Emitted versus detected frequencies

The only relation which is given on a single line in [1] reads:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G} \Big[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \Big]^{3/5}, \tag{2}$$

where $f = f(t)$ and $\dot{f} = \dot{f}(t)$ are the frequency of gravitational waves in time t and its time derivative, m_1 and m_2 are the masses of the components of the binary system for low frequencies, $\mathcal{M}$ is the chirp mass in the detector frame, $G = 6.674 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$ is the gravitational constant, and $c = 299\,792\,458$ m/s is the speed of light in vacuum.

According to [1, p. 3], f is the detected frequency. However, then the formula (2) cannot be true, in general, since f on its right-hand side essentially depends on the total redshift z, whereas the left-hand side of (2) is independent of z. The masses m_1 and m_2 cannot depend on z. Therefore, (2) can be valid only for $z = 0$. Let us emphasize that formula (2) was taken from reference [5]. However, in [5, p. 3516] and [12, p. 2663], the authors rightly consider the emitted frequency f_{em} (or the orbital frequency πf_{em}) in the source frame and not the detected frequency f as in [1]. Therefore, the detected frequency f should be replaced by the emitted frequency

$$f_{\mathrm{em}} = (z + 1) f, \tag{3}$$

where z is the total redshift of gravitational waves.

The masses m_1 and m_2 were estimated by a series of numerical simulations [17] with various input values m_1 and m_2. The numerical methods employed in constructing the templates for the merging black holes can also be found in [9] and [19]. However, no gravitational redshift is mentioned in these references.

Let us look at the equality (2) in more detail. This equality was derived from the relation (3) of the 1995 paper [5] in a manner such that many higher order terms were neglected. In spite of that, the authors of [1] kept the equality sign in (2).

Furthermore, notice that the left-hand side of quality (2) does not depend on time t, while the right-hand side is time dependent. Thus the equality (2) is satisfied if and only if the product $f^{-11/3}\dot{f}$ is a suitable constant C (e.g. $C = 0.00015142\ldots\ \mathrm{s}^{5/3}$ for (1)). This leads to the solution of an ordinary differential equation of the first order

$$\dot{f} = Cf^{11/3} \tag{4}$$

whose general solution is

$$f(t) = \left(\frac{3}{8}\right)^{3/8} \frac{1}{(K - Ct)^{3/8}}, \tag{5}$$

where $K \in (-\infty, \infty)$ is an arbitrary integration constant, see also [12, p. 2663]. By [20, p. 14] the function (5) is the only solution of the differential equation (4) for $t < K/C$. Since this equation is autonomous, we may choose $K = 0$. The relation (5) thus determines how the frequency increases with time.

The authors of [1] had only approximate values of the detected frequency f from the interval 35–250 Hz for approximately 8 orbital periods. Finally note that a numerical computation of the derivatives from smoothed data is an ill-conditioned problem.

3. Neglected gravitational redshift

Unfortunately, in [1] (also in [2], [3], [4], [5], [16]) there is no mention about gravitational redshift of gravitational waves. Let us recall that redshift (or blueshift) of frequency of waves coming to us from the universe has three basic components:

1) a Doppler component caused by the movement of the source or the observer with respect to its neighborhood,

2) a cosmological component caused by the expansion of the universe,

3) a gravitational component caused by the change of frequency of waves in a gravitational field.

According to [1, p. 7], the luminosity distance of the considered binary system is 410^{+160}_{-180} Mpc which is in perfect agreement with the cosmological redshift (see e.g. [21])

$$z = 0.09^{+0.03}_{-0.04}, \tag{6}$$

that is stated by the authors. Thus for the two remaining components of the redshift it remains

$$z \approx 0. \tag{7}$$

Let us note that the gravitational redshift for the surface of a neutron star is $z \approx 0.3$ which is greater than (6), which can be derived from (10) below. For the horizon of a single black hole with mass m with Schwarzschild radius

$$r = \frac{2Gm}{c^2} \tag{8}$$

we even have

$$z = \infty. \tag{9}$$

From this and the relation (7) it follows that the authors of [1] did not consider a large gravitational redshift caused by the binary black hole system.

The gravitational redshift is a direct consequence of Einstein's time dilatation. Time in a gravitational potential hole flows more slowly than outside. A photon has to spend some energy to leave a gravitational field of a mass object. Its frequency is indirectly proportional to the speed of flowing of time. Therefore, the frequency of electromagnetic waves decreases when leaving a large gravitational potential hole of the binary system. A similar phenomenon holds for gravitational waves that carry away energy and thus their frequency will decrease as well.

So let us recall the well-known formula that can be derived from the Schwarzschild solution of Einstein's equations. It expresses the change of frequency of a photon leaving the gravitational field of a single black hole at the distance $R > r$ from its center

$$f = f_{\rm em}\sqrt{1 - \frac{r}{R}}, \tag{10}$$

where r is given by (8), $f_{\rm em}$ is the emitted frequency of a photon and f is the detected frequency by a distant observer. From this and (3) we obtain the limiting relation (9) for $R \to r$.

Setting for instance $R = 2r$ in (10) (cf. [1, p. 3]), we find that $f = 2^{-1/2} f_{\rm em}$. By (3) the corresponding gravitational redshift is

$$z = \sqrt{2} - 1 = 0.414 \tag{11}$$

which is much larger than the value in (6). Similarly, for $R = 3r,\ 4r$, and $5r$, we get $z = 0.225$, 0.155, and 0.118, respectively, which are also larger than (6).

Just before the collision of the two black holes, the spacetime between them exhibited the largest deformations. By the measured data of LIGO detectors it produced gravitational waves with an increasing recorded frequency 35–250 Hz. The distance between both the black holes was only a few Schwarzschild radii [1, p. 3]. From (8)–(11) we may deduce that the gravitational redshift of the emitted gravitational waves will be quite essential and probably larger than that in (6). Unfortunately, the true analytical solution of Einstein's equations for two orbiting bodies is not known. However, a common gravitational potential hole of the two black holes is deeper than that of each of its components (see Fig. 1).

From equality (2) it is obvious that the sought masses of the black holes and also the constant C of (4) depend on the emitted frequency $f_{\rm em} = (z+1)f$. Consequently, a proper determination of the total redshift z is essential. By relation (3) we obtain $\dot f_{\rm em} = (z+1)^2 \dot f$, where the additional factor $(z+1)$ is due to Einstein's time dilatation. Substituting this and (3) into (2), we get the missing factor

$$(z + 1), \tag{12}$$

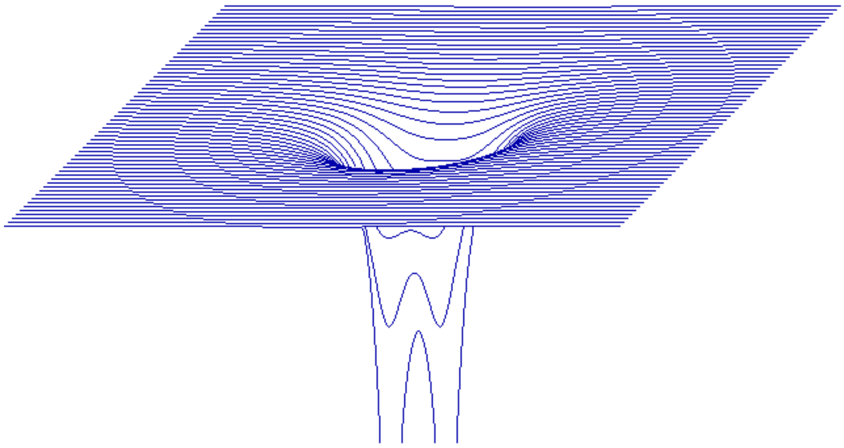

Figure 1: Schematic illustration of the gravitational potential of a binary back hole system. The largest deformation of the spacetime arises close to the central saddle point. According to [1, p. 3] the highest amplitude of the detected signal was reached for the separation $R = \frac{3}{2}r$.

because

$$\Big(f^{-11/3}\dot{f}\Big)^{3/5} = (z+1)\Big(f_{\rm em}^{-11/3}\dot{f}_{\rm em}\Big)^{3/5} \tag{13}$$

and because the remaining factors in (2) are constants. In [1] only the cosmological redshift (6) was considered, but the total redshift is larger (cf. e.g. (11)).

Finally, let us note that the Doppler shift may also not be negligible. Unfortunately, we do not know the orientation of the orbital plane of the black holes and its local movement. Therefore, we cannot reliably establish the corresponding Doppler redshift.

4. Other arguments

The mechanism of the origin of the binary black hole system (1) is not known. According to the recent survey paper [11, Fig. 8], all the 17 known X-ray binaries detected in our Galaxy have components with masses less than $10M_\odot$. Masses of all known single stellar mass black holes are in the interval $5M_\odot - 20M_\odot$ (cf. e.g. [7], [8], [18]). Hence, from a statistical point of view a system of two much larger black holes such as (1) seems to be quite exceptional, even though some selection effects may be present, since larger masses imply stronger signals. Moreover, its evolution path is unknown. All its parameters should be tuned very finely.

Chen et al. in [10] have also found that the important gravitational redshift of GW150914 was not accounted for. They assume that the black hole binary system was located in close vicinity of a supermassive black hole possessing a large gravitational redshift. Their results indicate that the mass of each component of such a pair of black holes is not greater than $10M_\odot$.

The wavelength $\lambda = c/f$ corresponding to the highest detected frequency $f = 250\,\mathrm{Hz}$ is equal to $\lambda = 1200\,\mathrm{km}$. This is a much larger value than the diameter of the wave zone given by the associated Schwarzschild radii of the black holes. Nevertheless, we observe that the emitted frequency $f_{\mathrm{em}} > f$ would yield a much more reliable size of the wave zone. This also shows that the gravitational redshift was missing.

5. Conclusions

The paper [1] presents an important analysis of signals from LIGO detectors. However, due to the lack of details on the mathematical tools used in this analysis, it can hardly be reproduced. The statement that two black holes with masses (1) have merged seems to be somewhat too strong. The main reason is that the detected frequency f in (2) should be replaced by the emitted frequency f_{em}, i.e., the chirp mass (2) should be divided by the missing factor $(z+1)$ given in (12). From Section 3 we may deduce that the total redshift z could be larger than $\frac{1}{2}$, see (6) and (11). Hence, the masses (1) were overestimated. To see this, suppose for simplicity that $m_1 = m_2$. Then from (2) we observe that the corresponding chirp mass $\mathcal{M} = m_1^{6/5}/(m_1 + m_1)^{1/5} = 2^{-1/5}m_1$ linearly depends on m_1. Hence, the masses m_i are affected by the same redshift correspondingly. In this special case for $z + 1 \geq \frac{3}{2}$ the masses m_i could be at least 33 % smaller.

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