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COSMOLOGICAL COINCIDENCES IN THE EXPANDING UNIVERSE

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Abstract: In the study of dimensionless combinations of fundamental physical constants and cosmological quantities, it was found that some of them reach enormous values. In addition, the order of magnitude of some of them appears to be the same or in an arithmetic relation to others. Because attempts to calculate these quantities and other (smaller ones) from first principles were unsuccessful, Paul A. M. Dirac attempted to base cosmological theory on these coincidences. The result of his relations resulted in a decreasing gravitational constant. After many more similar attempts to explain these coincidences, and after the creation of alternative theories of gravity to the general theory of relativity, it has been shown that the required degree of variability of the gravitational constant can be experimentally ruled out. With a combination of cosmological coincidences however, it is possible to establish a relationship that relates the ratio of the total mass of the observable universe to its radius. This relationship is independent of time and is given by the ratio of the square of the speed of light and the gravitational constant. The mass of the observable part of the universe thus increases in the same way as its radius. In an expanding universe, this can be explained simply by the fact that the horizon recedes and new matter enters the observable region.

Keywords: Cosmological coincidences, cosmological numbers, Dirac's Large Numbers Hypothesis, Dirac's cosmology, Eddington number, expanding universe, Einstein-de Sitter model

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1. Introduction

The dimensions and evolution of cosmic objects are determined by dimensionless combinations of fundamental physical constants, see [1]. In this paper, we will deal only with the following: speed of light c in vacuum, the reduced Planck constant $\hbar = h/(2\pi)$, the gravitational constant G, the elementary charge e, as well as the mass of proton m_{p} and the mass of electron m_{e} at rest (see Table 1).

name	symbol	value	remark
speed of light in vacuum	c	299 792 458 m/s	by definition
Planck constant	h	$6.62607015 \cdot 10^{-34}$ Js	by definition
reduced Planck constant	$\hbar$	$1.05457182 \cdot 10^{-34}$ Js	$\hbar = h/(2\pi)$
gravitational constant	G	$6.674 \cdot 10^{-11} \mathrm{m^3 kg^{-1} s^{-2}}$	measured value
elementary charge	e	$1.602176634 \cdot 10^{-19}$ C	by definition
mass of the proton	m_p	$1.67262192 \cdot 10^{-27}$ kg	measured value
mass of the electron	m_e	$9.10938370 \cdot 10^{-31}$ kg	measured value
Boltzmann constant	k	$1.380649 \cdot 10^{-23}$ J/K	by definition
Avogadro constant	N_A	$6.02214076 \cdot 10^{23}\ \mathrm{mol}^{-1}$	by definition
vacuum permittivity	ε_0	$8.854187812 \cdot 10^{-12}$ F/m	measured value

Table 1. Values of some fundamental constants and other quantities

In addition to the above-mentioned fundamental physical constants, quantities derived from the dimensions of the observable universe or the number of particles in it can also enter dimensionless combinations. All these dimensionless combinations form groups of order of magnitude separated by size — most combinations have a value from 10^{-3} to 10^3, but some reach extreme values of 10^{40} or even 10^{80}, see [2]. Most dimensionless quantities constructed in this way have their own names (see Table 2).

name	symbol	value	relationship
fine-structure constant	α	1/137.036	$\mathrm{e}^2/(4\pi\varepsilon_0\hbar c)$
grav. fine-structure constant	α_G	$5.90 \cdot 10^{-39}$	$Gm_\mathrm{p}^2/(\hbar c)$
elmag. to grav. intensity ratio	-	$1.24 \cdot 10^{36}$	$Gm_\mathrm{p}^2/(4\pi\varepsilon_0) = \alpha/\alpha_\mathrm{G}$
proton to electron mass ratio	-	1836.15	$m_\mathrm{p}/m_\mathrm{e}$
Compton wavelength of electron	λ_e	$3.86 \cdot 10^{-13}$m	$\hbar/(m_\mathrm{e}c)$
classical electron radius	r_e	$2.82 \cdot 10^{-15}$m	$\mathrm{e}^2/(4\pi\varepsilon_0 m_\mathrm{e}c^2) = \lambda_\mathrm{e}\alpha$
1st Bohr radius	r_B	$5.29 \cdot 10^{-11}$m	$4\pi\varepsilon_0\hbar^2/(m_\mathrm{e}\mathrm{e}^2) = \lambda_\mathrm{e}/\alpha$

Table 2. Values of some dimensionless and length quantities

Dimensionless quantities of the order of 10^{10} are referred to as *cosmological numbers* (sometimes also *cosmic numbers*). This group also includes the binding constant of gravity α_G and the ratio of the intensity of electromagnetic and gravitational interaction α/α_G. However, there are exceptions, because only they contain mere microphysical constants (if we consider the gravitational constant G as a microphysical constant). The relationships between cosmological numbers are then referred to as *cosmological coincidences*.

One of the basic physical questions from the beginning of the 20th century was whether the whole universe affected local physics or not, and whether the very small value of the gravity coupling constant α_{G} is not conditioned by the size of the observable universe or the number of particles in it. Cosmological coincidences may have indicated whether such a possibility was realistic.

Cosmological coincidences are usually assessed only in the order of magnitude, and coefficients close to one tend to be neglected in the relationships between cosmological numbers. In our work, however, we will respect such coefficients and take them into account.

In the older literature, where the CGS system is used, the expression e^2 corresponds to our expression $\mathrm{e}^2/(4\pi\varepsilon_0)$ appearing in Coulomb's law, where ε_0 is the permittivity of vacuum. Thus the CGS system has a different unit of charge than the SI system, it is not rationalized and the permittivity of the vacuum in it is by definition equal to one (in terms of one).

Note that for the dimensionless coupling constant of the electromagnetic interaction α, called the *fine-structure constant* or *Sommerfeld's constant*, the relation $\alpha = \mathrm{e}^2/(4\pi\varepsilon_0\hbar c)$ applies in the SI system, but in the CGS system $\alpha = \mathrm{e}^2/(\hbar c)$. Since it is a dimensionless quantity, its value is independent of the selected system of units. It pays a role in the relationships for the splitting of spectral lines, which is caused by relativistic phenomena and spin-orbital interactions.

We can construct a similar dimensionless quantity, α_{G}, for the gravitational interaction. Its notation is identical in the SI system and in the CGS system. Unlike the constant of the fine structure α, in the definition of which the elementary charge unambiguously appears, we have to decide which elementary particle and its mass we choose here to be fundamental. Usually the proton is chosen here, because it is a much more massive particle than the electron and it gravitationally dominates in the universe among stable particles. Then $\alpha_{\mathrm{G}} = Gm_{\mathrm{p}}^2/(\hbar c)$.

2. Weyl and Eddington

Let N_1 denote the ratio of the intensity of the electromagnetic and gravitational interaction between the proton and electron, which does not change with distance. We will write this relationships in the SI system as follows

$$N_1 = \frac{\alpha}{\alpha_{\mathrm{G}}}\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} = \frac{\dfrac{\mathrm{e}^2}{4\pi\varepsilon_0\hbar c}}{\dfrac{Gm_{\mathrm{p}}m_{\mathrm{e}}}{\hbar c}} = \frac{\dfrac{\mathrm{e}^2}{4\pi\varepsilon_0}}{Gm_{\mathrm{p}}m_{\mathrm{e}}} = 2.27 \cdot 10^{39}. \tag{1}$$

Futher, let N_2 denote the ratio of the *radius R of the observable universe* and the *classical electron radius* r_{e} (the classical electron radius $r_{\mathrm{e}} = \mathrm{e}^2/(4\pi\varepsilon_0 m_{\mathrm{e}}c^2)$ is a formal quantity denoting the radius of a sphere with uniformly spatially distributed charge, whose total electrostatic energy is equal to the rest energy of the electron).

We replace the radius R of the observable universe by the product cT, where T is the present age of the universe, which is quite famous quantity today ($13.8 \cdot 10^9$ years, i.e. $4.35 \cdot 10^{17}$ s), see [3]. We get a value of $1.31 \cdot 10^{26}$ m, i.e. cca 4200 Mpc. More precisely, there should be kcT, where the size of the constant k depends on a specific model of the universe. For instance, $k = 3$ in the Einstein-de Sitter model of the universe, which is spatially uncurved, has zero cosmological constant, and contains only material dust. We have

$$N_2 = \frac{R}{r_e} = \frac{R}{\frac{e^2}{4\pi\varepsilon_0 m_e c^2}} = \frac{4\pi\varepsilon_0 m_e c^2 R}{e^2} = \frac{4\pi\varepsilon_0 m_e c^3 T}{e^2} = 4.63 \cdot 10^{40}. \tag{2}$$

The coincidence order of the dimensionless quantities N_1 and N_2, both of which have a value of about 10^{40}, was discovered in 1919 by H. Weyl in [4] and discussed in more detail in [5]. F. Zwicky called it Weyl's hypothesis [6], but later this name was not adopted.

Another cosmological number N_3 was introduced by A. S. Eddington, see [7, 8]. In the literature, it is often referred to as the *Eddington number*. It represents the number of nucleons in the observable universe and can be expressed as the ratio of the mass M of the observable part of the universe to the mass of one proton. Its size is of the order of 10^{80}. We will also use this value,

$$N_3 = \frac{M}{m_p}. \tag{3}$$

Then

$$M = m_p 10^{80} = 1.67 \cdot 10^{-27} \cdot 10^{80} \text{ kg} = 1.67 \cdot 10^{53} \text{ kg}.$$

The order of magnitude of N_3 is thus roughly equal to the square of the numbers N_1 or N_2. Throughout his life, Eddington tried to create a theory that would allow the theoretical calculation of the number N_3 and other dimensionless quantities, such as the fine-structure constant α or the ratio of the masses of the proton and electron m_p/m_e, see [9]. The resulting Eddington theory was not published until after his death, by E. T. Whittaker in 1946 in the book Fundamental Theory (cf. [10]), but did not receive a favorable recognition, and the general opinion is that it is wrong [11]. Eddington's successive attempts to calculate the number N_3 gave results of the order 10^{79} to 10^{80}. His formula was based on the multiplication of the factors 136 and 2^{256}, where 136 is the approximate value of the reciprocal fine-structure constant $\alpha \approx 1/137$. All there numbers N_1, N_2, and N_3 are constants in Eddingon's theory. Attempts to calculate theoretically the exact values of α and m_p/m_e continues up to now, but without any noticeable success.

3. Dirac and Gamow

In 1937 and 1938, Paul A. M. Dirac published two works [12] and [13], in which he introduces a completely new approach to the problem of cosmological numbers. He formulates the *Large Numbers Hypothesis* (LNH, latter called *Dirac's LNH*) in which he assumes that all very large numbers in physics are connected together by simple arithmetic relations in which coefficients of magnitude of units occur. The hypothesis was based, among other things, on the fact that obtaining theoretically very large numbers is quite difficult. LNH-based cosmology is called *Dirac's cosmology.*

Thus, according to Dirac's LNH, it should hold $N_1 = N_2$ and $N_3 = N_2^2$. Because the number N_2 increases with time, the number N_1 should grow with time, too, and the number N_3 should grow with time quadratically, namely, we have

$$N_1(t)N_2(t) = N_3(t). \tag{4}$$

Since the number N_1 contains only microphysical constants, in order to satisfy the LNH, it is necessary that some of them are variable in time. Dirac chose the gravitational constant G as a variable. First, this constant is measured with very low relative accuracy, but an important argument was that many physicists at that time believed that the marked weakness of gravity was related to the large number of particles in the observable universe. Later, in 1974 (see [14]) and is 1979 (see [15]), Dirac added to his cosmology the assumption of the formation of particles in the observable universe so that the LNH would also be satisfied for the number N_3. He considered two possibilities – matter could increase in the universe either uniformly or preferentially in areas where it already exists.

The first objection to the change of the gravitational constant over time was as early as in 1948 published by E. Teller [16]. He stated that the luminosity of the Sun depends on the 7th power of the gravitational constant, and if Dirac's LNH were valid, the Sun in the Precambrian Era would have to shine much more strongly than it does today. Water could not exist on Earth in a liquid state, and the origin of life at that time would be impossible.

Second, Dirac's hypothesis had a large number of responses in the literature. On one hand, it was a partial inspiration for constructing alternative theories of gravity, such as Jordan's theory [17], Brans-Dicke's theory [18], and Hsieh-Canuto's theory [19]. Furthermore, various modifications of its occurrence have appeared, and still appear in the literature, and several other coincidences have been studied.

Third, Dirac's hypothesis led to an experimental effort to directly measure the limits for a possible change in the gravitational constant, which led to the definitive refutation of its original version in the 1980s. It was based on the measurements of the Viking spacecraft, located on Mars, which made it possible to measure very accurately the distance of Mars from Earth. It has been shown that the orbits of the planets in the Solar system do not increase to the extent that Dirac's hypothesis assumes, see [20].

In the late 1960s, G. Gamow published an alternative hypothesis that gravity does not weaken, but the strength of the electromagnetic interaction increases as the magnitude of the elementary charge increases over time [21, 22]. However, this version was very soon refuted, see [23, 24, 25, 26].

4. Dicke and Crater

Another opinion was expressed in 1961 by R. H. Dicke [27], who calculated that the average lifetime of stars roughly corresponds to the present age of the universe. This explains the order of magnitude of the number N_2, which must reach the value of N_1 in order for the interstellar matter to be enriched with heavier elements. It is mainly the carbon needed for the development of life and then for the formation of the observer. This view was an important impulse for the formulation of the anthropic principle by B. Carter in 1974, see [28, 29]. The anthropic principle formulated in various versions is very often discussed today in both cosmological and philosophical works. In its consequences, it also led to considerations about the possible existence of a multiverse, see [30].

In Dicke's view, therefore, N_1 is constant, N_2 increases with time as in Dirac's theory, and N_3, which is their product, also increases, but only proportionally to N_2.

$$N_1 N_2(t) = N_3(t). \tag{5}$$

5. Machian condition

Below we show how (5) arises. Substituting into equation (5) from relations (1), (2), and (3), we get

$$\frac{\dfrac{\mathrm{e}^2}{4\pi\varepsilon_0\hbar c}}{\dfrac{Gm_\mathrm{p}m_\mathrm{e}}{\hbar c}} \cdot \frac{R}{\dfrac{\mathrm{e}^2}{4\pi\varepsilon_0 m_\mathrm{e}c^2}} = \frac{M}{m_\mathrm{p}} \tag{6}$$

which means

$$\frac{\mathrm{e}^2 4\pi\varepsilon_0 m_\mathrm{e}c^2 m_\mathrm{p}}{4\pi\varepsilon_0 G m_\mathrm{p} m_\mathrm{e}\mathrm{e}^2} = \frac{M}{R},$$

and finally,

$$\frac{c^2}{G} = \frac{M}{R}. \tag{7}$$

We see that the M/R ratio should remain constant for the duration of the universe and should be equal to the constant $c^2/G = 1.27 \cdot 10^{27}$ kg/m. Calculated quantities $M = 1.67 \cdot 10^{53}$ kg and $R = 1.31 \cdot 10^{26}$ m satisfy this relationship very well. Since we are only looking for an order of magnitude match, the result is fully in line with expectations.

Dividing equation (5) by the right-hand side, we get (compare also with [31])

$$\frac{N_1 N_2}{N_3} = \frac{c^2 R}{GM} = 1,$$

that is

$$\frac{GM}{Rc^2} = 1. \tag{8}$$

This equality should be considered only approximately, of course. Thus, we obtain a result corresponding to the so-called *Machian initial condition*, the fulfillment of which is assumed for any realistic model of the universe, see [32]. Since Mach's principle is not included in Einstein's General Theory of Relativity, some authors consider relation (8) to be the selection principle for a proper choice of a realistic model of the universe, see [33].

6. Solution in an expanding universe

The question remains how it is possible for the number of particles in the observable universe to grow linearly with time. The answer is very simple. The observable universe seems to be flat, as found in 2000 in the BOOMERANG experiment [34]. Because the influence of the cosmological constant was negligible in the first billions of the universe's existence, the universe behaved more or less according to the Einstein-de Sitter model. This model describes a spatially flat infinite universe with a critical mass density and zero cosmological constant [35]. The radius of the observable region R increases roughly as the product of cT. Because this model is Euclidean, the volume of the observable universe will be proportional to t^3. However, in this model, the mass density decreases at the same time proportionally to t^2, see [36]. Therefore, the number of particles in the observable part of the universe will increase proportionally with time as

$$N_3 \sim V\rho \sim t^3 t^{-2} = t,$$

where ρ is the mean density.

7. Conclusions

We have to state that from today's point of view, it is clear that since the properties of the Einstein-de Sitter model of the universe were already known in 1961 (which also applies in 1937), it was possible to withdraw from the model with decreasing gravitational constant and not try to supplement or to modify the original LNH, or to create other theories of gravity competing with the General Theory of Relativity, which sought to describe gravity as an interaction whose intensity decreases with time (although the author of this article is aware that the main reason for constructing such theories was the incorporation of Mach's principle into the theory of gravity, not only the fulfillment of LNH). It is noteworthy that the development of Dirac's LNH in terms of development of the expanding universe appears in the literature only in note 26 to Chapter 4 of the book by J. D. Barrow *The Book of the Universes*, see [37]. The main goal of this article was to draw an attention to Barrow's evaluation.

Remark. Due to the redefining of the basic units of the SI system in 2019 (especially kilograms and amperes), the quantities c, $\hbar$, e (as well as the Boltzmann constant k and the Avogadro's constant N_{A}) are already defined as real constants [38, 39], cf. also Table 1. A possible variability of the coupling constants α and

α_G, which is still being sought, would be reflected in the variability of the vacuum permittivity ε_0 and the gravitational constant G.

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