


Cosmological consequences of the Lorentz and Doppler transformations

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The common opinion that the Lorentz transformation distorts the Minkowski spacetime arises from a mathematically incorrect neglect of its non-diagonal terms. When the non-diagonal Lorentz transformation is properly treated, the time dilation and the space distortion disappear. Consequently, the Lorentz metric produced by the Lorentz transformation is identical to the Minkowski metric. The spacetime distortion in moving inertial frames is properly described by the so-called Doppler metric produced by the Doppler transformation. This transformation assumes the existence of a preferred frame and depicts the Doppler shift of light for a moving light source and/or observer. The Doppler metric is Lorentz invariant and leaves the Maxwell's equations unchanged from their form in the Minkowski spacetime. Moreover, the Doppler metric is experimentally confirmed because it yields the null result in the Michelson–Morley experiment and other interferometric experiments. The suitability of the Doppler metric for describing light properties in Special and General Relativity is supported by recent cosmological observations of the cosmic microwave background.

Keywords: Doppler effect; Lorentz transformation; time dilation; conformal metric; Michelson–Morley experiment; cosmic microwave background.

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1. Introduction

The Lorentz transformation was introduced by Lorentz^{1,2} to explain the null result of the Michelson–Morley experiment, which aimed to detect the luminiferous ether.³ Based on the assumption of the constant speed of light c in all inertial systems, the Lorentz transformation is a linear transformation (without absolute terms) that relates two inertial frames in spacetime moving at a velocity v relative to each other.

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It preserves the interval ds in Minkowski space⁴

$$ds^2 = -c^2 dt^2 + dx^2, \quad (1)$$

and forms the so-called Lorentz group. The invariance of ds in Eq. (1) plays a crucial role in studying light propagation problems, and the Lorentz transformation became a fundamental pillar of Special Relativity (SR) and General Relativity (GR) developed by Einstein.^{5–9} The Lorentz factor $\gamma = 1/\sqrt{1 - \beta^2}$ (with $\beta = v/c$) standing in the transformation between two inertial frames was interpreted as a factor quantifying time dilation and length contraction characterizing the spacetime distortion of moving inertial systems. In non-inertial systems, the time dilation and space deformation is additionally affected by gravity and other forces and physical fields.^{7,10}

Obviously, the assumption of the constant speed of light c in all inertial frames has substantial consequences for transforming physical quantities between moving frames, including observations of the Doppler shift of photons emitted by a moving light source or observed by a moving receiver. Consequently, the classical Doppler shift is modified by multiplying it by the Lorentz factor γ to include relativistic effects.^{8,9,11} This modification appears straightforward but is actually confusing. First, if the Lorentz transformation describes the relationship between two mutually moving inertial frames why is the Doppler effect not predicted directly by this transformation? Second, the apparent change of the frequency or wavelength of emitted/received waves described by the Doppler effect occurs only when the source is moving relative to the observer. As a consequence, the wave speed must be different in the frames of the source and the observer in the classical theory. If we assume no difference between the speed of light in both frames in SR, it is questionable whether the shift of photons can arise at all.¹²

The origin of the confusion about the Doppler effect and the constant speed of light in SR is both physical and mathematical. For example, Dingle^{13–15} pointed out an inconsistency in the kinematical part of SR and initiated a debate among physicists,^{16–18} including Max Born,¹⁹ on this topic in *Nature*. However, the debate was inconclusive and ended with no consensus.²⁰ Other physicists have also raised doubts about the consistency of SR and the postulate of the constancy of the speed of light.^{12,21–29}

The aim of this paper is to clarify the relationship between the Lorentz transformation and the Doppler effect and to determine whether observations of the Doppler effect are consistent with SR. It is shown that, while the Lorentz transformation is mathematically correct, its common physical interpretation is flawed. This results in contradictory statements about the spacetime distortion produced by the Lorentz transformation. It is demonstrated that the Doppler metric is more appropriate for describing spacetime distortion in moving inertial frames. Moreover, the Doppler metric, like the Lorentz metric, yields the null result of the Michelson–Morley experiment and other interferometric experiments. Recent cosmological observations of cosmic microwave background (CMB) also support the usefulness of the Doppler metric in solving light propagation problems in the Universe.

2. Theory

2.1. Coordinate transformation and metric tensor

Transformation $\Lambda^\mu{}_\nu$ from the Minkowski space with coordinates $x^\alpha = (ct, \mathbf{x}) = (ct, x, y, z)$ to a new coordinate system with coordinates $x'^\alpha = (ct', \mathbf{x}') = (ct', x', y', z')$ reads^{4,30}

$$\Lambda^\mu{}_\nu = \frac{\partial x^\mu}{\partial x'^\nu} \quad \text{where } \mu, \nu = 0, 1, 2, 3. \quad (2)$$

Consequently, metric tensor $g'_{\alpha\beta}$ produced by transformation $\Lambda^\mu{}_\nu$ is (see Weinberg,¹⁰ his Eq. 3.2.7)

$$g'_{\alpha\beta} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta \eta_{\mu\nu}, \quad (3)$$

where $\eta_{\mu\nu}$ is the metric tensor of the Minkowski space, $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$. The distance element, $ds'^2 = g'_{\alpha\beta} dx'^\alpha dx'^\beta$, can be split into the time and space components

$$ds'^2 = -g'_{tt} c^2 dt' dt' + g'_{ii} dx'^i dx'^i, \quad (4)$$

where the Einstein summation convention over index $i = 1, 2, 3$ is applied. The term g'_{tt} defines the time dilation and terms g'_{ii} , $i = 1, 2, 3$, define the space deformation of the metric tensor $g'_{\alpha\beta}$ with respect to the Minkowski space.

2.2. Lorentz transformation and Lorentz metric

Let us consider two frames with coordinate systems x^α and x'^α moving each to the other at constant velocity v . In SR, the coordinates in these frames are assumed to be related by the Lorentz transformation⁴

$$\Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where $\gamma = 1/\sqrt{1 - \beta^2}$ is the Lorentz factor, $\beta = v/c$, velocity v is directed along the x -axis, and $\det \Lambda = 1$.

The Lorentz transformation (5) is non-diagonal; hence, the time and space coordinates are not transformed independently. Obviously, the fact that the Lorentz transformation is non-diagonal complicates understanding the behavior of time and space in the mutually moving frames. However, using the following substitution:

$$\gamma = \cosh u, \quad \gamma\beta = \sinh u, \quad (6)$$

the Lorentz transformation can be viewed as a rotation of coordinates (see Carroll,³⁰ his Eq. 1.32)

$$\Lambda^\mu{}_\nu = \begin{bmatrix} \cosh u & -\sinh u & 0 & 0 \\ -\sinh u & \cosh u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (7)$$

where $u = \operatorname{arctanh}\beta$ is called the rapidity, and $\det \Lambda = 1$. Here, hyperbolic functions instead of trigonometric functions are used because the time coordinate is imaginary and the rotation of coordinates is performed in pseudo-Euclidean space.

Since Eq. (7) represents a rotation matrix in the Minkowski space, time and space are not distorted by this transformation. This is confirmed by calculating the Lorentz metric tensor $g'_{\alpha\beta}$ produced by the Lorentz transformation using Eqs. (3) and (5) that transform the spacetime coordinates between two inertial systems x^α and x'^α :

$$g'_{\alpha\beta} = \eta_{\alpha\beta}. \tag{8}$$

Hence, the Lorentz metric $g'_{\alpha\beta}$ of the new coordinate system x'^α also describes the Minkowski space. Metrics of both frames are identical, and analogously to Eq. (1) we get

$$ds'^2 = -c^2 dt' dt' + dx'^i dx'^i, \tag{9}$$

where the mixed time-space terms of the metric tensor $g'_{\alpha\beta}$ disappear. Consequently, Eq. (9) describes a diagonal metric tensor, where the transformation of time and space components between two frames is fully separated.

Equation (9) is not surprising because the Lorentz transformation was derived under the fundamental postulate of SR that the speed of light is constant in all inertial frames. Hence, the term g'_{tt} of the metric tensor must be 1. If $g'_{tt} \neq 1$, the speed of light in the system x'^α will be varying and dependent on β .

2.3. Time dilation in the Lorentz metric

Using Eqs. (8) and (9), we readily obtain for the time dilation in the moving coordinate system x'^α ,

$$\frac{dt'}{dt} = 1. \tag{10}$$

Hence, no time dilation should be observed in the frame with coordinates x'^α . This looks strange and contradictory to the standard result in SR, where time is assumed to be dilated as

$$\frac{dt'}{dt} = \gamma. \tag{11}$$

The issue with Eq. (11) arises from overlooking the fact that the Lorentz transformation is non-diagonal, meaning the time differential dt' depends not only on dt but also on dx , as indicated in Eq. (5). The standard derivation of Eq. (11) found in textbooks contains two mathematical inaccuracies. First, it neglects a non-diagonal term in the first row of the Lorentz matrix in Eq. (5) to obtain Eq. (11). Second, it imposes the space contraction

$$\frac{dx'}{dx} = 1/\gamma, \tag{12}$$

to keep the determinant of the Lorentz transformation to be 1. Hence, in SR, the Lorentz transformation between frame systems x^α and x'^α is commonly assumed to

have actually the following diagonal form:

$$(\Lambda^\mu{}_\nu)_{\text{diag}} = \begin{bmatrix} \gamma & 0 & 0 & 0 \\ 0 & 1/\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (13)$$

Consequently, the resultant Lorentz metric tensor would read from Eqs. (3) and (13)

$$ds^2 = -\gamma^2 c^2 dt' dt' + \gamma^{-2} dx' dx' + dy' dy' + dz' dz'. \quad (14)$$

The difficulties with the standard derivation of the time dilation in SR have become apparent: First, it is not possible to derive the Lorentz transformation in the diagonal form, as written in Eq. (13), from its non-diagonal form in Eq. (5). Second, the metric described by Eq. (14) does not fulfil the SR postulate of the constancy of the speed of light. The equation for a light ray along the x' -axis obeys the null geodesics, $ds'^2 = 0$; therefore, we get from Eq. (14)

$$\gamma^2 c^2 dt'^2 = \gamma^{-2} dx'^2. \quad (15)$$

Consequently, the contravariant speed of light C reads

$$C = \frac{dx'}{dt'} = \gamma^2 c. \quad (16)$$

However, the speed C is not a coordinate-invariant quantity measurable in physical experiments. In order to express the physical (proper) speed of light \hat{C} , which is coordinate invariant, we have to express the speed of light in the orthonormal coordinate basis^{7,31,32}

$$\hat{C} = \sqrt{g_{xx}} \frac{dx'}{dt'} = \gamma c. \quad (17)$$

Surprisingly, Eq. (17) predicts a varying proper speed of light. This implies that the Lorentz transformation itself (defined in Eq. (5)) is consistent with the SR postulate of the constant speed of light, but the derivation of time dilation from the Lorentz transformation in Eq. (11) is flawed. Equation (11) is thus inconsistent with the Lorentz transformation as well as with the SR postulate of the constant speed of light c .

2.4. Doppler transformation and Doppler metric

Since the correctly applied Lorentz transformation predicts no time dilation, it also does not account for the Doppler effect, which involves changes in the frequency and wavelength of photons emitted by a moving light source or received by a moving observer. Therefore, it is necessary to derive an appropriate transformation, referred to as the Doppler transformation, which relates two frames moving relative to each other at velocity v .

For simplicity, the light source will be at rest in the frame with coordinates x^α and the observer will recede from the source being situated at the frame with coordinates

x^α . Let us assume the Doppler transformation $D^\mu{}_\nu$ in the following form, which is a modification of the Lorentz transformation in Eq. (5)

$$D^\mu{}_\nu = \begin{bmatrix} \varepsilon\gamma & -\varepsilon\gamma\beta & 0 & 0 \\ -\varepsilon\gamma\beta & \varepsilon\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (18)$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor, $\beta = v/c$, and ε is the Doppler factor which should be determined. The Doppler metric tensor $g'_{\alpha\beta}$, produced by the transformation $D^\mu{}_\nu$, reads from Eq. (3)

$$g'_{\alpha\beta} = D^\mu{}_\alpha D^\nu{}_\beta \eta_{\mu\nu}, \quad (19)$$

and the distance element, $ds'^2 = g'_{\alpha\beta} dx'^\alpha dx'^\beta$, is

$$ds'^2 = -\varepsilon^2 c^2 dt' dt' + \varepsilon^2 dx' dx' + dy' dy' + dz' dz'. \quad (20)$$

The physical meaning of Eq. (20) is straightforward: if the observed period of waves increases (or decreases) due to the motion of the source relative to an observer, the observed wavelength must also increase (or decrease). Considering the well-known formula for the frequency shift and wavelength transformation due to the Doppler effect³³

$$\frac{dt'}{dt} = \frac{1}{1-\beta} \quad \text{and} \quad \frac{dx'}{dx} = \frac{1}{1-\beta}, \quad (21)$$

we readily obtain the Doppler factor ε in the form

$$\varepsilon = \frac{1}{1-\beta}. \quad (22)$$

The Doppler transformation yields the following form:

$$D^\mu{}_\nu = \begin{bmatrix} \frac{\gamma}{1-\beta} & -\frac{\gamma\beta}{1-\beta} & 0 & 0 \\ -\frac{\gamma\beta}{1-\beta} & \frac{\gamma}{1-\beta} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (23)$$

Analogously, we can derive the Doppler factor ε and the Doppler transformation $\hat{D}^\mu{}_\nu$ for an observer at rest in the frame with coordinates x^α and the source receding from the observer being situated at the frame with coordinates x'^α ,

$$\varepsilon = 1 - \beta, \quad (24)$$

$$\hat{D}^\mu{}_\nu = \begin{bmatrix} (1-\beta)\gamma & -(1-\beta)\gamma\beta & 0 & 0 \\ -(1-\beta)\gamma\beta & (1-\beta)\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (25)$$

2.5. Speed of light in Doppler metric

Equation for the light ray along the x' -axis obeys the null geodesics, $ds'^2 = 0$; hence, we get from Eq. (20)

$$\varepsilon^2 c^2 dt'^2 = \varepsilon^2 dx'^2. \quad (26)$$

The contravariant speed of light C reads

$$C = \frac{dx'}{dt'} = c, \quad (27)$$

and the physical speed of light (i.e., the speed of light expressed in the system of orthonormal base vectors) is^{7,31,32}

$$\hat{C} = \sqrt{g_{xx}} \frac{dx'}{dt'} = \varepsilon c, \quad (28)$$

where the Doppler factor ε is defined in Eqs. (22) and (24) for the case of the receding observer and receding source, respectively.

Equation (27) implies that the Doppler transformation is consistent with the SR postulate of the constant speed of light, provided the contravariant (comoving) speed of light is considered. However, as seen from Eq. (28) the physical (proper) speed of light is not constant but varies, being $\hat{C} = (1 - \beta)c$ for the receding observer and $\hat{C} = c/(1 - \beta)$ for the receding light source.

The dependence of the proper speed of light on the velocity of a light source or an observer in Eq. (28) suggests that the postulate of the constancy of the proper speed of light is no longer valid in the Doppler metric. Unlike the Lorentz transformation and Lorentz metric, which maintain a constant proper speed of light in all inertial frames, the Doppler transformation and Doppler metric imply the existence of a preferred reference inertial frame. This exclusive frame can be identified through observations of the Doppler shift, which is absent only when the source and observer are at rest relative to each other.

2.6. Michelson–Morley experiments in Doppler metric

Equation (20) shows that the Doppler metric is conformal with the Minkowski (Lorentz) metric in the direction of motion of inertial frames. This means that both metrics differ only by a scalar factor $\Phi(x^\alpha)$, which is common for the time and space components in the metric tensor $g'_{\alpha\beta}$:

$$g'_{\alpha\beta} = \Phi(x^\alpha)g_{\alpha\beta} \quad (\text{no summation over } \alpha). \quad (29)$$

Conformal transformations and metrics are extensively studied in the fields of general relativity, gravitation, and cosmology.^{34–41} They possess exceptional properties because they are Lorentz invariant for light and leave Maxwell's equations unchanged from their form in Minkowski spacetime.^{34,42,43} This makes

electromagnetic calculations in spacetimes described by conformal metrics particularly simple.

This applies also to the Doppler metric, where $\Phi(x^\alpha) = \varepsilon^2$. Using Eqs. (1) and (20) with $dy' = dz' = 0$, we obtain the following equation for the distance elements ds'^2 and ds^2 in the Doppler and Minkowski (Lorentz) metrics along the motion direction of frames (see Gron and Johannesen,³⁷ their Eq. 17)

$$ds'^2 = -c^2 dt'^2 + dx'^2 = \varepsilon^2(-c^2 dt^2 + dx^2) = \varepsilon^2 ds^2. \quad (30)$$

Hence, the null geodesics equations $ds'^2 = 0$ and $ds^2 = 0$ yield identical solutions in the Doppler and Minkowski metrics. This confirms the Lorentz invariance of the Doppler metric.

It is important to note that Lorentz invariance is a fundamental principle of SR, closely tied to the constancy of the speed of light. This invariance is considered essential for ensuring that physical laws are identical in all local inertial frames, regardless of their velocity and orientation.⁴⁴

Importantly, the Lorentz invariance of the Doppler metric implies that the light travel time measured in the Minkowski space remains unchanged when the Minkowski metric (or Lorentz metric) is transformed into the Doppler metric. This finding has fundamental consequences for interpreting travel time delays and fringe patterns in all interferometric experiments with light propagating in different directions. This includes the Michelson–Morley experiment^{3,45} and other ether-drift experiments,^{46–50} modern optical experiments with masers^{51,52} or lasers,⁵³ and experiments with optical resonators focused on testing the isotropy of the speed of light.^{54–60} It is commonly believed that the existence of any preferred inertial reference frame would produce nonzero time delays and a varying fringe pattern in these experiments, indicating space anisotropy and a violation of Lorentz invariance. However, since these experiments are designed to test Lorentz invariance and the Doppler metric is Lorentz invariant, they are actually insensitive to the anisotropy inherent in the Doppler metric and produced by the Doppler effect.

It was assumed that the only Lorentz-invariant frames are those with a constant physical speed of light in all inertial frames. Instead, these experiments must yield the “null result” for all metrics characterized by the constant comoving speed of light. In Minkowski space, the comoving and physical speeds of light are equal, leading most authors to assume that Lorentz invariance implies a constant physical speed of light. However, Lorentz invariance applies to light not only in Minkowski space but in all spaces conformal with the Minkowski metric, including the Doppler metric. Therefore, these experiments cannot detect anisotropic proper speed of light produced by conformal metrics. Consequently, they cannot detect the ether drift. The authors overlooked that interferometric patterns in the Doppler metric, which are associated with the ether drift, depend on both the changes of the speed of light and the apparent distortion of space, which mutually cancel out.

3. Discussion

While the Lorentz transformation is mathematically correct and consistent with the postulate of SR concerning the constancy of the speed of light, its current physical interpretation is misleading. Intuitive arguments that disregard non-diagonal terms of the Lorentz transformation have led to the belief that the Lorentz transformation results in time dilation and length contraction in moving frames. In reality, the Lorentz transformation is a simple rotation in pseudo-Euclidean spacetime that does not distort the rotated coordinate axes. This is demonstrated by calculating the Lorentz metric produced by the Lorentz transformation between two inertial frames, which is identical to the metric of Minkowski space.

From a physical standpoint, the identity of the Minkowski and Lorentz metrics stems from the SR postulate about the constancy of the physical (proper) speed of light in all inertial systems. This condition contradicts any distortion of time. If time were distorted, the g_{tt} component of the metric tensor must be different from unity, and the physical (proper) speed of light must vary, as shown in GR.^{7,39–41,61} Consequently, the condition of constant speed of light rules out any time dilation. Furthermore, it also precludes the existence of the Doppler shift of photons in moving frames. The reasoning is clear: if a photon is emitted from a source at velocity c and received by an observer at the same velocity c , no shift of the photon's frequency can occur. This challenges the notion that the Doppler effect can be accommodated in SR by multiplying the standard Doppler formula by the Lorentz factor. Such an approach is not tenable because the Doppler shift is intrinsically tied to changes in the physical speed of waves due to either a moving source or a moving observer. Given that the Doppler shift can only arise when the physical speed of light varies, the SR postulate about the constancy of the *physical* speed of light should be abandoned. Instead, it should be replaced by the postulate of the constancy of the *comoving* (contravariant) speed of light. Under this postulate, the Lorentz invariance remains valid.


Since the Doppler metric is Lorentz invariant, experiments such as the Michelson–Morley experiment and other ether-drift experiments have to fail in detecting the ether drift. The null result obtained does not actually negate the ether hypothesis based on considering the existence of a preferred inertial frame. Most of the authors assumed that the ether drift would violate the Lorentz invariance. They overlooked the fact that the Lorentz invariance holds not only in the Minkowski metric but also in a broad class of metrics conformal to the Minkowski metric. When analyzing experimental data, the authors focused on detecting time dilation between rays propagating in perpendicular directions but neglected a change in the apparent wavelength and space distortion associated with the ether drift. However, both phenomena must be considered in proper interpretations of the experiments. The Doppler metric produces apparent time dilation as well as wavelength distortion of photons; these two effects cancel each other out, resulting in the null result in interferometric experiments.

Hence, the Doppler metric and other metrics conformal to the Minkowski space were not actually refuted by the Michelson–Morley experiments, as so far assumed. On the contrary, recent cosmological observations of the evolution of the Universe indicate the presence of a preferred reference system known as the cosmological coordinate system.^{39–43} The Earth is in motion relative to this system, and this motion can be measured, for example, through observations of the CMB. The CMB radiation uniformly and isotropically fills cosmic space in all directions, except for a dipole anisotropy.^{62–66} This CMB perturbation is produced by the Doppler effect linked to the motion of the Earth and our Galaxy in cosmic space.^{67,68} By analyzing the dipole anisotropy in the CMB, the drift, which was undetectable by the Michelson–Morley experiments, can be successfully measured. The application of the conformal metric to describe cosmic evolution also resolves some challenges to the Λ CDM cosmological model, such as observations of the SN Ia supernova dimming³⁹ and the flat rotation curves of galaxies.⁴¹

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