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Energy of Photons in Expanding Spacetime: Comparing FLRW and Conformal Cosmology Metrics

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Abstract

We investigate the behaviour of photons in Riemann spacetime, focusing on how their velocity and energy are affected by cosmic expansion. Specifically, we examine the differences in energy conservation depending on the cosmological model. Our findings indicate that photons exhibit fundamentally different behaviour based on the chosen metric. In the standard ΛCDM model, which relies on the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, the energy conservation law for redshifted photons is violated. However, in a cosmological model based on the conformal cosmology (CC) metric, this law remains valid. The CC metric offers additional advantages, as it accurately reproduces the cosmological redshift, cosmic time dilation observed in Type Ia supernova light curves, and flat galaxy rotation curves without requiring the introduction of dark matter. These findings underscore the potential significance of the CC metric in cosmological applications.

Keywords: cosmic expansion; conformal cosmology metric; cosmological redshift; FLRW metric; energy conservation law; photons

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1. Introduction

The cosmological redshift, first reported by Lemaître [1] and Hubble [2], is commonly interpreted as the stretching of photon wavelengths during their propagation through an expanding universe. In general relativity (GR), this phenomenon is described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, which forms the foundation of modern cosmology [3–5]:

$$ds^2 = -c^2 dt^2 + a^2(t) dl^2 \ , \tag{1}$$

where ds is the spacetime element, $a(t)$ is the relative scale factor defining cosmic expansion, c is the speed of light at present, t is the cosmic time, and l is the contravariant (comoving) spatial distance.

Unlike spatial coordinates, the time coordinate is assumed to remain unchanged in the FLRW metric during cosmic evolution. This assumption is unusual and surprising because other solutions in GR, such as the well-known Schwarzschild solution, involve distortions in both space and time [3,5,6]. The justification for keeping the lapse function constant, $g_{tt} = 1$, in the FLRW metric is unclear and likely stems from the belief that this form of the metric simplifies mathematical formulations in GR without physical consequences.

However, as demonstrated by Vavryčuk [7,8], this assumption is incorrect and leads to several fundamental inconsistencies when applying the FLRW metric to cosmological problems. One major issue is the inconsistency regarding the cosmological redshift. Although the FLRW metric is widely believed to predict the cosmological redshift, Vavryčuk [7,8] showed that the traditional mathematical derivation, originally proposed by Lemaître [1] and repeated in textbooks, is misleading (for the error in Lemaître's derivation; see Appendix A.1). Vavryčuk's analysis demonstrated that, while cosmic expansion increases the distance between galaxies at rest, it does not affect the wavelength of photons propagating through expanding space. A correct formulation shows that photon wavelengths remain constant in the FLRW metric, and their frequency does not change. In GR, any change in photon frequency is linked to a variation in the lapse function g_{tt} in GR, similarly as for the gravitational redshift. For the same reason, the FLRW metric also fails to adequately explain another relativistic time distortion effect, cosmic time dilation, manifested by the stretching of Type Ia supernova (SN Ia) light curves in the observer's frame [9–14].

These unresolved issues of the FLRW metric suggest that a more general metric tensor is required to describe the evolution of the universe—one that accounts for both spatial expansion and a time-dependent lapse function [7,8,15,16]. A promising candidate is the conformal cosmology (CC) metric, in which the lapse function g_{tt} is proportional to the scale factor $a(t)$ [17–20]. This metric evolves over time according to a conformal transformation, a concept intensively studied in GR in recent years [21–23]. In this framework, comoving and proper times differ analogously to comoving and proper distances. The CC metric is particularly intriguing because it preserves Lorentz invariance and leaves Maxwell's equations unchanged from their form in Minkowski spacetime [17,24,25].

In this paper, we investigate the behaviour of photons in an expanding universe described by both the FLRW and CC metrics. Specifically, we analyse how their energy is influenced by cosmic expansion. Our findings reveal that photons behave fundamentally differently in the CC metric compared to the FLRW metric. Notably, in the FLRW metric, the energy conservation law for redshifted photons is violated, whereas it remains valid in the CC metric. This result highlights the importance of the CC metric for cosmological applications.

2. Conformal Cosmology Metric

Let us consider an expanding universe described by the conformal cosmology (CC) metric [7,18]:

$$ds^2 = a^2(t)\left(-c^2dt^2 + dl^2\right), \tag{2}$$

where ds is the spacetime element, $a(t)$ is the relative scale factor defining cosmic expansion, c is the speed of light at present, t is the coordinate time, and l is the contravariant (comoving) spatial distance.

When comparing Equations (1) and (2), we see that the FLRW and CC metrics differ in the lapse function g_{tt}, which equals 1 in the FLRW metric but is time-dependent in the CC metric. A fundamental question arises: Is this difference merely formal, originated in a coordinate transformation, and thus purely mathematical, or do the two metrics represent physically distinct models of the universe? The prevailing view supports the former, based on the principle that physical laws should hold independently of the coordinate system used. According to this interpretation, a change in coordinates does not alter the underlying physics. However, in this paper, I advocate the latter perspective and argue that the FLRW and CC metrics describe two fundamentally different cosmological models.

The physical consequences of employing different metrics can be illustrated by comparing the Minkowski metric, which describes a static universe, with the FLRW metric, which describes an expanding universe. Clearly, the FLRW metric can be derived from the Minkowski metric

$$ds^2 = -c^2dt^2 + dl'^2 , \tag{3}$$

by rescaling the spatial distance element dl' through the transformation

$$dl' = a(t)dl \ . \tag{4}$$

However, Equation (4) does not imply the physical equivalence of the static and expanding universe models. The key point is that in a purely mathematical coordinate transformation, dl' is a physical distance element in Equations (3) and (4), but dl is not a physical distance in Equation (4) but a rescaled quantity. Under such a transformation, Equations (1) and (3) would still describe a static universe. However, if both distance elements dl' and dl in the Minkowski and FLRW metrics are interpreted as physical, the two metrics represent different physical models of the universe, and the argument for their equivalence no longer holds.

The same reasoning applies when comparing the FLRW and CC metrics. The CC metric can be derived from the FLRW metric by rescaling the time coordinate:

$$dt' = a(t)dt \ . \tag{5}$$

Yet, Equation (5) does not imply physical equivalence between the FLRW and CC models. While the FLRW metric describes spatial expansion only, the CC metric incorporates both spatial expansion and time dilation throughout cosmic history. As demonstrated by Vavryčuk [7,8], assuming a time-varying lapse function $g_{tt} = a(t)$ in the CC metric is essential for accurately predicting the cosmological redshift and time dilation observed in supernova light curves. As shown below, the velocity and energy of photons also behave fundamentally differently in the CC metric compared to the FLRW metric.

3. Velocity of Photons

3.1. Contravariant (Comoving) Velocity

The propagation of photons in the CC metric follows the null geodesic equation, $ds^2 = 0$, leading to

$$a^2(t)\left(-c^2dt^2 + dl^2\right) = 0 \, . \tag{6}$$

Thus, the comoving velocity of photons $\hat{c}$ is given by

$$\hat{c} = \frac{dl}{dt} = c \, , \tag{7}$$

where t is the coordinate time (travel time) along the null worldline.

The velocity of photons $\hat{c}$ depends on the choice of coordinates, as it is evaluated in curvilinear coordinates with a non-orthonormal vector basis. To determine the physical velocity of photons C, which is coordinate-invariant and experimentally measurable (e.g., using atomic clocks and rigid rods), an orthonormal coordinate basis must be used. These quantities are independent of the choice of the coordinate system as explained by Hartle (pp. 153–154, [26]) and Cook [27].

3.2. Physical (Coordinate-Invariant) Velocity

The physical velocity C is obtained by rescaling the comoving velocity $\hat{c}$ as follows [7,15]:

$$C = \sqrt{g_{ll}} \, \hat{c} = a(t)\hat{c} = a(t)c \, . \tag{8}$$

Thus, the physical velocity of photons C is not constant, unlike in the FLRW metric (see Appendix A.2). Instead, it increases with cosmic expansion, following the relation $C = a(t)c$, where c is the present-day speed of light (i.e., with scale factor $a_0 = 1$).

This result highlights a fundamental difference between the CC and FLRW metrics: in the FLRW metric, the speed of light remains constant over cosmic time, in the CC metric, the speed of light evolves with the expansion of the universe.

Note that we consistently use the term 'physical' velocity instead of the more commonly used term 'proper' velocity, as physical velocity has a broader and more general meaning. The physical velocity is a coordinate-invariant quantity that can be measured using an atomic clock and a rigid rod in any frame (free-falling or non-inertial). In contrast, the term proper velocity typically refers to the physical velocity measured in a free-falling frame only.

4. Energy of Photons

4.1. High-Frequency Electromagnetic Waves

Considering photons as high-frequency electromagnetic waves propagating locally as plane waves through a smoothly curved spacetime, the electromagnetic 4-potential A^α is expressed as (§22.5, [6])

$$A^\alpha = \mathrm{Re}\left(a^\alpha e^{\mathrm{i}\theta}\right) , \tag{9}$$

where a^α is the slowly varying complex amplitude, and θ is the rapidly varying phase.

Applying Equation (9), the electromagnetic (Faraday) tensor $F^{\alpha\beta}$ and the electromagnetic stress–energy tensor $T^{\alpha\beta}$,

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha , \tag{10}$$

$$T^{\alpha\beta} = \frac{1}{\mu_0}\left(F^{\alpha\mu}F^\beta{}_\mu - \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right) , \tag{11}$$

are given as (box 22.4E, [6])

$$F^{\alpha\beta} = \mathrm{i}\left(k^\alpha a^\beta - k^\beta a^\alpha\right) , \tag{12}$$

$$T^{\alpha\beta} = \frac{1}{\mu_0} a^2 k^\alpha k^\beta , \tag{13}$$

where $k^\alpha = \partial^\alpha\theta$ is the 4-wave vector (eq. 22.26d in [6]), which defines the ray direction that is perpendicular to the wavefront, $a = \sqrt{a^\mu a^*_\mu}$ is the scalar amplitude, and a^*_μ is the complex-conjugate value of a_μ. In deriving Equations (12) and (13), derivatives of the vector a^α are neglected because the phase θ varies with time much faster than a^α.

The stress–energy tensor $T^{\alpha\beta}$ in Equation (13) satisfies the conservation of energy and momentum,

$$\nabla_\alpha T^{\alpha\beta} = 0 , \tag{14}$$

expressed for high-frequency waves in curved spacetime as

$$\nabla_\alpha\left(a^2 k^\alpha k^\beta\right) = 0 , \tag{15}$$

where ∇_α denotes the covariant derivative, meaning the derivative along a ray path. Subsequently, using the geodesic equation in the form $k^\alpha\nabla_\alpha\left(k^\beta\right) = 0$, we get (box 22.4D in [6]):

$$\nabla_\alpha\left(a^2 k^\alpha\right) = 0 . \tag{16}$$

Equation (16) is called the transport equation for radiation energy and physically means that energy flux is conserved along light rays when measured in a fixed frame.

4.2. Light as an Ensemble of Photons

Considering light as an ensemble of free photons moving through spacetime along geodesic worldlines without collisions, and applying Liouville's theorem in curved spacetime (eq. (22.44) in [6]) together with the collisionless Boltzmann equation (eq. (22.47) in [6]), Equation (16) can be rewritten as the law of conservation of photon number (eq. (22.35) in [6]):

$$\nabla_\alpha (N^\alpha) = 0 \,, \tag{17}$$

where the vector

$$N^\alpha = \frac{1}{\hbar} a^2 k^\alpha \,, \tag{18}$$

is the conserved photon number current,

$$N^0 = \frac{1}{\hbar} a^2 k^0 \,, \tag{19}$$

represents the conserved number of photons inside a ray tube, and $\hbar$ is the reduced Planck constant.

Equation (17) implies that both the photon flux and the number of photons are adiabatic invariants, meaning they are conserved in a spacetime whose radius of curvature is much larger than the photon wavelength. In other words, *no photons are created or destroyed in weak gravitational fields if they propagate along a geodesic worldline without collisions, scattering, or absorption*.

4.3. Conservation of Photon Energy

From Equation (16), we immediately find that the energy density u of free photons propagating in an expanding universe evolves as

$$u = u_0 a(t)^{-3} \,, \tag{20}$$

where u_0 is the present-day energy density of photons, and $a(t)$ is the scale factor with $a(0) = 1$. Similarly, Equation (17) implies that the number density of photons n evolves as

$$n = n_0 a(t)^{-3} \,, \tag{21}$$

where n_0 is the present-day number density of photons.

Since $u = nE$ and $u_0 = n_0 E_0$, it follows that the energy of individual photons is conserved during cosmic expansion:

$$E = E_0 \,, \tag{22}$$

where E_0 is the present-day energy of individual photons. This result holds for any cosmological metric used to describe the expansion of the universe, including both the FLRW and CC metrics [28].

4.4. Cosmological Redshift

The energy E_0 of a photon at present can be expressed in terms of its momentum:

$$E_0 = p_0 c \,, \tag{23}$$

where $p_0 = \hat{h}\nu_0$ is the present-day photon momentum, $\hat{h} = h/c$ is the Planck constant normalized by c, and ν_0 is the present photon frequency. Similarly, at cosmic time t, the energy E of a photon is

$$E = pC \,, \tag{24}$$

where $p = \hat{h}\nu$ is the photon momentum, and C is the physical velocity of photons at time t. Since the energy of individual photons is conserved, $E = E_0$ (Equation (22)), and the velocity of photons increases with the scale factor as $C = a(t)c$ in the CC metric (Equation (8)), we obtain from Equations (23) and (24) the frequency of photons as

$$\nu = \nu_0 a(t)^{-1} \,. \tag{25}$$

which is the well-known expression for the cosmological redshift: the photon frequency decreases with cosmic expansion as $1/a(t)$. This decrease is a direct consequence of the conservation of photon energy and the increasing velocity of photons during cosmic expansion.

Thus, in the CC metric, photons increase their velocity with the scale factor while maintaining constant energy. The increase in photon velocity exactly compensates for the decrease in frequency:

$$E = \hat{h}\nu C = \hat{h}\nu_0 c = E_0 \,. \tag{26}$$

In contrast, within the FLRW metric, where the photon velocity remains constant during expansion, $C = c$, no redshift is predicted by Equation (26) (see Appendix A.2). Therefore, the observed decrease in photon frequency due to redshift implies a violation of energy conservation in the FLRW metric. This represents a significant issue for the ΛCDM model, as it lacks a physical mechanism to explain the energy loss of redshifted photons freely propagating through the expanding universe.

5. Discussion

5.1. Tensions in the ΛCDM Model

The FLRW metric serves as the foundational theoretical framework of general relativity (GR) embedded in the standard ΛCDM model, which aims to describe the universe and its evolution. However, the ΛCDM model is known to suffer from several tensions and inconsistencies [29–32] that may possibly originate in the FLRW metric's inability to fully capture the relativistic aspects of an expanding universe. These challenges include the following:

- The need to introduce dark matter to explain flat galaxy rotation curves,
- The requirement for dark energy to account for the dimming of Type Ia supernovae luminosity,
- The failure to describe the expansion of galaxies and other local gravitationally bound systems.
- The unresolved question of what happens to the energy of photons lost due to cosmological redshift during their propagation through expanding space.

These issues underscore the necessity of revisiting and potentially revising the relativistic models that describe cosmic expansion.

5.2. The Conformal Cosmology Metric

An alternative framework is offered by the conformal cosmology (CC) metric, which accounts not only for the expansion of the universe but also for a varying rate of cosmic time throughout the cosmic evolution. The CC metric has fundamentally different physical properties compared to the FLRW metric. One key implication is its prediction of a time-varying physical velocity of photons during cosmic expansion. This effect also appears in several recently developed cosmological models [33–37] and is analogous to the behaviour of photons in the Schwarzschild metric, where the physical velocity of photons decreases near a black hole due to spacetime curvature [38].

In both the CC and Schwarzschild metrics, the varying velocity of photons arises from a changing lapse function g_{tt}, which reflects time distortion due to a gravitational field. Equivalently, in the Newtonian framework, it corresponds to a varying gravitational potential. In the case of a local static gravitational field produced by massive bodies, the gravitational potential depends on spatial position, specifically, on the distance of a photon from the central mass. However, for the gravitational field of the universe, the gravitational potential evolves over time as the universe expands. This time-dependence leads to an increase in the velocity of photons in the CC metric.

5.3. Cosmological Redshift and Cosmic Time Dilation

Cosmic expansion affects both the velocity and frequency of photons, resulting in the cosmological redshift. A rigorous general relativistic analysis shows that spatial expansion alone cannot alter photon frequency, contrary to the common assumption presented in textbooks [4,39–41]. The notion that spatial expansion by itself predicts cosmological redshift was originally proposed by Lemaître [1], and the error in his derivation is discussed in Appendix A.1. Therefore, the FLRW metric does not predict redshift, and thus fails to fully account for this key cosmological observation [7]. In contrast, the CC metric naturally predicts the observed redshift.

Likewise, the CC metric offers a consistent explanation for cosmic time dilation [8], as observed in the time stretching of Type Ia supernova light curves [9,11,12,14,42]. This dilation is a direct consequence of the time-dependent lapse function in the CC framework.

5.4. Energy Conservation of Photons

A major advantage of the CC metric is that it preserves the conservation of photon energy despite cosmological redshift. As the photon frequency decreases with cosmic expansion, the photon velocity increases proportionally, resulting in constant photon energy throughout propagation of photons in expanding space. By contrast, in the FLRW metric, where the photon velocity remains fixed, photon energy appears to decline due to redshift. Notably, there is no physical mechanism to explain this loss. Since no interaction between freely moving photons and a gravitational field is admissible in Einstein's field equations, this raises a critical issue for the ΛCDM model. Note that applying energy conservation within the CC metric to the cosmic microwave background (CMB) leads to the conclusion that the total energy of the CMB remains constant over cosmic time, in contrast with the assumptions made in standard cosmology.

5.5. Observational Support for the CC Metric

Significant differences emerge when applying the CC and FLRW metrics to the dynamics of galaxies. The FLRW metric fails to account for the expansion of galaxies [6,43,44], whereas the CC metric describes galaxies as non-stationary objects expanding according to the Hubble flow. As shown by Vavryčuk [15], the CC model predicts that galaxy sizes increase with redshift—without requiring changes in the galaxy mass. This prediction has been observationally confirmed by numerous studies [45–49].

Moreover, the CC metric explains flat galaxy rotation curves without invoking dark matter. In this model, the physical velocity of massive particles remains constant, regardless of cosmic expansion. This also applies to the rotational velocity of stars in galaxies. As galaxies expand, stars move outward from the galaxy centre to its outer regions while maintaining their rotational velocity. This naturally produces flat rotation curves, as modelled by Vavryčuk [15]. This result challenges the necessity of dark matter to explain this phenomenon.

Finally, the CC metric also resolves the supernova dimming discrepancy, i.e., the mismatch between the observed luminosities of Type Ia supernovae [50,51] and the predictions of the standard cosmological model in the absence of dark energy. This discrepancy has been addressed either by modifying the time-redshift relation [52,53] or by introducing dark energy within the ΛCDM model. In contrast, the model based on the CC metric yields modified Friedmann equations that naturally fit the supernova data without requiring any change to the time–redshift relation or the inclusion of dark energy [7].

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Appendix A

Appendix A.1. Cosmological Redshift Inconsistency in the FLRW Metric

The common derivation presented in textbooks was originally proposed by Lemaître [1] in the following form. Light travels along the null geodesic, $ds^2 = 0$; hence, from Equation (1) we get

$$c^2 dt^2 = a^2(t) dl^2 \,, \tag{A1}$$

and consequently,

$$\frac{cdt}{a(t)} = dl \,. \tag{A2}$$

Suppose the distant galaxy emits photons at a constant rate Δt_e and with wavelength λ_e. The photons are observed at a rate Δt_r and with wavelength λ_r. The first photon is emitted at time t_e and received at time t_r. Taking into account that the comoving distance between the galaxy and the observer is the same for two successive photons,

$$\int_{t_e}^{t_r} \frac{cdt}{a(t)} = \int_{t_e+\Delta t_e}^{t_r+\Delta t_r} \frac{cdt}{a(t)} \,, \tag{A3}$$

and subtracting the integral

$$\int_{t_e+\Delta t_e}^{t_r} \frac{cdt}{a(t)} \,, \tag{A4}$$

we obtain

$$\int_{t_e}^{t_e+\Delta t_e} \frac{cdt}{a(t)} = \int_{t_r}^{t_r+\Delta t_r} \frac{cdt}{a(t)} \,. \tag{A5}$$

Since the scale factor $a(t)$ varies slowly during the emission and reception of the two successive photons, we can write

$$\frac{1}{a(t_e)} \int_{t_e}^{t_e+\Delta t_e} cdt = \frac{1}{a(t_r)} \int_{t_r}^{t_r+\Delta t_r} cdt \,. \tag{A6}$$

Hence,

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_r}{a(t_r)} \,, \tag{A7}$$

where $\lambda_e = c\Delta t_e$ and $\lambda_r = c\Delta t_r$ are the wavelengths of two successive photons at the emitter and the receiver, respectively.

However, this derivation is incorrect, because using Equation (A7) we readily obtain

$$\frac{\Delta t_e}{a(t_e)} = \frac{\Delta t_r}{a(t_r)} \,, \tag{A8}$$

which indicates that the time rate depends on the scale factor $a(t)$. This is inconsistent with the FLRW metric in Equation (1), where the time rate is invariant under cosmic expansion.

The difficulty in Lemaître's derivation lies in the assumption that Equation (A3) expresses the comoving distance between the emitter and the observer. This is incorrect, because the element of the comoving distance is

$$dl = \frac{dL}{a(t)} \neq \frac{cdt}{a(t)}, \tag{A9}$$

where dL is the element of proper distance and cdt is the element of light travel distance. While these quantities are identical in a static universe ($dL = cdt$), they differ fundamentally in an expanding universe ($dL \neq cdt$).

Appendix A.2. Velocity and Energy of Photons in the FLRW Metric

The propagation of light is governed by the null geodesic equation, $ds^2 = 0$. For the FLRW metric, this equation takes the form (see Equation (1))

$$c^2 dt^2 = a^2(t)\, dl^2\,. \tag{A10}$$

Consequently, the contravariant (comoving) speed of light $\hat{c}$ is given by

$$\hat{c} = \frac{dl}{dt} = \frac{c}{a(t)}, \tag{A11}$$

where t is the parameter along the null worldline.

Since the basis vectors of the FLRW metric are not orthonormal, the comoving speed of light and the physical speed of light differ. The comoving speed depends on the choice of coordinates because it is evaluated in a non-orthonormal basis. To determine the physical speed of light C, which is coordinate-invariant and experimentally measurable (e.g., using atomic clocks and rigid rods), we must use an orthonormal coordinate basis as described by [26,27,38]. This yields

$$C = a(t)\,\hat{c} = c\,. \tag{A12}$$

Thus, in the FLRW metric, the physical velocity of photons C remains constant,

$$C = c\,, \tag{A13}$$

being independent of cosmic expansion.

Now, considering the energy of individual photons E is conserved during cosmic expansion (see Equation (22)),

$$E = E_0\,, \tag{A14}$$

and applying the Planck–Einstein relation,

$$E = \hbar\nu\,,\;\; E_0 = \hbar\nu_0\,, \tag{A15}$$

we obtain

$$\nu = \nu_0\,, \tag{A16}$$

where $\hbar$ is the reduced Planck constant.

Equation (A16) implies that the FLRW metric does not predict any change in photon frequency with cosmic expansion. This is in clear contradiction with observed cosmological redshift. The inability of the FLRW metric to properly account for this effect was further analysed by Vavryčuk [7] using an independent approach.

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