∞-tilting theory

Leonid Positselski – University of Haifa

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Joint work with Jan Šťovíček:

L. Positselski, J. Šťovíček. The tilting-cotilting correspondence. arXiv:1710.02230

L. Positselski, J. Šťovíček. ∞-tilting theory. arXiv:1711.06169

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(The notation means: S acts in $M \in \mathrm{C}$ on the right, and we consider the category of left S-modules).

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$\mathrm{B} = \mathbb{Z}_p\text{-}\mathrm{Contra}$ is equivalent to the full subcategory in Ab consisting of all the abelian groups B such that $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}[p^{-1}], B) = 0 = \mathrm{Ext}^1_{\mathbb{Z}}(\mathbb{Z}[p^{-1}], B)$.

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Set $W = \Psi(J) \in \mathrm{B}$. □

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An ∞-tilting pair (T, E) in A

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This means that, for any ∞-tilting pair (T, E), one has $\mathrm{E}_{\min}(T) \subset \mathrm{E} \subset \mathrm{E}_{\max}(T)$.

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$$\begin{tikzcd}
\mathrm{D^{co}}(\mathrm{A}) \arrow[d, two heads] & \mathrm{D^{ctr}}(\mathrm{B}) \arrow[d, two heads] \\
\mathrm{D}(\mathrm{E}) \arrow[r, equal] \arrow[d, two heads] & \mathrm{D}(\mathrm{F}) \arrow[d, two heads] \\
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The related derived equivalence is

$$\mathrm{D}^{\mathrm{co}}(\mathrm{A}) \simeq \mathrm{Hot}(\mathrm{A}_{\mathrm{inj}}) = \mathrm{Hot}(\mathrm{B}_{\mathrm{proj}}) \simeq \mathrm{D}^{\mathrm{ctr}}(\mathrm{B}).$$

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$$\begin{tikzcd}
\mathrm{D}(\mathrm{E}_{\min}) \arrow[r, equal] & \mathrm{D}^{\mathrm{co}}(\mathrm{A}) \arrow[r, equal] \arrow[d, two heads] & \mathrm{D}^{\mathrm{ctr}}(\mathrm{B}) \arrow[r, equal] \arrow[d, two heads] & \mathrm{D}(\mathrm{F}_{\min}) \\
\mathrm{D}(\mathrm{E}_{\max}) \arrow[r, equal] & \mathrm{D}(\mathrm{A}) \arrow[r, equal] & \mathrm{D}(\mathrm{B}) \arrow[r, equal] & \mathrm{D}(\mathrm{F}_{\max})
\end{tikzcd}$$

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