The Very Flat Conjecture

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In this case we will say that G is a transfinitely iterated extension of the left R-modules G_{i+1}/G_i, $0 \leqslant i < \alpha$ (in the sense of the inductive limit). Sometimes one simply says that the R-module G is filtered by the R-modules G_{i+1}/G_i.

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The applications of the Very Flat Conjecture

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In order to prove the Finitely Very Flat Main Theorem for R, S, and F, one applies Noetherian induction to the ring $\overline{R}$ and the Finitely Very Flat Main Lemma to quotient rings of R by ideals generated by finite sets of elements from $\overline{R}$.

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where U is a free R-module and V is a free $R[r^{-1}]$-module.

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The Flat Main Lemma and Toy Main Lemma are much easier to prove than the Noetherian Very Flat Main Lemma and the Finitely Very Flat Main Lemma.

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Let R be an associative ring and H be a left R-module. Suppose that there is an ordinal α and a projective system of R-modules $H_i \longleftarrow H_j$, $0 \leqslant i < j \leqslant \alpha$, such that $H_0 = 0$, $H_\alpha = H$, the triangle diagrams $H_i \longleftarrow H_j \longleftarrow H_k$ are commutative for all $i < j < k \leqslant \alpha$, $H_j = \varprojlim_{i<j} H_i$ for all limit ordinals $j \leqslant \alpha$, and the maps $H_i \longleftarrow H_{i+1}$ are surjective with the kernels L_i.

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The proofs of the Main Lemmas

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The proofs of the Main Lemmas are based on results describing the right classes of modules in certain cotorsion pairs as the classes of all modules right 1-obtainable from certain "seed" classes.

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Let us also mention that it follows from the Main Proposition that all R-modules are right 2-obtainable from contraadjusted R/rR-modules and contraadjusted $R[r^{-1}]$-modules.

Proof of Bounded Torsion Very Flat Main Lemma

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Proof of Bounded Torsion Very Flat Main Lemma

The Noetherianity assumption in the Main Proposition can be weakened to the assumption on the r-torsion in R: the r-torsion in R should be a sum of bounded r-torsion and r-divisible r-torsion. This allows to deduce the Bounded Torsion Very Flat Main Lemma.

Proof of Finitely Very Flat Main Lemma

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So the ring R_J is obtained by inverting all the elements r_j, $j \in J$, Let R be a commutative ring and $\mathbf{r} = \{r_1, \ldots, r_m\}$ be a finite set of elements in R. Recall that an R-module is said to be **r**-contraadjusted if it is r_j-contraadjusted for all $j = 1, \ldots, m$. An R-module F is said to be **r**-very flat if $\mathrm{Ext}^1_R(F, C) = 0$ for all **r**-contraadjusted R-modules C.

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Put $K = \{1, \ldots, m\} \setminus J$. Denote by R_J the ring

$$R[r_J^{-1}]/(\textstyle\sum_{k \in K} r_k R[r_J^{-1}]).$$

So the ring R_J is obtained by inverting all the elements r_j, $j \in J$, and annihilating all the elements r_k, $k \in K$ in the ring R.

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The $\mathbf{r}^\times$-Very Flat Theorem is deduced from the following Main Proposition.

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$\mathbf{r}^\times$-Contraadjusted Main Proposition

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Let R be a commutative ring and $\mathbf{r} = \{r_1, \ldots, r_m\} \subset R$ be a finite subset of its elements. Let F be a flat R-module. Then the R-module F is $\mathbf{r}^\times$-very flat if and only if for every subset of indices $J \subset \{1, \ldots, m\}$ the R_J-module $R_J \otimes_R F$ is projective.

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Let R be a commutative ring and $\mathbf{r} = \{r_1, \ldots, r_m\} \subset R$ be a finite subset of its elements. Then an R-module is $\mathbf{r}^\times$-contraadjusted if and only if it is right 1-obtainable

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Let R be a commutative ring and $\mathbf{r} = \{r_1, \dots, r_m\} \subset R$ be a finite subset of its elements. Let F be a flat R-module. Then the R-module F is $\mathbf{r}^\times$-very flat if and only if for every subset of indices $J \subset \{1, \dots, m\}$ the R_J-module $R_J \otimes_R F$ is projective.

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Let R be a commutative ring and $\mathbf{r} = \{r_1, \dots, r_m\} \subset R$ be a finite subset of its elements. Then an R-module is $\mathbf{r}^\times$-contraadjusted if and only if it is right 1-obtainable from R_J-modules, where J runs over the all the subsets in the set of indices $\{1, \dots, m\}$.

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All r-contramodule R-modules (and more generally, all r-contraadjusted R-modules) are r-complete. All r-separated r-complete R-modules are r-contramodules. But the converse assertions do not hold.

Torsion Module and Contramodule Lemmas

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Let R be a commutative ring and $r \in R$ be an element. Then an R-module is an r-contramodule if and only if it is simply right obtainable from R/rR-modules.

Proof of Toy Main Proposition

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$$R \longrightarrow R[r^{-1}] \longrightarrow K^\bullet \longrightarrow R[1],$$

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we obtain an exact sequence of R-modules

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Conversely, all R-modules simply right obtainable (or even right 1-obtainable) from R/rR-modules and $R[r^{-1}]$-modules are r-contraadjusted, because all the R/rR-modules and $R[r^{-1}]$-modules are r-contraadjusted.

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where ${}_tA \subset A$ denotes the submodule of all elements annihilated by an element $t \in R$. The projective system $(A/r^nA)_{n \geqslant 1}$ is formed by the natural projection maps $A/r^{n+1}A \longrightarrow A/r^nA$, while the projective system $({}_{r^n}A)_{n \geqslant 1}$ is formed by the multiplication maps $r\colon {}_{r^{n+1}}A \longrightarrow {}_{r^n}A$.

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First of all, all R/r^nR-modules are finitely iterated extensions of R/rR-modules, so they are simply right obtainable. Set $D'_n = \operatorname{im}(\varprojlim_m D_m \to D_n) \subset D_n$.

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We have an exact sequence of R-modules

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In order to show that C is right 1-obtainable, it suffices to check that $\mathrm{Hom}_R(R[r^{-1}], C)$ and $\mathrm{Ext}^1_R(K^\bullet, C)$ are right 1-obtainable and that $\mathrm{Ext}^0_R(K^\bullet, C)$ is right 2-obtainable.

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The R-module $\mathrm{Hom}_R(R[r^{-1}], C)$ is already a contraadjusted $R[r^{-1}]$-module.

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The R-module $\mathrm{Hom}_R(R[r^{-1}], C)$ is already a contraadjusted $R[r^{-1}]$-module.

The R-module $\mathrm{Ext}^0_R(K^\bullet, C)$ is an r-contramodule. According to the Contramodule Lemma, it is simply right obtainable from R/rR-modules. Any R/rR-module can be embedded into a contraadjusted

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The R-module $\mathrm{Ext}^0_R(K^\bullet, C)$ is an r-contramodule. According to the Contramodule Lemma, it is simply right obtainable from R/rR-modules. Any R/rR-module can be embedded into a contraadjusted (e.g., injective) R/rR-module, and the quotient R/rR-module is also contraadjusted.

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The R-module $\mathrm{Hom}_R(R[r^{-1}], C)$ is already a contraadjusted $R[r^{-1}]$-module.

The R-module $\mathrm{Ext}_R^0(K^\bullet, C)$ is an r-contramodule. According to the Contramodule Lemma, it is simply right obtainable from R/rR-modules. Any R/rR-module can be embedded into a contraadjusted (e.g., injective) R/rR-module, and the quotient R/rR-module is also contraadjusted. Hence any R/rR-module is right 2-obtainable from contraadjusted R/rR-modules.

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The R-module $\mathrm{Ext}^1_R(K^\bullet, C)$ is a contraadjusted r-contramodule R-module.

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The R-module $\mathrm{Ext}^0_R(K^\bullet, C)$ is an r-contramodule. According to the Contramodule Lemma, it is simply right obtainable from R/rR-modules. Any R/rR-module can be embedded into a contraadjusted (e.g., injective) R/rR-module, and the quotient R/rR-module is also contraadjusted. Hence any R/rR-module is right 2-obtainable from contraadjusted R/rR-modules.

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Beyond Very Flat and Contraadjusted Modules

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