Randomness and Computation, Spring'22
Homework 1

Navid Talebanfard

Deadline: 24/2 - 04:33

Let $G = (V, E)$ be a simple undirected graph. For any two disjoint subsets $A, B \subseteq V$, let $E(A, B)$ be the set of all edges with one endpoint in A and one in B. We say that G is $(d, 2)$-*Ramsey* if for every family of d pairwise disjoint subsets $S_1, \ldots, S_d \subseteq V$ (not necessarily partitioning V) it holds that

$$\{|E(S_i, S_j)| \pmod 2 : 1 \leq i < j \leq d\} = \{0, 1\}.$$

1. Prove that for every $\epsilon > 0$ there exists n_0 such that for every $n \geq n_0$ there exists an n-vertex graph which is $(n^{\frac{1}{2}+\epsilon}, 2)$-Ramsey.
2. Give an explicit graph which is $(\frac{n}{2} + 1, 2)$-Ramsey.
3. (Bonus) Give an explicit graph which is $(\delta n, 2)$-Ramsey for some constant $\delta < 1/2$.