Randomness and Computation, Spring'22
Homework 2

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Deadline: 17/3 - 23:59

1. Prefix-free codes:

   (a) **Kraft's inequality.** Let F be a collection of binary strings of finite lengths, and assume that no member of F is a prefix of another one. For a binary string x, let $|x|$ be the length of x. Show that $\sum_{x \in F} 2^{-|x|} \leq 1$.

   (b) Use this to show that if $\frac{1}{|F|} \sum_{x \in F} |x| \leq t$, then $|F| \leq 2^t$.

2. Even cycles:

   (a) Let G be an n-vertex digraph with minimum out-degree at least $\log n + 1$. Show that G has contains an even length directed cycle.

   (b) Let G be a digraph with minimum out-degree δ and maximum in-degree Δ. Show that if $e(\Delta\delta + 1)2^{-\delta} < 1$ then G contains an even length directed cycle.