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GENERALIZED BOUNDARY VALUE PROBLEMS WITH ABSTRACT SIDE CONDITIONS AND THEIR ADJOINTS II

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0. PRELIMINARIES

Let \(-\infty < a < b < \infty\). Let \(A\) be an \(m \times m\)-matrix valued function essentially bounded on \([a, b]\). Let \(F\) be a locally convex topological vector space and let \(H\) be a linear continuous mapping of the Sobolev space \(W^{1,\infty}_m\) into \(F\).

For \(u \in W^{1,\infty}_m\), \(\ell u\) denotes the value of the differential expression

\[
\ell u := u' + A(t) u.
\]

This expression is defined a.e. on \([a, b]\) and \(\ell u \in L^\infty_m\) for any \(u \in W^{1,\infty}_m\). The symbol \(\ell\) will be also used for the "maximal" operator

\[
\ell : u \in W^{1,\infty}_m \to \ell u \in L^\infty_m.
\]

Under our assumptions the graph

\[
(0,1) \quad G = G(\ell) = \{(u, \ell u) \in L^\infty_m \times L^\infty_m : u \in W^{1,\infty}_m\}
\]

of \(\ell\) is certainly closed in \(L^\infty_m \times L^\infty_m\). Hence when endowed with the usual operations and with the norm of \(L^\infty_m \times L^\infty_m\)

\[
(u, \ell u) \in G \to \|u\|_\infty + \|\ell u\|_\infty,
\]

\(G\) becomes a Banach space.

We shall consider the linear differential operator \(L\) acting on \(L^\infty_m\) defined on

\[
D(L) = \{u \in L^\infty_m : u \in W^{1,\infty}_m \text{ and } Hu = 0\}
\]

by

\[
Lu := \ell u.
\]

We shall use the notation introduced in the first part [1] of the paper. Given locally convex topological vector spaces \(X, Y\) and a linear operator \(T\) with the definition domain \(D(T) \subset X\) and the range \(R(T) \subset Y, N(T)\) denotes its null space and \(G(T)\)
its graph. \(X^*\) is the dual space to \(X\) and \([\cdot, \cdot]_X\) denotes the linear continuous functional on \(X\) corresponding to \(u \in X^*\). For \(M \subset X\) and \(N \subset X^*\) the symbols \(M^\perp\) and \(N^\perp\) are defined by

\[
M^\perp = \{ u \in X^* : [x, u]_X = 0 \text{ for all } x \in M \}
\]

and

\[
N^\perp = \{ x \in X : [x, u]_X = 0 \text{ for all } u \in N \},
\]

respectively. Furthermore, \(\text{cl}^*(N)\) denotes the weak*-closure of \(N\) in \(X^*\) (with respect to the duality \([\cdot, \cdot]_X\)). If \(X\) is normed, then the norm on \(X\) is denoted by \(\| \cdot \|_X\) and \(\overline{M} = \text{cl}^*(\text{conv}(M))\) is the corresponding norm closure of \(M \subset X\). In such a case it is possible also to equip \(X^*\) with the norm \(\| u \|_{X^*} = \sup_{\| x \|_X \leq 1} |[x, u]|\). The corresponding norm closure of \(N \subset X^*\) is denoted by \(\overline{N}\).

Let \(S\) be a linear operator acting from \(Y^*\) into \(X^*\) (\(D(S) \subset Y^*\), \(R(S) \subset X^*)\). We say that the set \(G(*S)\) is the graph of the pre-adjoint relation \(*S\) to \(S\) if

\[
G(*S) = \{ (x, y) \in X \times Y : [x, Su]_Y = [y, u]_Y \text{ for all } u \in D(S) \},
\]

i.e. \(G(*S) = N(-S)\), where the orthogonal complement of the graph \(G(-S) = \{ (-Su, u) : u \in D(S) \subset Y^* \}\) of \(-S\) is considered with respect to the duality \([\cdot, \cdot]_{X \times Y}\) on \((X \times Y) \times (X^* \times Y^*)\).

\[
[(x, y), (u, v)]_{X \times Y} = [x, u]_X + [y, v]_Y.
\]

\(D(*S) = \{ x \in X : (x, y) \in G(*S) \text{ for some } y \in Y \}\) is the definition domain of \(*S\), \(R(*S) = \{ y \in Y : (x, y) \in G(*S) \text{ for some } x \in X \}\) its range, \(N(*S) = \{ x \in X : (x, 0) \in G(*S) \}\) its null space and

\[*S x = \{ y \in Y : (x, y) \in G(*S) \}\] for \(x \in D(*S)\).

\(*S\) is an operator if \(*S x = 0\) for \(x = 0\).

### 0.1. Lemma (cf. [2], Theorem 2.3).

Let \(X, Y\) be Banach spaces. If \(S : D(S) \subset \subset Y^* \rightarrow X^*\) is weakly*-closed in \(X^* \times Y^*\) and \(\overline{R(S)} = R(S)\), then \(R(S)\) is weakly*-closed in \(X^*\), \((*S)^* = S\) and

\[
R(S) = N(*S)^\perp, \quad \perp R(S) = N(*S),
\]

\[
R(*S) = \perp N(S), \quad \perp R(*S)^\perp = N(S).
\]

\(C^m\) denotes the space of complex row \(m\)-vectors, \(\| \cdot \|\) is the norm on \(C^m\), \(x^*\) denotes the conjugate transposition of \(x \in C^m\); \(L^p_m\) \((1 \leq p \leq \infty)\) is the space of functions \(x : [a, b] \rightarrow C^m\) for which

\[
\| x \|_p = \left( \int_a^b |x(t)|^p \, dt \right)^{1/p} < \infty \quad \text{if} \quad 1 \leq p < \infty.
\]
or
\[ \|x\|_{\infty} = \sup_{t \in [a,b]} |x(t)| < \infty \quad \text{if} \quad p = \infty ; \]

$W^{1,p}_m$ is the Sobolev space of functions $x : [a, b] \to C^m$ absolutely continuous on $[a, b]$ and such that their derivatives $x'$ belong to $L^p_m$, 

\[ \|x\|_{1,p} = |x(a)| + \|x'\|_p . \]

Let $(1/p) + (1/q) = 1$ if $1 < p < \infty$, $q = \infty$ if $p = 1$, then $L^\infty_m$ is the dual space to $L^p_m$ with respect to the duality

\[ [x, u]_L = \int_a^b u^* x \, dt \quad \text{for} \quad x \in L^1_m \quad \text{and} \quad u \in L^\infty_m \]

and $W^{1,q}_m$ is the dual space to $W^{1,p}_m$ with respect to the duality

\[ [x, v]_W = v^*(a) x(a) + [x', v']_L \quad \text{for} \quad x \in W^{1,p}_m \quad \text{and} \quad v \in W^{1,q}_m . \]

1. Normal Solvability of $L$

In the first part of the paper we proved that under our assumptions $L$ has a closed range in $L^\infty_m$, i.e. it is normally solvable in the usual sense. However, since we have no proper analytic representation of the dual space to $L^\infty_m$ we cannot obtain an analytic form of the adjoint $L^*$ to the operator $L$. This means that the relations (Fredholm Alternatives)

\[ R(L) = \frac{1}{2} N(L^*) , \quad R(L)^\perp = N(L^*) \]

which follow from the normal solvability give us no useful information. Nevertheless, we have a chance to obtain similar but more useful Fredholm type relations using the pre-adjoint $*L$ of $L$. Since $L^\infty_m$ is the dual space to $L^1_m$, the pre-adjoint $*L$ to $L$ is a linear relation in $L^1_m \times L^1_m$ with the graph

\[ (1,1) \quad G(*L) = \{(x, y) \in L^1_m \times L^1_m : [x, \ell u]_L = [y, u]_L \text{ for all } u \in D(L)\} , \]

definition domain 

\[ (1,2) \quad D(*L) = \{x \in L^1_m : (x, y) \in G(*L) \text{ for some } y \in L^1_m\} , \]
	null space

\[ (1,3) \quad N(*L) = \{x \in L^1_m : [x, \ell u]_L = 0 \text{ for all } u \in D(L)\} \]

and values

\[ (1,4) \quad *Lx = \{y \in L^1_m : (x, y) \in G(*L)\} \quad \text{for} \quad x \in D(*L) . \]

If we show that $L$ is weakly*-closed in $L^\infty_m \times L^\infty_m$ (with respect to the duality

\[ [(x, y), (u, v)] = [x, u]_L + [y, v]_L \quad \text{for} \quad x, y \in L^1_m \quad \text{and} \quad u, v \in L^\infty_m , \]

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then by Lemma 0.1 we obtain the formulas

\[(1.5) \quad R(L) = N(\ast L)^\perp, \quad R(L) = N(\ast L), \]

\[ R(\ast L) = N(L), \quad R(\ast L)^\perp = N(L). \]

After proving this we shall in the following section derive the analytic form of the pre-adjoint relation \( \ast L \) to \( L \). The following assumptions will be kept.

1.1. Assumptions. \( A \) is an \( m \times m \)-matrix valued function essentially bounded on \( [a, b], -\infty < a < b < \infty \); \( F \) is a locally convex topological vector space such that \( F = (\ast F)^* \) for some locally convex topological vector space \( *F; H \) is a linear continuous mapping of the space \( W^{1, \infty}_m \) into \( F \) such that \( H = (\ast H)^* \) for some linear continuous mapping \( *H \) of \( *F \) into \( W^{1,1}_m \).

1.2. Notation. We denote by \( J \) the linear operator (cf. \( (0, 1) \))

\[ J : (u, \ell u) \in G \subset L^\infty_m \times L^\infty_m \to u \in W^{1, \infty}_m. \]

Obviously,

\[(1.6) \quad J_{-1}(N(H)) := \{ (u, \ell u) \in G : Hu = 0 \} = G(L) \]

is the graph of \( L \).

1.3. Lemma. \( \text{cl}^* (N(H)) = N(H) \) (the weak*-closure in \( W^{1, \infty}_m \) with respect to the duality \( [, , , ]_w\).)

Proof. Let \( u \in \text{cl}^*(N(H)) \). Then for each finite set \( Z = \{ z_1, z_2, \ldots, z_k \} \subset W^{1,1}_m \) there exists a sequence \( \{ u_j^{(Z)} \})_j \subset N(H) \) such that

\[ [z, u_j^{(Z)}]_w \to [z, u]_w \quad \text{as} \quad j \to \infty \]

holds for any \( z \in Z \). Let us choose an arbitrary \( \varphi \in *F \). Then there exists a sequence \( \{ u_j^{(\varphi)} \})_j \subset N(H) \) such that

\[ [\ast H \varphi, u_j^{(\varphi)}]_w \to [\ast H \varphi, u]_w \quad \text{as} \quad j \to \infty. \]

This means that

\[ [\varphi, Hu]_w = [\varphi, H(u - u_j^{(\varphi)})]_w = [\ast H \varphi, u - u_j^{(\varphi)}]_w \to 0. \]

Since \( \varphi \in *F \) was arbitrary, this implies that \( Hu = 0 \), i.e. \( u \in N(H) \). This completes the proof.

1.4. Lemma. The mapping \( J \) defined in 1.2 is continuous with respect to the corresponding weak*-topologies.

Proof. Let \( \varepsilon > 0 \) be given and let \( Z \) be an arbitrary finite subset of \( W^{1,1}_m \). To prove the lemma we have to show that there exist \( \delta > 0 \) and a finite subset \( W \) of \( L^1_m \times L^1_m \)
such that for every \( u \in W_{m,\infty}^1 \) satisfying
\[
[[x, u]]_L + [[y, \ell u]]_L < \delta \quad \text{for all} \quad (x, y) \in W
\]
we have
\[
[[z, u]]_W < \varepsilon \quad \text{for all} \quad z \in Z.
\]
Recall that
\[
[z, u]_W = u^*(a) z(a) + \int_a^b u^* z' \, dt
\]
and
\[
[x, u]_L + [y, \ell u]_L = \int_a^b u^* x \, dt + \int_a^b (u' + Au)^* y \, dt = \]
\[
= \int_a^b u^*(x + A^* y) \, dt + \int_a^b u^* y \, dt = \]
\[
= u^*(a) \int_a^b (x + A^* y) \, dt + \int_a^b u^* \left[ \int_t^b (x + A^* y) \, d\tau + y \right] \, dt.
\]
Now we shall prove

**Auxiliary Assertion.** For any \( z \in W_{m,1}^1 \) there exist \( x, y \in L_m^1 \) such that
\[
\int_a^b (x + A^* y) \, dt = z(a) \quad \text{and} \quad y(t) + \int_t^b (x + A^* y) \, d\tau = z'(t) \quad \text{a.e. on} \quad [a, b].
\]

**Proof (of Auxiliary Assertion).** We have to show that for any \( d \in C_m^1 \) and \( w \in L_m^1 \) there exist \( x, y \in L_m^1 \) such that
\[
\int_a^b (x + A^* y) \, dt = d,
\]
\[
y(t) + \int_t^b (x + A^* y) \, d\tau = w(t) \quad \text{a.e. on} \quad [a, b].
\]
If \( x, y \) satisfy (1,9), then there certainly exists \( \xi \in W_{m,1}^1 \) such that \( \xi = w - y \) a.e. and
\[
\xi(t) = \int_t^b (x + A^* (w - \xi)) \, d\tau \quad \text{on} \quad [a, b], \quad d = \int_a^b (x + A^* (w - \xi)) \, dt.
\]
Notice that then \( \xi(a) = d \) and \( \xi(b) = 0 \).

On the other hand, if \( \xi \in W_{m,1}^1 \) and \( x \in L_m^1 \) fulfill (1,10), then the couple \( (x, y) \), \( y = w - \xi \), fulfils (1,9).
Differentiating (1.10) we further obtain that our assertion holds if for any $g \in L^*_m$ and $d \in C^m$ there exists $x \in L^1_m$ such that the two-point boundary value problem

\begin{equation}
-\xi' + A^*(t) \xi = g(t) + x(t) \quad \text{a.e. on} \quad [a, b],
\end{equation}

\[ \xi(a) = d \quad \text{and} \quad \xi(b) = 0 \]

has a solution $\xi \in W_{m}^{1,1}$.

Given $g \in L^*_m$ and $d \in C^m$, let us put

\[ \xi(t) = \frac{b - t}{b - a} d \quad \text{for} \quad t \in [a, b] \]

and

\[ x(t) = -\xi'(t) + A^*(t) \xi(t) - g(t) \quad \text{for a.e.} \quad t \in [a, b]. \]

Then evidently $\xi \in W_{m}^{1,1}$, $\xi(a) = d$, $\xi(b) = 0$ and $\xi$ is a solution to the system (1.11). This completes the proof of Auxiliary Assertion.

Proof of Lemma 1.4 (continuation). Let $Z$ be an arbitrary finite subset of $W_{m}^{1,1}$. Then by Auxiliary Assertion for any $z \in Z$ there exist $x_z, y_z \in L^1_m$ such that (1.8) holds when the symbols $x, y$ are replaced by $x_z$ and $y_z$, respectively. Let us denote

\[ W := \{(x_z, y_z) : z \in Z\}. \]

Let $u \in W_{m}^{1,\infty}$ be such that

\[ \left[ [x, u]_L + [y, \ell u]_L \right] < \varepsilon \quad \text{for all} \quad (x, y) \in W. \]

Then for any $z \in Z$ we have in virtue of (1.7)

\[ \left| [z, u]_W \right| = \left| [x_z, u]_L + [y_z, \ell u]_L \right| < \varepsilon. \]

This completes the proof of Lemma 1.4.

Now we can prove the following assertion.

1.5. Theorem. Under Assumptions 1.1 the graph $G(L)$ of $L$ is weakly*-closed in $L^\infty_m \times L^\infty_m$.

Proof. By (1.6), $G(L) = J_{-1}(N(H))$. Since $N(H)$ is weakly*-closed in $W_{m}^{1,\infty}$ by Lemma 1.3 and $J : G \subset L^\infty_m \times L^\infty_m \to W_{m}^{1,\infty}$ is continuous with respect to the corresponding weak*-topologies by Lemma 1.4, it follows immediately that $G(L)$ is weakly*-closed in $L^\infty_m \times L^\infty_m$.

Since $R(L)$ is closed in $L^\infty_m$ (cf. Theorem 4.3 of the first part [1] of this paper) and $L$ is weakly*-closed in $L^\infty_m \times L^\infty_m$, it follows from Lemma 0.1 that $R(L)$ is weakly*-closed in $L^\infty_m$.

1.6. Theorem. Under Assumptions 1.1, $R(L)$ is weakly*-closed in $L^\infty_m$, $(\ast L)^* = L$ and the relations (1.5) hold.
1.7. **Remark.** The results of this section also hold if we only assume the operator \( H : W^{1, \infty}_m \to F \) to be continuous and such that its pre-adjoint relation \( *H \) is densely defined in \( *F \), i.e. \( \overline{D(*H)} = *F \). (The last condition is fulfilled e.g. if \( H \) is weakly*-closed in \( W^{1, \infty}_m \times F \). In fact, in this case we have \( \overline{D(*H)} = \{0\} \), cf. [2], Theorem 2.3.) The proof of Lemma 1.3 should be modified as follows:

Let \( u \in \text{cl}\ast(N(H)) \). Then for each \( \varphi \in D(*H) \subset *F \) and each value \( z \in *H\varphi \subset W^{1, 1}_m \) there exists a sequence \( \{u_j^{(j)}\}_{j=1}^{\infty} \subset N(H) \) such that

\[
[z, u_j^{(j)}]_w \to [z, u]_w \quad \text{as} \quad j \to \infty.
\]

Consequently

\[
[\varphi, Hu]_F = [\varphi, H(u - u_j^{(j)})]_F = [z, u - u_j^{(j)}]_w \to 0,
\]

i.e. \( [\varphi, Hu]_F = 0 \) for any \( \varphi \in D(*H) \). Since \( \overline{D(*H)} = *F \), this implies that \( Hu = 0 \) and \( u \in N(H) \).

2. **PRE-ADJOINT RELATION**

We want to find an analytic description of the pre-adjoint relation \( *L \) to \( L \).

Let us assume \( 1.1 \).

2.1. **Theorem.** The graph \( G(*L) \) of the pre-adjoint relation \( *L \) to \( L \) is the set of all couples \((y, v) \in L^1_m \times L^1_m\) for which there exists \( \psi \in L^1_m \) such that

\[
y + \psi \in W^{1, 1}_m \),
\]

\[
v = \ell^+(y, \psi) := -(y + \psi)' + A^*y,
\]

\[
[y + \psi]_m(a) = 0
\]

and

\[
u^*(a) \left[ y + \psi \right](a) + \int_a^b u^* \psi \, dt = 0 \quad \text{for all} \quad u \in D = D(L).
\]

**Proof.** a) Let \((y, v) \in L^1_m \times L^1_m\) belong to \( G(*L) \). Then

\[
0 = [y, \ell^u]_L - [v, u]_L = \int_a^b \left[ (u^* + Au)^* y - u^*v \right]_L \, dt =
\]

\[
u^*(a) \left( A^*y - v \right)_L + \int_a^b u^* \left[ y + \int_a^t (A^*y - v) \, dt \right]_L \, dt
\]

*) The functions \( y, \psi \) are supposed to be defined everywhere on \([a, b]\).
for all \( u \in D(L) \). Let \( \psi \in L^1_m \) be such that
\[
[y + \psi](t) + \int_t^b (A^*y - v) \, dt = 0 \quad \text{for any} \quad t \in [a, b].
\]
Then \( y + \psi \in W^{1,1}_m, \ [y + \psi](b) = 0, v = -(y + \psi)' + A^*y \) a.e. on \([a, b] \). Consequently, the couple \((u, v)\) fulfills (2.1)–(2.3). Furthermore, since
\[
\int_a^b (A^*y - v) \, dt = [y + \psi](a),
\]
it follows from (2.5) that it fulfills also (2.4).

b) Let \((y, v) \in L^1_m \times L^1_m \) and let \( \psi \in L^1_m \) be such that (2.1)–(2.4) hold. Then for any \( u \in D(L) \) we have
\[
\int_a^b u^*v \, dt = -\int_a^b u^*(y + \psi)' \, dt + \int_a^b u^*Ay \, dt =
\]
\[
= -u^*[y + \psi]_a^b + \int_a^b u^*[y + \psi] \, dt + \int_a^b u^*Ay \, dt =
\]
\[
= \int_a^b (u' + Au)^* \, y \, dt.
\]
Hence \((y, v) \in G(*L)\).

Let \( D'_0 \) again denote the set of all derivatives \( u' \in L^\infty_m \) of functions \( u \) from \( D_0 = \{u \in D : u(a) = u(b) = 0\} \). Analogously as we obtained in the first part of this paper ([1]) the analytic description 4.6 of the adjoint relation \( L^*_0 \) to the restriction \( L_0 \) of \( L \) on \( D_0 \) for the case \( 1 \leq p < \infty \) from Theorem 4.5, we also can obtain in our present situation from Theorem 2.1 an analytic description of the pre-adjoint \( *L_0 \) to \( L_0 \),
\[
L_0 : u \in D_0 \to \ell u \in L^\infty_m \quad (D(L_0) = D_0).
\]

2.2. Corollary. \( G(*L_0) \) is the set of all \((y, v) \in L^1_m \times L^1_m \) for which there exists \( \psi \in L^1 D_0 \) (the set of all \( \chi \in L^1_m \) such that \( [\chi, u']_L = 0 \) for all \( u \in D_0 \)) such that (2.1) and (2.2) hold.

The following assertion is analogous to Theorem 4.8 of the first part [1] of this paper.

2.3. Theorem. Let us assume 1.1. \( G(*L) \) is the set of all \((y, v) \in L^1_m \times L^1_m \) for which there exist \( \zeta \in W^{1,1}_m \) and its derivative \( \zeta' \in L^1_m \) such that
\[
y + \zeta' \in W^{1,1}_m, \quad (2.6)
\]
\[
v = \ell^+(y, \zeta') \quad \text{a.e. on} \quad [a, b], \quad (2.7)
\]
\[
[y + \zeta'](a) = \zeta(a), \quad [y + \zeta'](b) = 0 \quad (2.8)
\]

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and

\[(2,9) \quad \zeta \in \overline{R(H)} \quad (\text{the closure in } W^{1,1}_m). \]

**Proof.** a) Let \( y, v \in L^1_m, \zeta \in W^{1,1}_m \) and \( \zeta' \in L^1_m \) be such that \( (2,6) - (2,9) \) hold. Obviously \( y, v \) and \( \psi := \zeta' \) fulfil \( (2,1) - (2,3) \). Since \( H \) is weakly*-closed in \( W^{1,\infty}_m \times F \), \( \overline{R(H)} = \overset{\perp}{N(H)} = \overset{\perp}{D} \) (with respect to the pairing \([\cdot, \cdot]_w\)). Thus \( (2,9) \) implies that

\[ u^*(a) [y + \psi] (a) + \int_a^b u^* \psi \, dt = 0 \quad \text{for all} \quad u \in D, \]

i.e. \( (2,4) \) holds and \( (y, v) \in G(L) \) according to Theorem 2.1.

b) On the other hand, if \( (y, v) \in G(L) \), then by Theorem 2.1 there exists \( \psi \in L^1_m \) such that \( (2,1) - (2,4) \) hold. Let us put

\[(2,10) \quad \zeta(a) = [y + \psi](a), \quad \zeta(t) = \zeta(a) + \int_a^t \psi \, d\tau \quad \text{on} \quad [a, b]. \]

Then the relations \( (2,6) - (2,8) \) follow directly from \( (2,1) - (2,3) \). Furthermore, we have by \( (2,4) \) and \( (2,10) \)

\[ u^*(a) \zeta(a) + \int_a^b u^* \zeta' \, dt = 0 \quad \text{for all} \quad u \in D. \]

It means that \( \zeta \in \overset{\perp}{D} \subset W^{1,1}_m \) (with respect to the pairing \([\cdot, \cdot]_w\)). Since \( \overset{\perp}{D} = \overset{\perp}{N(H)} = \overline{R(H)} \), the relation \( (2,9) \) follows immediately.

**2.4. Remark.** Notice that from the assumptions in 1.1 concerning \( H \) we have exploited in this section only the weak*-closedness of \( H \) in \( W^{1,\infty}_m \times F \).

**References**


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